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MURNGIN

A MATHEMATICAL SOLUTION

PIN-HSIUNG LIU

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PREFACE

In this paper I present the result of my mathematical study of the Murngin system. The work was began in spring 1967. During that summer a short essay "A note on the Murngin system" was published in the *Bulletin of the Visiting Scholars Association, Harvard-Yenching Institute, Harvard University, China Branch* Vol. 5–6 (1967) and appeared also in the *Newsletter of Chinese Ethnology* No. 7 of the same year. By the end of the following summer, the first draft of the present study in Chinese was prepared. The major part concerned with the section system was summarized in English under the title of "Formal analysis of prescriptive marriage system: the Murngin case" for presentation at the VIIIth International Congress of Anthropological and Ethnological Sciences held in Tokyo and Kyoto, September 1968. In this résumé a new mathematical device was adopted, which permitted me to revise the original draft. This second draft was published under the title of "Mathematical study of the Murngin system" in Chinese in the *Bulletin of the Institute of Ethnology, Academia Sinica,* No. 27 in 1969. The present English version makes some changes and refinements necessary.

In this paper I deal with mathematical models, but since I am a social anthropologist, I am fully aware that the method by which the results are reached may not be as concise and direct as that of a mathematician.

The mathematical analysis of the Murngin System proves that the joint operation of anthropology and mathematics leads to the solution of a hitherto insolvable problem. The application of mathematical methods is in fact not an unsurmountable barrier facing the anthropologist as generally assumed. It is hoped that mathematicians may become aware of the dilemma and lend their vital assistance to anthropologists, for it is only by a combination of the two disciplines that solutions to such problems may be reached. For a general discussion of this question, attention is drawn to my paper "Theory of groups of permutations, matrices and kinship: a critique of mathematical approaches to prescriptive marriage systems" which was published in the *Bulletin of the Institute of Ethnology, Academia Sinica*, No. 26 in 1968, is included here as an appendix to the present study.

During the entire period of research, the work was supported by the Harvard-Yenching Institute, Harvard University. The author is much obliged to his colleagues of the Institute of Ethnology, Academia Sinica, for their helpfulness throughout this study. Special thanks are due to Mrs. Inez de Beauclair and Mr. Raleigh Ferrell who kindly examined the English text. Publication expenses were generously provided by the National Council for the Development of Sciences.

PIN-HSIUNG LIU

Nankang, Taipei
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WORKING HYPOTHESIS AND METHOD

One of the most remarkable achievements in recent kinship study is the establishment of kinship algebra, a long-pending problem for social anthropology. As early as the end of the last century, social anthropologists recognized the applicability of mathematics to the analysis of the Australian section systems regulated by prescribed marriage rules. It was Francis Galton (1889) and Émile Durkheim (1898) who first proposed the theory of 'double descent' as a clue to the mathematical study of kinship. Since that time not a few scholars have been engaged in this study, but unfortunately, owing to their restricted mathematical knowledge, kinship algebra was never realized as a established science.

It was André Weil who first applied pure algebra to the study of certain types of marriage laws, namely, the section system. Weil proposed the following three rules as basic properties of the matrilateral cross-cousin marriage system (Lévi-Strauss 1969: 221–222):

(A) For any individual, man or woman, there is one and only one type of marriage which he (or she) has the right to contract.
(B) For any individual, the type of marriage which he (or she) may contract depends solely on sex and the type of marriage from which he (or she) is descended.
(C) Any man must be able to marry his mother's brother's daughter.

Based on the special character of the 'marriage types' proposed by him, in which indication of the marriage type of the children's generation is nothing but a rearrangement of that of the parent's generation, Weil points out that the theory of groups of permutations is applicable to the study of the section system. Thus this method is applied to prove that the four-section system proposed by Claude Lévi-Strauss as the implicit system of the Murngin could meet rule (C). For the Murngin's present eight-subsection system, owing to the contradiction between its marriage regulations and rule (A), Weil introduces another mathematical device, the addition of an n-tuple modulo two system, to demonstrate that Lévi-Strauss's hypothesis is mathematically constructable. Weil's unique suggestion has shaped the current mode of mathematical approaches to kinship study.

