中国科学院院士文集

$$\mathrm{d}Q_i = \frac{\partial H}{\partial P_i}\mathrm{d}t$$

$$\mathrm{d}P_i = -\left(\frac{\partial H}{\partial Q_i} + \varepsilon m_{ij}(\boldsymbol{Q},\boldsymbol{P})\frac{\partial H}{\partial P_i} - u_i(\boldsymbol{Q},\boldsymbol{P})\right)\mathrm{d}t + \varepsilon^{1/2}\sigma_{ik}(\boldsymbol{Q},\boldsymbol{P})\mathrm{d}B_k(t)$$

$$i, j = 1,2,\cdots,n;\ k = 1,2,\cdots,m$$

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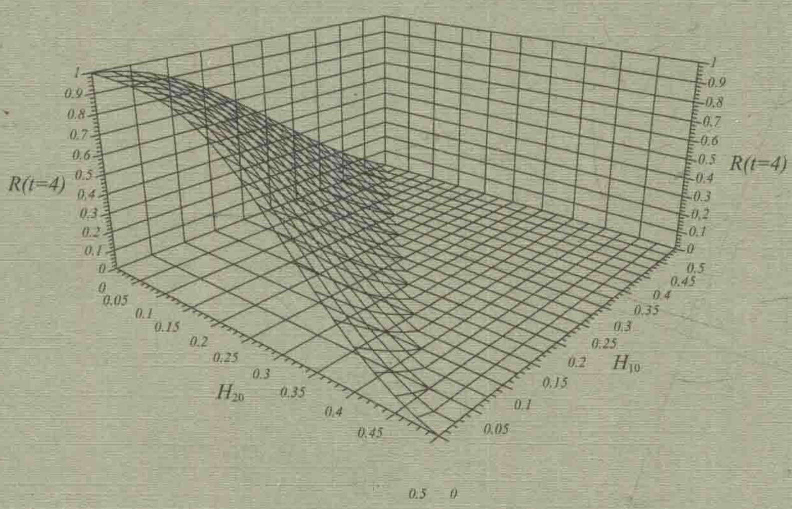

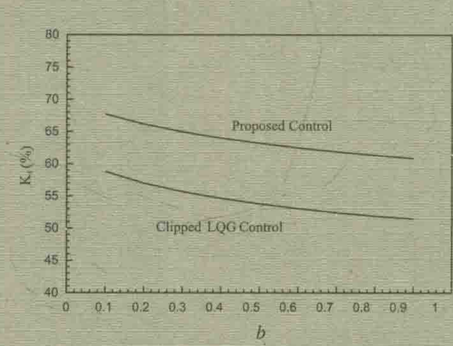

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《朱位秋论文选集》编委会　编

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编 委 会

序

朱位秋先生是随机动力学与控制领域的国际著名科学家，更是我敬重的学术前辈。因此，我非常荣幸能为他的论文集作序。

二十世纪八十年代，我在南京航空航天大学攻读博士学位期间，多次聆听导师张阿舟先生赞誉朱位秋先生的理论功底和学术成就。获得博士学位后留校任教不久，我担任张江监同学的博士学位论文答辩委员会秘书，有幸与前来参加答辩评审的朱先生相识，首次目睹了他的学术风采。此后三十年里，通过与朱先生的学术交往，尤其是多次长谈，我深深感受到他热爱祖国、探求真理、严谨求实、提携后学的高尚品格。通过阅读朱先生的学术论著，聆听他的学术报告，我不断从他的精辟见解和杰出成果中汲取学术营养。

本论文集精选了朱位秋先生的近四十篇代表性论文，所涉及的研究历程主要分为如下两个阶段。

二十世纪八十年代，朱先生及其合作者将标量随机场的局部平均理论推广于矢量随机场，提出了基于矢量随机场局部平均的随机有限元法；提出将随机载荷作用下结构疲劳破坏看成疲劳累积损伤的首次穿越问题和随机载荷作用下的疲劳裂纹尺寸的首次穿越问题，得到随机载荷作用下结构的可靠性和疲劳寿命的概率密度等。

