



普通高等教育“十一五”国家级规划教材

数量  
经济  
学  
系列  
丛书

# 协整理论与波动模型

金融时间序列分析及应用(第二版)

张世英 樊智 著

Q  
U  
A  
N  
T  
I  
T  
A  
T  
I  
V  
E  
E  
C  
O  
N  
O  
M  
I  
C  
S

清华大学出版社



数量经济学系列丛书

# 协整理论与波动模型 金融时间序列分析及应用 (第二版)

张世英 樊智 著

清华大学出版社  
北京

## 内 容 简 介

本书论述了时间序列的协整理论和金融时间序列波动性模型的原理、方法和实际应用。在时间序列的协整理论方面,包括单位根过程的极限分布和检验,单方程和系统方程协整关系的估计和检验,非线性、长记忆协整关系的建模和检验问题,协整系统的贝叶斯分析及变结构协整的理论、方法等。在金融时间序列波动模型方面,包括自回归条件异方差(ARCH)模型各类一维和多维模型体系及各类随机波动(SV)模型的性质、模型参数估计和检验问题,讨论了变结构波动模型的建模及其应用等。金融波动性问题是当今金融分析中的重要课题,本书探讨了金融波动及其持续性的市场机制,建立了在金融波动持续性基础上的资本资产定价模型和金融风险规避策略等。书中详细讨论了高频金融时间序列分析与建模问题,研究了各类高频时间序列已实现波动率的计算方法和统计性质,讨论了超高频数据持续期的ACD类和SCD类两类模型。书中还讨论了小波方法在金融时间序列波动分析和建模方面的应用;讨论了各类连续时间资产收益模型及参数估计的MCMC方法。

本书可作为数量经济学研究人员、有关教师、经济和金融工作者的参考书,亦可作为相关领域研究生的教学参考书。

本书封面贴有清华大学出版社防伪标签,无标签者不得销售。

版权所有,侵权必究。侵权举报电话:010-62782989 13701121933

### 图书在版编目(CIP)数据

协整理论与波动模型:金融时间序列分析及应用/张世英,樊智著. —2版. —北京:清华大学出版社,2009.5  
(数量经济学系列丛书)

ISBN 978-7-302-19697-6

I. 协… II. ①张… ②樊… III. 金融—时间序列分析 IV. F830

中国版本图书馆CIP数据核字(2009)第036510号

责任编辑:龙海峰

责任校对:王凤芝

责任印制:李红英

出版发行:清华大学出版社

地 址:北京清华大学学研大厦A座

<http://www.tup.com.cn>

邮 编:100084

社 总 机:010-62770175

邮 购:010-62786544

投稿与读者服务:010-62776969, [c-service@tup.tsinghua.edu.cn](mailto:c-service@tup.tsinghua.edu.cn)

质 量 反 馈:010-62772015, [zhiliang@tup.tsinghua.edu.cn](mailto:zhiliang@tup.tsinghua.edu.cn)

印 装 者:三河市春园印刷有限公司

经 销:全国新华书店

开 本:185×260 印 张:30.25 插 页:1 字 数:720千字

版 次:2009年5月第2版 印 次:2009年5月第1次印刷

印 数:1~3000

定 价:49.00元

本书如存在文字不清、漏印、缺页、倒页、脱页等印装质量问题,请与清华大学出版社出版部联系  
调换。联系电话:(010)62770177 转 3103 产品编号:032895-01

# 前

第二版

PREFACE

# 言

《协整理论与波动模型——金融时间序列分析及应用》第一版出版后,金融时间序列研究和应用在国内外都取得了多方面的进展,我们也获得许多新的研究成果。在另一项国家自然科学基金项目——“多变量矩序列长期均衡关系及动态金融风险规避策略研究”(No. 70471050)的资助下,我们在金融高频时间序列波动性分析、金融波动分析的小波和频域方法、时间序列矩的持续和持同持续性研究、高阶矩风险建模方法与动态组合投资研究、Copula 技术及其在金融时间序列分析上的应用,等诸多方面都进行了创新性与探索性的研究,并获得大量研究成果。在此基础上,对第一版作了补充和修订。

