

NONLINEAR
PHYSICAL
SCIENCE

39

Dimitri Volchenkov

Grammar of Complexity

From Mathematics to a Sustainable World

复杂性内在逻辑
从数学到可持续世界



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From Mathematics to a Sustainable World

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从数学到可持续世界

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NONLINEAR PHYSICAL SCIENCE

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Preface

“Grammar does not tell us how language must be constructed in order to fulfil its purpose, in order to have such-and-such an effect on human beings. It only describes and in no way explains the use of signs,”¹ wrote Ludwig Wittgenstein in his *Philosophical Investigations*.

In this book, we only describe and no way explain the “use of signs” for determining the transmission of information from the micro- to the meso- and the macro-scales highlighted as central to the behavior of complex systems [Kolmogorov, 1958, Kolmogorov, 1959, James *et al.*, 2016].

In particular, we discuss

- how local and global properties of complex systems are related to each other (i.e., *geometry*);
- how uncertainty in the environmental conditions and immediate transitions are related to uncertainty of the infinitely long paths (or sequences of transformations) in complex systems (i.e., *predictability*);
- which part of information is lost in transitions and which part is stored and have repercussions in the future evolution of the complex system (i.e., *selectivity*).

In order to enhance readability of the text,



the important conclusions are displayed in large print to make reading easier.

The concepts of the book have been developed following the lectures delivered by us during the spring semester 2017 in the Artificial Intelligence Key Laboratory of Sichuan Province, School of Automation and Information

¹ Wittgenstein, Ludwig (1963). *Philosophische Untersuchungen (Philosophical Investigations)*, transl. G. E. M. Anscombe, Oxford, Basil Blackwell.

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Chapter 1

Perplexity of Complexity¹

Systems composed of many components interacting with each other are considered *complex* since their behavior may not be expressed as a direct sum of individual behaviors of their parts. More complex systems naturally involve more parts and potentially integrate more diverse interactions between their components compared to simpler ones. Envisioning complex systems is an observer dependent practice [Johnson, 2014], since a knowledge of more detailed characteristics can make a system appear more complex.

Phenomena which emerge from a collection of interacting objects are often termed *complex phenomena* [Johnson, 2009]. The study of complex phenomena is perplexing because of emergent properties of a complex system which arise at a higher level of organization, above the level of its individual components. In this chapter we informally introduce some of the basic concepts of the modern approach to complex systems and explain why they are important.

1.1 A Compositional Containment Hierarchy of Complex Systems and Processes

On the one hand, individual components of a complex system may be themselves complex, on the other hand the entire system embracing them may be just an elementary constituent of even more complex systems. An ordering of the parts that make up a complex system is called a *compositional containment hierarchy*. Each level of the compositional containment hierarchy is characterized by the certain emergent properties that may not be seen at the lower levels of hierarchy.

¹ In his highly influential paper “*From Complexity to Perplexity*”, J. Horgan [Horgan, 1995] addressed the question whether science can achieve a unified theory of complex systems.

Atoms, the most fundamental and stable units of matter, are bonded into molecules. Aggregates of very large molecules formed by polymerization may be composed into organelles, the simplest level of organization of living things.

Different organelles may further be organized into cells. Related cells although not identical, but working together to accomplish specific functions are collectively referred to as a tissue. Tissues varying in their compositions are then joined in a higher level structural unit serving as a common function, an organ.

Groups of organs working together to perform one or more functions form organ systems. Contiguous groups of organ systems, being capable of some degree of response to stimuli, reproduction, growth, development, and homeostasis inside a defined environment, form organisms.

Collections of organisms of a species, sharing a particular characteristic and having the capability of interbreeding, are populations. Groups of populations, sharing a common environment, form ecological communities that form ecosystems functioning as a unit with their environments. The global sum of all ecosystems, largely self-regulating, forms the biosphere.

Although we stop climbing this hierarchy there, the absence of evidence is not the evidence of absence of higher levels in that. It is also obvious that the hierarchy of organization of living things is by no means a unique logical path linking the events and things in this world at the micro-, meso- and macroscopic levels.



An ordering of the parts that make up a complex system is a compositional containment hierarchy, each level of which is characterized by the certain emergent properties that are not seen at the lower levels.

On the one hand, complex systems can be fragile, since strong coupling between levels in the hierarchy can lead to cascading failures once a critical breakdown occurs in some of the system's levels causing catastrophic consequences on the functioning of the entire system [Buldyrev *et al.*, 2010]. On the other hand, a multi-level complex system forming a complex network of many interacting components can be extremely resilient, as being able to adapt to the rapidly changing environments [Gao *et al.*, 2016].

1.2 Top-Down and Bottom-Up Processes Associated to Complex Systems and Processes

A hierarchical organization of complex systems suggests the existence of two opposite processes linking all levels in the compositional containment hierarchy by bi-directional causation [Lane, 2006].



The compositional containment hierarchy of complex systems is characterized by bi-directional causation associated to the processes of speciation and adaptation.