二十世纪八十年代至今，朱先生及其合作者创造性地将多自由度非线性随机系统由拉格朗日体系转化为哈密顿体系，利用哈密顿系统的可积性和共振性，将系统分为不可积、完全可积非共振、完全可积共振、部分可积非共振、部分可积共振五类，相应地得到了五类非线性随机系统的精确平稳解（其中四类是在世界上首次得到的非能量等分解），并利用该精确平稳解和多种等效准则得到了五类非线性随机系统的等效非线性系统法；提出并发展了多种随机激励（如高斯白噪声、宽带噪声、有界噪声等）下具有滞迟、时滞和含分数阶导数阻尼等的多自由度强非线性系统的随机平均法，并用于系统的随机响应、随机稳定性、可靠性等研究；提出并发展了分别以响应最小、稳定性裕度最大、可靠度最大、平均寿命最长和给定平稳概率密度为目标的非线性随机最优控制设计方法，进一步发展了解决部分可观测与不确定，控制力具有时滞、有界和不能完全执行最优控制律等情况下的多自由度强非线性随机振动系统的最优控

制理论方法。这些研究构成了一个较为完整的非线性随机振动系统动力学与控制理论体系，为解决科学与工程中广泛存在且十分困难的强非线性随机振动系统的动力学与控制问题提供了一整套崭新而有效的理论方法。

上述部分研究成果体现在为美国《应用力学杂志》创刊五十周年和美国《应用力学评论》撰写的四篇特邀综述以及在科学出版社出版的两部学术专著中。部分研究成果曾获得2002年度国家自然科学奖二等奖。

本论文集较为系统地汇总了朱位秋先生长期从事随机动力学与控制研究过程中所形成的学术思想和取得的学术成果。我相信，这本论文集不仅将引导青年学者进一步推进该学科的发展，还将有利于工程界将这些理论方法应用于实际问题的解决，促进经济和社会发展。

中国科学院院士
北京理工大学校长 胡海岩

2017年4月

前 言

今年是朱位秋院士八十华诞。师恩如山，师德似海，朱院士的弟子们发起并组织编辑了这本论文集，从朱院士学术生涯三百余篇论文里选录了三十八篇代表作集结成册。其中大部分是我辈在恩师的悉心指导下合作完成的研究成果，内容涉及非线性随机系统的精确平稳解、等效非线性系统法、随机稳定性、随机分岔、可靠性、随机最优控制、随机有限元及随机疲劳等，涵盖了随机动力学与控制的主要研究方向。本论文集不仅是恩师的学术思想和学术成就的集中展现，也是师生深厚情谊的永恒见证。

人间正道是沧桑，一路走来，几多艰辛。1938年8月，恩师出生在浙江义乌的陇头朱村。本科前两年就读于西北工业大学航空发动机系，后经选拔进入工程力学班。1961年11月，考取季文美教授的硕士研究生，开始从事非线性振动理论研究。1975年10月，调浙江大学任教。1981年2月，赴美国威斯康星大学工程力学系访问，同年9月转入美国麻省理工学院机械系，合作导师是随机振动学科的奠基人S.H.Crandall院士。期间，朱老师在民主德国Frankfurt/Oder召开的IUTAM关于随机振动与可靠性讨论会上报告了关于能量包线随机平均法的论文，并在很长一段时间内是IUTAM（关于随机动力学系列讨论会）唯一受邀的国内学者；他协同Crandall院士为纪念*Journal of Applied Mechanics*创刊五十周年撰写了特邀评述论文，对随机振动做了全面而系统的评述。1984年，Crandall院士在写给时任浙江大学力学系主任谢贻权的信里这样评价朱老师："他是一位能进入一个新的领域并很快对该领域做出重大贡献的第一流的研究者。"