在第二版中,本书基本结构保持不变,增加了三章新的内容:第 9 章高频金融时间序列分析与建模;第 10 章金融时间序列分析的小波方法;第 11 章连续时间模型及其应用。并对第一版中第 5 章,第 6 章和第 7 章作了必要的补充和修订。第一版第 9 章所展望的几个新方向我们都获得丰富的研究成果,改写为第二版的第 9 章、第 10 章内容,所以第一版第 9 章就取消了。在本书的修订过程中,仍以我们的研究成果为中心展开讨论。为了进一步说明我们的工作,同时也提到国外学者的一些相关工作。另外,Copula 方法的讨论已单独成书——《Copula 理论及其在金融分析上的应用》,由清华大学出版社出版。

本书的修订工作由张世英和樊智共同完成。在该书的修订中耿克红、李胜歌、胡素华、徐梅同志也给予了帮助。该书在出版过程中得到清华大学出版社的大力支持,在此一并表示感谢。

张世英

2008 年 7 月 25 日

本书是我们研究集体近十多年来在协整理论和时间序列波动性分析两个领域的研究成果。这项研究工作得到国家自然科学基金项目——“多变量时间序列波动持续性及其在金融系统上的应用”(No. 70171001)和教育部博士点基金项目——“社会经济系统中协整建模方法研究”(No. 9505621)的资助。结合这两项基金的研究工作,在协整理论、方法和金融时间序列波动性分析两个方面都获得了一系列创造性的成果。

在协整理论和方法方面,Engle 和 Granger 所建立的协整理论反映了非平稳时间序列之间的长期线性均衡关系,所以是线性协整。但在经济系统中,许多经济变量具有长期记忆的特点,而且这些序列本身及它们之间的关系往往是非线性的。为了揭示非线性与长记忆时间序列之间的长期均衡关系,我们全面地研究了非线性协整的理论、方法,以及非线性协整关系的拟合和检验问题;研究了长记忆向量分整序列的线性协整和非线性协整问题,并利用吸引子的概念解释了长记忆向量非线性时间序列之间的协整关系,这样就将协整理论在 Engle 和 Granger 工作的基础上作了全面提升。为了拓宽线性协整理论和方法的应用,我们提出并研究了非协整系统中分量序列的非线性变换问题,这样就使得一些看似不存在协整关系的序列经过变换后,可以利用协整方法来处理。对于协整系统(包括季节性协整)的检验问题,我们发展了贝叶斯检验,并进行了实证研究。利用协整技术来提高经济预测精度是协整技术的重要应用领域,通过系统的实证研究指出,协整方法对于预测精度的提高在中长期预测时表现得最为明显,对短期预测同样也可以提高其预测精度,但与其他方法相比改善不大,因此协整技术在经济预测中的作用主要应是提高中长期预测精度。

在时间序列波动性模型研究方面,自从 Engle(1982)首创 ARCH 模型以来,国际上已有十多种各类扩展的 ARCH 类模型用以描述不同特点的 ARCH 效应,这对 ARCH 类模型的应用、检验、参数估计和变结构研究既方便又有不足。为了统一现有的各种 ARCH 类模型,我们提出了分整增广 GARCH-M 模型,该模型除包容了国际上目前所发展的 11 种 ARCH 类模型外,还提出了 21 种新的长、短记忆的 ARCH 类模型,这些新模型都具有明确的经济含义,因此,分整增广 GARCH-M 模型在 ARCH 模型族的设定检验中具有重要作用。为了解决这一复杂模型的参数估计问题,我们提出了禁忌-递阶遗传算法并用之进行分整增广 GARCH-M 模型的参数估计问题。此外,对于向量 ARCH 类模型以及另一类波动性模型——SV 模型的估计问题,我们也采用了禁忌-递阶遗传算法。实证表明,采用禁忌-递阶遗传算法解决复杂模型的参数估计问题,比一些常规方法,如 BHHH 算法,具有明显的优势。

波动持续性问题金融波动性研究中的重要问题,Engle 和 Bollerslev 等在这一领域作出了重要贡献。对于多变量时间序列的波动持续性问题,Bollerslev 和 Engle