1.2.1 The Top-Down Process of Adaptation (Downward Causation)

Definition 1. The *downward causation* is a causal relationship from higher levels of a complex system to its lower-level parts [Campbell, 1974].

The corresponding top-down process enforces certain constraints from a higher level development on the behavior of lower level components of the system, making them move in ways that may be unpredictable, even the complete information about these components is given. Since the mechanisms operating at the higher levels of organization of a complex system generally fail to accomplish the tasks at the lower levels of organization directly, the top-down process of the downward causation usually takes effect indirectly, through the environment [Galaaen, 2006]. Therefore, a complex system is profoundly influenced by the environment, facing a succession of environmental challenges and actively *adapting* behavior to get along in the environment with greatest success.

Remark 1. Since Herbert Spencer had first used the phrase “*survival of the fittest*” [Spencer, 1864], we used to think that evolution is about the ability of species to adapt to the changing environment. The process of adaptation whereby a living organism is able to “leave the most copies of itself in successive generations” [Spencer, 1864] plays the central role in our contemporary understanding of the key mechanisms of natural selection favoring the species “better designed for an immediate, local environment” (as was clarified by Darwin himself in the fifth edition of his famous work *On the Origin of Species* published in 1869) [Freeman, 1977].

1.2.2 The Bottom-Up Process of Speciation (Upward Causation)

Definition 2. The opposite (to adaptation) bottom-up process related to the *upward causation* involves interactions between the lower level components that causes emergent properties arising at the level of the entire system, which are not seen at the level of its basic constituents.

Emergence is particularly common when things are assembled to form new wholes [Johnson, 2014]. Emergent phenomena can occur in hierarchi-

cal systems that are far from the thermodynamic equilibrium due to a self-organization process between their parts [Prigogine *et al.*, 1977]. The development path of a self-organizing complex system encounters bifurcation points, where the system is forced to take one of several possible ways.

The complex systems are subject to innovation, demonstrating *sensitive dependence on initial conditions* when a small change in the initial state of a system results in the system diverging considerably. The initially identical, but reproductively isolated biological populations can evolve to become distinct species in the process of *speciation* taking place over the course of evolution [Sulloway, 1982].

Remark 2. Speciation triggers the mechanism driving biodiversity and populating the levels of the pyramid shown in Fig. 1.1 with new traits, species, and ecosystems arising due to divergent natural selection among different habitats. Once identical populations become subjected to dissimilar selective pressures and/ or undergo genetic drift independently, the fixation of incompatible mutations might make them no longer capable of exchanging genes provided occasionally they come back into contact [Baker, 2005].

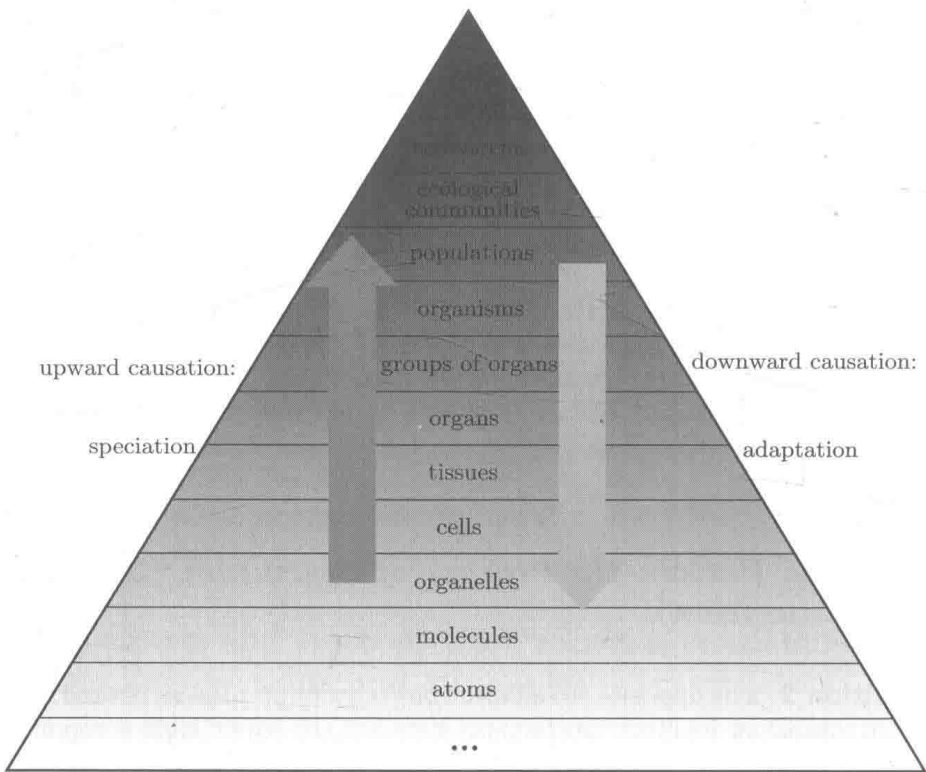


Fig. 1.1 A hierarchical organization of complex systems suggests the existence of two oppositely directed processes linking all levels in the hierarchy by the bi-directional causation