1992年秋，朱老师的首本专著《随机振动》出版，美国随机动力学权威专家、美国工程院院士Y.K.Lin作序称"现今欧美日各国尚无类似专著可以比拟"。到今天，这本专著已是国内该领域被引用次数最多的学术专著。

进入二十世纪九十年代中期，恩师开始了非线性随机动力学的哈密顿理论体系的系统研究。他通过将多自由度强非线性随机系统转化为随机激励的耗散的哈密顿系统，利用相应哈密顿系统的可积性与共振性，分成不可积、可积非共振、可积共振、部分可积非共振及部分可积共振五类；对每一类得到其精确平稳解，并发展了相应的等效非线性系统法及随机平均法，

进而研究其响应、稳定性、分岔、可靠性等。

2001年，随机激励的耗散的哈密顿系统理论获中国高校科学技术一等奖。2002年，获国家自然科学二等奖。2003年，集二十年研究之大成，专著《非线性随机动力学与控制——Hamilton理论体系框架》终于付梓。Y.K.Lin院士称其“实属学术上重要贡献”，“书中理论之发展，以统一的哈密顿框架为基础，乃朱位秋教授之首创，尤属独特可贵”。随机振动专家方同教授评论本书“反映了这一领域中当代的最新成就，可谓非线性随机动力学发展过程中的一个新的里程碑”。

2003年，朱老师主持召开了第五届国际随机结构动力学会议。同年11月，当选为中国科学院院士。

三十多年来，朱院士指导了五十余名研究生（已毕业博士生：刘雯彦、罗明、黄志龙、邓茂林、吴勇军、刘中华、宦荣华、王永、吴禹、冯长水、李雪平、陈林聪、曾岩、赵明、罗银淼、冯驹、胡昉、朱晨烜、谷旭东、刘伟彦、贾万涛、胡荣春；已毕业硕士生：余金寿、雷鹰、吴笛、吴伟强、何亚飞、马银江、王建泉、鲁民清、杨勇勤、徐伟国、梅春晖、何世民、郑小刚、董雷、程慧、茅樊、宋海山、尹路、潘国峰、仲勤芳、刘平、王庆、腾俊超、陈逸安；已出站博士后：甘春标、王文林、杨礼康；在读研究生：熊豪、潘杉杉、吕强锋、雷灵宏、王雪峰、胡凌俊、卢可；在站博士后：胡凯明），如今早已是桃李满天下，大部分在高校或研究机构从事相关研究，或在创业方面颇有成就，其中不乏国家杰出青年科学基金项目获得者、“青年千人计划”入选者等。

恩师是随机动力学与控制领域的国际著名学者，虽已年近耄耋，但他对工作仍未有丝毫懈怠，坐言立行，耳濡目染，让我们一众弟子领悟到了一个科学家开拓创新的科研精神、严谨踏实的学术操守以及担当谦和的处事态度。希望本文集能促进更多的青年学者投身随机振动领域，并为他们提供学术营养、精神鼓舞和思维启迪。

《朱位秋论文选集》编委会
2017年8月

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1 综　述

朱位秋院士在美国麻省理工学院(MIT)访问期间(1981—1983年),与随机振动学科的奠基人 S.H.Crandall 院士一起为美国机械工程师学会(ASME)的《应用力学杂志》(*Journal of Applied Mechanics*)创刊五十周年撰写了特邀论文,对随机振动学科前三十年的发展做了全面而系统的评述。之后,他先后三次应邀为美国机械工程师学会的《应用力学评论》(*Applied Mechanics Reviews*)撰写综述论文。其中,1988年和1996年的两篇文章综述了随机平均法及其在随机振动分析中的应用,2006年的一篇文章概述了他自己创立的非线性随机动力学与控制的哈密顿理论体系。本章收录了这四篇综述论文。通过阅读这些论文,您将会对随机振动、随机平均法、随机动力学与控制的哈密顿理论体系有较全面的了解。