(1993)提出了波动协同持续这一概念,即通过对多个变量的线性组合来消除波动的持续性,这一问题对于资产组合理论以及金融风险防范问题无疑具有重要意义,但是,此后国外文献中很少有进一步的研究。我们深入研究并发展了国外相关的研究成果,证明了波动持续性与波动非协方差平稳性之间的等价关系,给出了市场组合意义下波动协同持续性存在与否的条件,同时建立了时间序列协同持续性与线性协整之间的关系。另一方面,从单整的角度,我们也提出了波动持续性和协同持续性的定义,并在此基础上讨论了向量 GARCH 过程和向量 SV 过程的持续性和协同持续性问题。进一步,我们将协同持续概念扩展为非线性协同持续,提出非线性协同持续的概念及其算法。线性协同持续与非线性协同持续概念与方法的提出,为从动态角度研究金融风险的持续性及其规避策略提供了理论基础。基于金融波动持续性和协同持续性分析,我们系统地研究了金融动态风险的影响问题以及资产组合中的风险规避策略和途径。研究了存在方差持续性条件下资本资产定价模型和套利定价模型的性质,为证券投资分析提供一种新的方法和手段。

迄今,ARCH 类模型和 SV 类模型是广泛应用于金融时间序列波动性分析的两类重要模型。我们从建模理论以及模型对于金融时间序列实际刻画能力两个角度研究了两类模型各自的特点以及二者之间的联系,从而为金融波动性分析和实际应用提供基础。

本书利用分形理论探讨了金融波动特性的市场机制,指出波动的持续性反映了市场的分形和非线性特性,分析了传统有效市场理论的缺陷和不足,指出在金融分析中引入分形市场理论的必要性。分形和多重分形理论可以作为金融风险分析与管理的理论基础,在这方面特别研究了分形市场中的资本资产定价问题。

变结构建模是社会经济系统建模中的一个重要问题,可以说模型结构变化是社会经济系统中模型的基本特征。在 20 世纪 80 年代初,我们系统地建立了变结构经济计量模型的建模理论和方法。在协整模型的变结构分析中,对于线性协整模型,通常的结构突变和结构渐变问题可以沿用一般的变结构分析方法来处理,而对非线性向量时间序列而言,系统内部动态均衡结构的变化不仅体现在空间结构上,而且具有一定的时间结构特性。为此,提出了一种新的变结构分析理论,即无模型的非线性系统变结构分析思想,给出与模型无关的系统变结构的定义,并利用非参数的神经网络技术和基于递归遗传规划的智能化变结构分析方法,解决了非线性复杂系统的变结构分析问题。

对于 ARCH 类模型的变结构问题,国外目前的一些研究是基于 ARCH 类模型中的某一种模型形式进行的,但这未能解决包含许多模型形式的 ARCH 类模型族的变结构问题。为此,我们充分发挥所提出的分整增广 GARCH-M 模型的包容性,利用分整增广 GARCH-M 模型进行 ARCH 类模型的变结构分析,这就是我们解决 ARCH 类模型变结构问题的基本途径。

本书主要是以我们的研究成果为中心展开讨论的。为使本书体系完整,也提到国外相关的工作,但这些内容只起到进一步说明我们工作的作用。

在本书定稿之际,得知协整理论以及 ARCH 模型的原创者美国经济学家 Engle 和英国经济学家 Granger 荣膺 2003 年度诺贝尔经济学奖,我们也受到鼓舞。本书内容正是在他们工作的基础之上,提出新的研究课题,获得的一些新成果。

在这里我特别感谢十多年来和我一道从事这项研究的我的博士生和硕士生们。他们在协整理论、方法以及时间序列波动性分析两个领域进行了大量的研究工作,本书所反映的只是他们的部分工作(在参考文献中列出),还有相当多的成果不可能包括在一本书中。而且我们的研究集体目前还在继续从事着这项研究工作,就该领域的一些国际前沿问题进行探讨。尽管一些

研究是与国际同步进行的,但也具有自己的特色。

樊智同志和我一道完成本书的编写工作,书中第5章和第8章的稿子是由他完成的(第5章我也参加了部分内容的编写)。在该书写作中,韦艳华、徐梅、许启发和徐正国同志也给予了帮助。全书由我统纂定稿。本书在出版过程中得到清华大学出版社的大力支持,刘显等同志给予了许多帮助。在此谨向有关同志致以衷心的感谢。

张世英

于天津大学

2003年11月13日

## Preface

This book is the fruit of over ten years' research on cointegration theory and volatility of time series of our research team. The research was sponsored by National Science Foundation of China—"Persistence in Volatility of Multivariate Time Series and its Applications in Financial System"(No. 70171001), and Doctoral Foundation of Ministry of Education—"Research on Cointegration Modeling in Social Economic System"(No. 9505621). Integrating the research work of these two foundations, we have achieved a series of creative results in cointegration theory and analysis of volatility in financial time series.

In the research of cointegration, the cointegration theory established by Engle and Granger indicated long term linear equilibrium relationship among non-stationary time series, and it could be defined as linear cointegration. While in the economic system, many variables have long memory. The univariate time series and the relationship among them are usually nonlinear. To disclose the long term equilibrium relationship among nonlinear and long memory time series, we did a thorough research on the theory and methodology, as well as fitting and testing of nonlinear cointegration. We studied the linear and nonlinear cointegration of long memory fractional time series, and demonstrated the mechanism of cointegration by introducing the conception of attractor, thus enhanced the cointegration theory based on the work of Engle and Granger to a new level. To broaden the applications of linear cointegration theory, we raised and studied the issue of nonlinear transformation of time series in non-cointegrated system, and made it possible to apply cointegration analysis to these time series. As for testing of cointegrated system, including seasonally cointegrated system, we developed Bayesian testing method and carried out empirical studies. One important application of cointegration analysis is to improve the forecasting precision. Empirical studies in this book have shown prominent improvement in mid and long term forecasting by using cointegration technique, while no such evident improvement in short term forecasting compared with other forecasting methods, although it does help to some extent. Therefore, the main aim of cointegration technique is to improve mid and long term forecasting precision.

Since ARCH model was presented by Engle in 1982, more than ten kinds of extended ARCH class models have been used to describe different ARCH effects. The complication of these ARCH models has brought conveniences, as well as difficulties, to their applications, tests, parameters estimation and study of structure change. To consolidate the current ARCH class models, we presented fractionally integrated augment GARCH-M model. It included not only 11 existing ARCH class models, but also 21 new kinds of long



and short memory ARCH models which have explicit economic meanings, and this new model played an important role in specification of ARCH class models. In order to estimate the parameters in the complicated models, we proposed tabu-hierarchy genetic algorithm (THGA) and applied it to parameter estimation of fractionally integrated augmented GARCH-M model. Furthermore, THGA was used in parameter estimation of vector ARCH class models and another type of volatility models—SV models. Empirical studies indicated that THGA had more advantages than regular algorithm does, e. g. BHHH algorithm.

Persistence in volatility is an important problem in financial volatility study, to which Engle and Bollerslev have made great contributions. In multivariate time series analysis, Engle and Bollerslev proposed the conception of co-persistence in variance, i. e. to reduce the volatility persistence by linear combination of assets. This proposition had significant influences on assets portfolio theory and financial risk management, but little further research on this topic has been carried out since then. Based on thorough studying and developing of relevant research work of foreign researchers, we proved the equivalence of persistence in variance and non-stationarity of covariance for volatility, and introduced the conditions for existence of co-persistence in assets portfolio. Meanwhile, we established the relationship between cointegration and co-persistence. We also proposed the definitions of persistence and co-persistence in variance from integration point of view, and discussed persistence and co-persistence in vector GARCH process and vector SV process. Furthermore, we extended the conception of co-persistence into nonlinear co-persistence and proposed the algorithm for it. These work constituted the theoretical foundations for the research on persistence in financial risk and the dynamically avoiding strategies. Based upon this analysis, we systematically studied the dynamics of financial risk and the avoidance strategies in portfolio. Considering the persistence in variance, we investigated the characteristics of CAPM and APT, which would provide a new way for portfolio allocation.