Random vibration: a survey of recent developments

S. H. Crandall and W. Q. Zhu

Journal of Applied Mechanics, 1983, 50(4b): 953-962

Abstract A general overview of the problems, methods, and results achieved in random vibration since its inception as a technical discipline nearly 30 years ago is given with particular emphasis on developments during the past 15 years. Research areas of current interest include development of improved probabilistic models for sources of random excitation, development of more effective random response prediction procedures for nonlinear systems and systems with parametric excitation, development of improved procedures for estimating reliability of systems undergoing random vibration, and development of improved techniques for identification of system parameters from measurements made during random vibration.

1 Introduction

Random vibration is the name given to the body of theory associated with dynamic systems responding to random excitations. The ultimate purpose of the theory is to provide a sound basis for improving the reliability of structures, vehicles, and equipment that must withstand randomly fluctuating loads. Within the general framework of random vibration theory, three principal problems can be identified depending on whether attention is focused on the response, the excitation, or the dynamic system. Most commonly the system and its excitation are taken to be known and the problem is to predict statistical information about the dynamic response or the reliability of the system. The inverse problem of characterizing the excitation where the system and its response are known is called the measurement problem. Associated with this are the statistical operations performed on the measurements (referred to as data processing) and the problem of constructing useful probabilistic models to represent the random processes and random fields under investigation. Finally there is the diagnostic or identification problem in which information about the dynamic system is sought based on knowledge of the random excitation and its response.

In retrospect, the first investigation of dynamic response to random excitation was Einstein's pioneering study of Brownian motion in 1905 ([72] of [16]). The words "random vibrations" were first used in the title of a technical paper by Lord Raleigh in 1919 ([3] of [18]) to describe an acoustical problem equivalent to the random walk in a plane. The present usage of the term random vibration arose in the middle 1950s in connection with three aerospace problems: buffeting of aircraft by atmospheric turbulence, acoustic fatigue of aircraft panels due to jet noise, and the reliability of payloads in rocket-propelled vehicles. The common factor in all three problems was the random nature of the excitation. The procedures developed to solve these problems [1] were largely based on existing theories of statistical mechanics, communication noise, and fluid turbulence (Wax, [184] of [16]).

Initially, most of the work was based on linear models. In the middle 1960s many investigators turned their attention to random vibration of nonlinear systems and to problems involving random parametric excitation. Random data processing was primarily performed by analog instruments until about 1970 when efficient digital data-processing instruments, triggered by the fast Fourier transform (FFT) breakthrough, became available. Random vibration theory was first applied to vehicles (aerospace vehicles, ships, trucks, trains, etc.) and then to nominally stationary structures such as tall buildings subject to random wind and earthquake loadings, off-shore structures subject to random wave loadings, and heat exchanger tubes subject to turbulent external flows.

During the development of random vibration theory, many books and.survey papers have been written. Introductions to random vibration theory are provided by Crandall and Mark [2], by Robson [3], by Bolotin [4], and by Newland [5]. The books by Lin [6] and by Bolotin [7] give deeper and more systematic descriptions of random vibration theory with emphasis on structural engineering. Further discussions of particular topics in random vibration are contained in the collections edited by Crandall [1] and in the proceedings of two IUTAM Symposiums [13, 14]. Random data processing with engineering applications is described in some depth in the books by Bendat and Piersol [8, 9]. Theoretical treatments of stochastic differential equations and stochastic stability (closely related to random vibration theory) are provided by Jazwinski [10], by Arnold [11], and by Khasminskii [12]. The results of the first decade of development of random vibration theory are described in survey papers by Crandall [15, 16]. Further developments in random vibration theory since 1966 are surveyed by Vanmarcke [19]. Special topics reviewed include random vibration of nonlinear systems [17, 18, 20-22], random parametric vibration [23, 24], random vibration of one and two-dimensional structures [25], and modern spectrum analysis [26].