So far, ARCH models and SV models are two kinds of important models used in financial volatility analysis. We studied the characteristics and relations of these two models from two aspects—modeling theory and the ability to simulate financial time series, which provided the basis for financial volatility analysis and applications.

This book discussed the market mechanism of financial volatility by utilizing the fractal theory, and showed that persistence in variance reflected the fractal and nonlinear characteristics of the financial market. We analyzed the insufficiency of efficient market theory and pointed out the necessity of introducing fractal market theory into financial analysis. Fractal and multifractal theories can be taken as the theoretical basis for financial risk analysis and management. We particularly set up capital assets pricing model in fractal market.

Structure change is an important issue in social economic system modeling, and

structure change can be considered as the basic characteristic of social economic system. In the early 1980's, we systematically set up the modeling theory and methodology for econometric models with structure changes. For cointegration models with structure changes, the breaks or gradual changeness of the models can be analyzed by using regular structure change analytical methods. However, for the nonlinear vector time series, structure changes of models are reflected not only in space structure, but also in time series structure. We gave the definition of model-free structure change of nonlinear system as a new structure change theory, and put forward structure change analysis of nonlinear complicated system through non-parameter neural network technique and intelligent structure change analysis, based upon recursive genetic programming.

For structure change analysis of ARCH class models, current research is based on a certain type of ARCH model, but this can not include structure changes of all ARCH class models. For this, we made full use of the fractionally integrated augment GARCH-M model and applied it to structure change analysis of ARCH class models, which was our way to study the structure changes of ARCH class models.

The contents of the book are centered on the research achievements of our research team. To ensure the integrity of the book, we also refer to the related works abroad, which just help to demonstrate our research.

Before the finalization of the book, we were excited to learn that American economist Engle and English economist Granger, the founders of cointegration theory and ARCH models, were awarded the Nobel Prize for Economics in 2003. The main contents of this book are new research topics and achievements based upon their works.

I would like to thank my Ph. D. and Master students who carried out the research together with me. They did plenty of work in cointegration theory and time series volatility analysis. This book just represents part of their works (listed in the references), and quite a lot can not be covered here. Our research work is continuing, focusing on some forefront issues in this field. Although our research work is in phase with those of the counterparts abroad, we have our own specialties.

Fan Zhi and I wrote this book together, in which, chapter 5 and 8 were written by him (part of chapter 5 was written by me). Wei Yanhua, Xu Mei, Xu Qifa, and Xu Zhengguo gave their help in writing the book. I was responsible for the compilation and finalization of the whole book. The publication of the book was greatly supported by Tsinghua University Press, with originalities from Liu Yu. Here I sincerely express my gratitude to those who gave their support for this book.

Zhang Shiyong  
in Tianjin University  
Nov. 13, 2003

第 1 章 时间序列分析 .....	1
1.1 时间序列的一般模型 .....	1
1.2 向量平稳时间序列·向量自回归模型 .....	11
1.3 非平稳随机过程与单整序列 .....	15
1.4 长记忆时间序列 .....	25
参考文献 .....	37
第 2 章 单位根检验 .....	40
2.1 单位根过程 .....	40
2.2 单整过程的极限分布 .....	42
2.3 单位根检验 .....	44
2.4 有单位根的向量自回归过程 .....	61
参考文献 .....	66
第 3 章 协整理论与方法 .....	68
3.1 协整与误差校正模型 .....	68
3.2 单一方程协整关系的估计和检验 .....	71
3.3 系统方程协整关系的估计和检验 .....	75
3.4 协整系统的贝叶斯分析 .....	80
3.5 向量分整序列的线性协整分析 .....	86
3.6 协整系统的预测 .....	91
3.7 单整时间序列的非线性变换 .....	100
参考文献 .....	104
第 4 章 季节单整和协整 .....	106
4.1 季节单整和协整及其检验 .....	106
4.2 贝叶斯季节协整检验 .....	109
附录 Lagrange 多项式近似定理 .....	115
参考文献 .....	116
第 5 章 非线性协整理论 .....	117
5.1 非线性协整的含义 .....	117

5.2	非线性协整关系的估计和检验 .....	118
5.3	非线性协整关系的存在性研究 .....	126
5.4	基于小波神经网络的非线性协整建模 .....	131
5.5	线性协整系统误差校正模型的非线性形式 .....	136
5.6	长记忆向量时间序列的非线性协整关系 .....	140
5.7	变结构协整与建模 .....	143
	参考文献 .....	156
<b>第 6 章</b>	<b>ARCH 模型 .....</b>	<b>158</b>
6.1	短记忆 ARCH 模型族 .....	158
6.2	长记忆 ARCH 模型 .....	166
6.3	分整增广 GARCH-M 模型 .....	168
6.4	面板数据的 GARCH 模型族 .....	174
6.5	GARCH 模型的若干统计性质 .....	176
6.6	ARCH 模型族的建模问题 .....	182
6.7	ARCH 模型诊断分析与变结构模型 .....	191
6.8	GARCH 过程对应的随机微分方程 .....	202
6.9	存在条件异方差的单位根检验 .....	205
6.10	向量 GARCH 模型及建模问题 .....	208
6.11	向量 GARCH 过程的持续性和协同持续性 .....	216
6.12	时间序列条件矩的持续和协同持续性 .....	229
	参考文献 .....	239
<b>第 7 章</b>	<b>随机波动模型 .....</b>	<b>243</b>
7.1	基本 SV 模型及其统计性质 .....	243
7.2	扩展 SV 模型 .....	245
7.3	SV 模型的参数估计方法 .....	251
7.4	基于禁忌遗传算法的伪极大似然估计方法与 Monte Carlo 研究 .....	263
7.5	长记忆 SV 模型的估计及应用 .....	266
7.6	变结构 SV 模型 .....	271
7.7	SV 过程的聚合及边际化 .....	278
7.8	SV 过程的持续性和协同持续性 .....	282
7.9	SV 模型与 GARCH 模型比较 .....	288
7.10	平方根随机自回归波动模型 .....	300
	参考文献 .....	304
<b>第 8 章</b>	<b>金融市场波动性分析 .....</b>	<b>308</b>
8.1	金融市场的波动特性 .....	308
8.2	分形市场理论与金融波动的市场机制 .....	311

8.3	分形市场中的资本资产定价研究 .....	324
8.4	金融波动持续性及金融风险规避策略 .....	328
8.5	具有方差持续性的资本资产定价研究 .....	334
8.6	具有方差持续性的套利定价研究 .....	336
8.7	VaR 的波动持续性研究 .....	339
	参考文献 .....	348
<b>第 9 章</b>	<b>高频金融时间序列分析与建模 .....</b>	<b>352</b>
9.1	日历效应及其计量方法 .....	352
9.2	已实现波动理论 .....	356
9.3	基于已实现波动的波动持续和协同持续性 .....	365
9.4	已实现极差波动与已实现双(多)幂次变差 .....	370
9.5	超高频金融时间序列的持续期模型(I) .....	377
9.6	超高频金融时间序列的持续期模型(II) .....	385
	参考文献 .....	394
<b>第 10 章</b>	<b>金融时间序列分析的小波方法 .....</b>	<b>397</b>
10.1	离散小波变换与多分辨分析 .....	397
10.2	基于小波方法的金融波动分析 .....	408
10.3	基于小波方法的协整分析 .....	416
10.4	多分辨持续及协同持续 .....	418
10.5	基于小波方法的 LMSV 模型分析 .....	420
	参考文献 .....	427
<b>第 11 章</b>	<b>连续时间模型及其应用 .....</b>	<b>429</b>
11.1	金融资产收益连续时间模型的一般分析 .....	429
11.2	资产价格的抛物线跳跃扩散模型 .....	438
11.3	连续时间 SV 模型及应用 .....	443
11.4	连续时间资产收益变结构模型 .....	448
	参考文献 .....	453
<b>附 录</b>	<b>.....</b>	<b>455</b>
附表 1	DF 检验临界值表 .....	455
附表 2	DF 的 $t$ 检验临界值表 .....	455
附表 3	似然比检验表 .....	456
附表 4	协整回归检验临界值表 .....	458
附表 5	协整检验的临界值响应面表 .....	459
附表 6	迹统计量检验临界值表 .....	460
附表 7	$\lambda$ -max 检验临界值表 .....	461
附表 8	季节单位根检验临界值表 .....	462
附表 9	季节协整检验临界值表 .....	463
<b>作者简介</b>	<b>.....</b>	<b>466</b>