The present paper gives a general description of the important concepts, problems, and methods of random vibration, and a survey of the most significant results in the field, especially those achieved in the last 15 years. Efforts also have been made to try to indicate directions for future work.

2 Random Processes and Random Fields

As a technical discipline random vibration is a combination of structural dynamics and probability theory. The key concepts from probability theory are those of random processes and random fields which are used as models for excitation and response time histories. A random process is an infinite population or ensemble whose samples are functions of a single variable (usually time) together with information concerning relative probabilities of sample values. A random field [27] is a similar ensemble whose samples are functions of more than one variable (usually time and space). In principle, random processes and random fields can be described completely in terms of infinite sequences of joint probability distributions of successively higher order, or in terms of infinite sequences of moment functions or cumulant functions. In practice, it is customary to deal with very incomplete descriptions; e.g., first and second-order probability distributions only, or with a limited number of statistical averages based on these two distributions.

For many applications it is acceptable to assume a kind of uniformity for the random process or random field under consideration that is expressed by the designations *stationary* in time and *homogeneous* in space. A stationary process is one whose probability distributions across the ensemble are invariant with respect to translations of the origin of time. Similarly a random field is homogeneous with respect to a particular spatial coordinate if its probability distributions are invariant with respect to translations of the origin along the axis of that coordinate.

The common statistical averages are the mean and mean square, which depend on the first-order probability distribution, and the correlation function, which depends on the second-order probability distribution. If $f(\mathbf{x}, t)$ is a random field, the space-time correlation function is the ensemble average of the product $f(\mathbf{x}_1, t_1)\, f(\mathbf{x}_2, t_2)$. In general this is a function of the two locations and the two times. If the field is stationary then the correlation no longer depends on the two times separately but only on their difference $\tau = t_2 - t_1$. The Fourier transform of the space-time correlation function of a stationary field with respect to the time lag τ is called the space-time cross-spectral density function which is a function of the two locations and a frequency variable. When only a single time history is involved the correlation function for a random process $x(t)$ is the ensemble average of the product $x(t_1)x(t_2)$. This (auto)correlation function R_x depends only on the the time lag $\tau = t_2 - t_1$ if the process is stationary. The Fourier transform of the correlation with respect to τ is called the (auto)spectral density function W_x which is a function of a single frequency variable f. The mean square $< x^2 >$ of the stationary random process is given by the value of $R_x(\tau)$ when τ is zero or by the total area under the spectrum

$$< x^2 > = \int_0^\infty W_x(f)df \qquad (1)$$

The importance of the correlation and spectral density functions is because they provide average amplitude and frequency information about the sample histories and that (*i*) they can be measured with available data-processing techniques; (*ii*) they are closed with respect to linear time-invariant operations in the sense that if these statistics are known for the excitation, then it is possible to obtain the corresponding statistics for the response of a linear time-invariant dynamic system; and (*iii*) they often provide adequate information about the response for making engineering decisions concerning the severity of the vibration and the reliability of the system.

Nonstationary random processes and fields are more awkward to deal with at every step: measurement, response prediction, and drawing conclusions about reliability. A nonstationary process can sometimes be modeled as a stationary random process modulated by a deterministic amplitude variation [7] or as a nonstationary shot noise [6]. Since the correlation function of a nonstationary process involves two independent time arguments the ordinary spectral density function for stationary processes loses its significance. Some of the manipulative advantages of a spectrum can be retained by introducing a generalized spectral density defined by a double Fourier transform [6, 8]. An evolutionary spectrum was proposed by Priestley ([28-30] of [19]) to describe relatively slow changes of the frequency content of nonstationary processes. Two time-dependent spectra, the physical spectrum and the instantaneous spectrum, were introduced by Mark ([31] of [19]).