# Contents

<b>Chapter 1 Time Series Analysis</b> .....	1
1.1 General time series models .....	1
1.2 Vector stationary time series • vector autoregressive model .....	11
1.3 Non-stationary stochastic processes and integrated time series .....	15
1.4 Long memory time series .....	25
References .....	37
<b>Chapter 2 Tests of Unit Root Processes</b> .....	40
2.1 Unit root processes .....	40
2.2 Limiting distribution of integrated processes .....	42
2.3 Tests of unit root processes .....	44
2.4 Vector autoregressive processes with unit root .....	61
References .....	66
<b>Chapter 3 Cointegration Theory and Methodology</b> .....	68
3.1 Cointegration and error correction model .....	68
3.2 Estimation and tests of cointegration relationship in single equation .....	71
3.3 Estimation and tests of cointegration relationship in simultaneous equation system .....	75
3.4 Bayesian analysis of cointegrated system .....	80
3.5 Linear cointegration analysis of fractionally integrated vector time series .....	86
3.6 Forecasting of cointegrated system .....	91
3.7 Nonlinear transformation of integrated time series .....	100
References .....	104
<b>Chapter 4 Seasonal Integration and Cointegration</b> .....	106
4.1 Seasonal integration, cointegration and tests .....	106
4.2 Bayesian tests of seasonal cointegration .....	109
Appendix: Lagrange polynomial approximation theorem .....	115
References .....	116

<b>Chapter 5 Nonlinear Cointegration Theory</b> .....	117
5.1 Definition of nonlinear cointegration .....	117
5.2 Estimation and tests of nonlinear cointegration relationship .....	118
5.3 Existence of nonlinear cointegration relationship .....	126
5.4 Nonlinear cointegration modeling based on wavelet neural network .....	131
5.5 Nonlinear error correction models of linearly cointegrated system .....	136
5.6 Nonlinear cointegration relationship in long memory vector time series .....	140
5.7 Cointegration with structure changes and modeling .....	143
References .....	156
<b>Chapter 6 ARCH Class Models</b> .....	158
6.1 Short memory ARCH class models .....	158
6.2 Long memory ARCH class models .....	166
6.3 Fractionally integrated augment GARCH-M model .....	168
6.4 GARCH class models for panel data .....	174
6.5 Statistical properties of GARCH model .....	176
6.6 Modeling of ARCH class models .....	182
6.7 Diagnostic analysis and structure change modeling of ARCH class models .....	191
6.8 Stochastic differential equation of GARCH process .....	202
6.9 Unit root tests with conditional heteroskedasticity .....	205
6.10 Vector GARCH models and modeling .....	208
6.11 Persistence and co-persistence in vector GARCH process .....	216
6.12 Persistence and co-persistence in conditional moments of time series .....	229
References .....	239
<b>Chapter 7 Stochastic Volatility Models</b> .....	243
7.1 Basic SV models and statistical properties .....	243
7.2 Extended SV models .....	245
7.3 Parameters estimation of SV models .....	251
7.4 QML estimation based on THGA and Monte Carlo .....	263
7.5 Estimation of long memory SV models and applications .....	266
7.6 SV models with structure changes .....	271
7.7 Aggregation and marginalization of SV models .....	278
7.8 Persistence and co-persistence in SV models .....	282
7.9 Comparison of SV and GARCH models .....	288
7.10 Square-root stochastic autoregressive volatility model .....	300
References .....	304