Another classification of random processes and fields can be made according to the influence of the past on the present probability distributions. There are processes with no memory at all; i.e., their present distributions are totally independent of the past, and there are processes whose present distributions are completely determined by the distributions at any single time in the past. An important example of the former class is white noise which is a stationary random process having uniform spectral density for all frequencies. Its correlation function is proportional to the Dirac delta function; i.e., it has no correlation with the past, no matter how recent. Such a process is an idealized concept that cannot be physically realized (the mean square of a white noise process is infinite) but it is a useful model for stationary processes with short correlation times in the same sense that ideal point loads are useful in the theory of elasticity.

A process whose present probability distribution depends on that at only one previous time is called a Markov process. The structure of a Markov process is completely determined for all future times by the distribution at some initial time and by a transition probability density function. The importance of Markov processes resides in the fact that when the excitation of a dynamic system (linear or nonlinear) is ideal white noise, the response is a Markov process, and a formal technique exists to obtain a partial differential equation satisfied by the transition probability density function of the process.

An important special class of random process (and fields) are the normal or Gaussian processes. These are processes whose probability distributions of all orders are completely determined by knowledge of the mean value for all times t and knowledge of the correlation function for all pairs of times t_1 and t_2. The importance of normal processes is because (i) they are closed with respect to linear time-invariant operations in the sense that if the excitation of a linear time-invariant system is a normal process, then so is the response process, and (ii) many real phenomena can be satisfactorily modeled by normal processes. A rationalization for the latter statement can be based on the central limit theorem which states that when a process is the sum of a very large number of small independent random processes it approaches a normal process as the number of independent constituents increases without limit.

3 Sources of Random Excitation

Random loading on a dynamic system can be modeled by a random process when it is applied at a single location or by a random field when it is applied as a distributed loading over a given length or given area of the system. In many cases it is assumed that the excitation is stationary and normal. This simplifies the problem of measuring the appropriate parameters to characterize the excitation and the problem of predicting the response and judging the reliability. Similar measurements made over a range of conditions together with physically based scaling laws sometimes provide a means for extrapolating the excitation parameters to estimate the levels of excitation that will occur under conditions beyond the range of previous experience. This procedure, however, has its limitations. One of the more important areas of future development of random vibration theory is the construction of improved models for random excitations.

In many cases the excitation is clearly nonstationary over a long period of time although for short intervals, which are still long compared to the response times of the dynamic system, the excitation appears to be stationary. A random process (or field) of this sort is called quasi-stationary [7] and may be described by a short-term (local) behavior and a long-term (global) behavior. For example, the short-term behavior of a quasi-stationary random process might be described by a spectral density function whose parameters were slowly varying functions of time. The long-term behavior might then be described by a joint probability distribution for the slowly varying parameters in the spectral density function. Several proposals for quantifying quasi-stationary or quasi-homogeneous random processes and random fields have been put forth. This is indicated in the brief descriptions of random excitations which follow. Because of the difficulty of making extensive measurements of random fields, simplifying assumptions are often built in to the models employed. For example, in cases of moving random loads due to turbulent flow about a vehicle it may be assumed, as in Taylor's hypothesis, that a "frozen" turbulence pattern is convected past the vehicle. In random fields with two space dimensions the fluctuations in one direction may be neglected, or it may be assumed that the randomness is isotropic so that measurements along any one direction can be used to infer the complete two-dimensional behavior.

3.1 Atmospheric Turbulence

Gust loading due to atmospheric turbulence can be a major design load for large commercial airplanes. It is commonly assumed that the vertical turbulent velocity component can be represented as a quasi-stationary normal process [29]. The short-term behavior is described by a spectral density function in which the RMS turbulent velocity is a slowly varying parameter whose long-term behavior is described by a probability density function. The assumption that the turbulent velocity pattern has no spanwise variation and is frozen as it is convected past the airplane, permits calculating the gust loads and the airplane response by using appropriate