<b>Chapter 8 Analysis of Financial Volatility</b> .....	308
8.1 Volatility characteristics in financial market .....	308
8.2 Fractal market theory and mechanism of financial volatility .....	311
8.3 Capital assets pricing in fractal market .....	324
8.4 Persistence in financial volatility and avoiding strategy of financial risk .....	328
8.5 Capital assets pricing model with persistence in variance .....	334
8.6 Arbitrage pricing theory with persistence in variance .....	336
8.7 Persistence in Value at Risk .....	339
References .....	348
<b>Chapter 9 Analysis and Modeling for High-Frequency Financial Time Series</b> .....	352
9.1 Calendar effects and its econometric approach .....	352
9.2 Realized volatility theory .....	356
9.3 Persistence and co-persistence in realized volatility .....	365
9.4 Realized range-based volatility and realized bipower (multipower) variation .....	370
9.5 Durations models for ultra-high frequency financial time series( I ) .....	377
9.6 Durations models for ultra-high frequency financial time series( II ) .....	385
References .....	394
<b>Chapter 10 Wavelet Methods for Financial Time Series Analysis</b> .....	397
10.1 Discrete wavelet transform and multiresolution analysis .....	397
10.2 Analysis of financial volatility based on wavelet methods .....	408
10.3 Cointegration theory based on wavelet methods .....	416
10.4 Multiresolution persistence and co-persistence .....	418
10.5 LMSV modeling based on wavelet analysis .....	420
References .....	427
<b>Chapter 11 Continuous Time Models and its Applications</b> .....	429
11.1 General analysis for continuous time financial asset return models .....	429
11.2 Hyperbolic jump-diffusion models for asset prices .....	438
11.3 Continuous time SV models and applications .....	443
11.4 Structural change model for continuous time asset return .....	448
References .....	453
<b>Statistical Tables</b> .....	455



## 1.1 时间序列的一般模型

### 1.1.1 时间序列及统计特征

设  $T$  为某个时间集, 对  $t \in T, x_t$  为随机变量, 对于该随机变量的全体  $\{x_t, t \in T\}$ , 当  $T$  取为连续集, 如  $T = (-\infty, +\infty)$ , 或  $T = (0, \infty), \dots$ , 称  $\{x_t\}$  为随机过程; 当  $T$  取为离散集, 如  $T = \{1, 2, \dots\}$ , 或  $T = \{1, 2, \dots, n\}, \dots$ , 则称  $\{x_t\}$  为随机序列。

对于一个随机序列, 一般只能通过记录或统计得到它的一个样本序列  $x_1, x_2, \dots, x_n$ , 称它为随机序列  $\{x_t\}$  的一个现实。随机序列的现实是一族非随机的普通数列, 在许多实际问题中, 往往仅能获得随机序列的一个现实。由于随机序列  $\{x_t\}$  的整数变量  $t$  一般代表等间隔的时间增长量, 所以常称随机序列为时间序列。

对于时间序列, 需要给出它的一些统计特征量以描述其性质。主要的特征量有三个。

#### 1. 均值函数

对于每个  $t$  而言, 时间序列  $\{x_t\}$  的均值函数为

$$\mu_t = E[x_t] \triangleq \int_{-\infty}^{+\infty} x f_t(x) dx \quad (1-1)$$

其中,  $f_t(x)$  为  $\{x_t\}$  的概率密度函数。

#### 2. 自协方差函数

为了考虑时间序列  $\{x_t\}$  中不同时刻随机变量之间的统计关系, 对任意  $t, s$  时, 取  $x_t$  和  $x_s$  的相关矩, 即

$$\gamma_{t,s} = \text{Cov}(x_t, x_s) \triangleq E[(x_t - Ex_t)(x_s - Ex_s)] \quad (1-2)$$

称  $\gamma_{t,s}$  为时间序列  $\{x_t\}$  的自协方差函数, 且有性质  $\gamma_{t,s} = \gamma_{s,t}$ 。

特别的, 当  $t=s$  时, 称  $\gamma_{t,t}$  为时间序列  $\{x_t\}$  的方差, 记作  $\gamma_{t,t} = \text{Var}(x_t)$ 。

#### 3. 自相关函数(auto correlation function, ACF)

时间序列  $\{x_t\}$  的自相关函数  $\rho_{t,s}$  为

$$\rho_{t,s} \triangleq \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t}\gamma_{s,s}}} \quad (1-3)$$

所以,  $\rho_{t,s}$  是  $\gamma_{t,s}$  的归一化, 有  $\rho_{t,s} = \rho_{s,t}$  且  $\rho_{t,t} = 1$ 。