Robert R. Bush, extending Weil's method, concludes that the algebra of permutations, special topics in group theory, matrix algebra, and operator algebra are appropriate for the study of the section system. Thus Bush introduces the concept of a mathematical 'operator' demonstrating that 'permutation matrices' are an effective tool for kinship analysis. One of the extraordinary merits of this method is the production of identity operators and other equations, which means the formulation of generation cycles of descent lines or marriage rules for the given society in mathematical formulae. (See White 1963, Appendix 2.)

After Bush, Kemeny, Snell and Thompson contributed to an algebraic analysis of the societies to be investigated an integrated set of axioms as follows (Kemeny, Snell and Thompson 1956: 343):

Axiom 1. Each member of the society is assigned a marriage type.

Axiom 2. Two individuals are permitted to marry only if they are of the same marriage type.

Axiom 3. The type of an individual is determined by the individual's sex and by the type of his parents.

Axiom 4. Two boys (or two girls) whose parents are of different types will themselves be of different types.

Axiom 5. The rule as to whether a man is allowed to marry a female relative of a given kind depends only on the kind of relationship.

Axiom 6. In particular, no man is allowed to marry his sister.

Axiom 7. For any two individuals it is permissible for some of their descendants to intermarry.

Both method and axioms are revised by Harrison C. White (1963). Perceiving that marriage type is not a concept to be found in either the field notes of anthropologists or the thinking of members of the societies, White adopts two new operators or generators. He represents the transformation of husband's section into wife's section by one matrix, and the transformation of father's section into children's section by another, instead of having one matrix representing the transformation of parent's marriage type into son's type, and another similar matrix to represent daughter's marriage type. Meanwhile, Kemeny-Snell-Thompson's axioms are revised as follows (1963: 34–35):

1. The entire population of the society is divided into mutually exclusive groups, which we call *clans*. The identification of a person with a clan is permanent. Hereafter *n* denotes the number of clans.
2. There is a permanent rule fixing the single clan among whose women the men of a given clan must find their wives.
3. By rule 2, men from two different clans cannot marry women of the same clan.

4. All children of a couple are assigned to a single clan, uniquely determined by the clans of their mother and father.
5. Children whose fathers are in different clans must themselves be in different clans.
6. A man can never marry a woman of his own clan.
7. Every person in the society has some relative by marriage and descent in each other clan: i.e., the society is not split into groups not related to each other.
8. Whether two people who are related by marriage and descent links are in the same clan depends only on the kind of relationship, not on the clan either one belongs to.

In White's axioms the term *clan* is used instead of *section*, but this substitution might cause conceptual confusion, and its unfitness has already been pointed out by Russell M. Reid (1967: 171). He insists that his proposed 'marriage cycles' is the essential feature of the model resulting from White's eight axioms, and therefore he proposes a ninth axiom as follows:

9. All marriage cycles in the same system must contain the same number of segments.

Though the axioms and methods are incessantly being refined and improved, they still contain some flaws in themselves, so the effective range of applicability is still limited. First, as recognized by the mathematician himself, the method is not applicable to the analysis of societies practicing matrilateral cross-cousin marriage, such as the Murngin or Prums, owing to the contradictions between their marriage rules and the axioms mentioned above (White 1963: 145). Secondly, such systems as those of uncle/niece marriage, or absurdities such as father/daughter or mother/son marriage and others are taken for matrilateral cross-cousin marriage systems in mathematicians' treatments (Liu 1968). Once the deficiencies of the method are clarified, methodological improvement should follow at once. However, no matter how far the method may be refined, the Murngin problem remains insolvable as long as the axiomatic contradiction persists. Without the establishment of new axioms or hypotheses and a new proposal of mathematical method, the Murngin problem will remain an enigma forever.

In the author's recent study of kinship, the mathematical model of kinship or genealogical space is discussed (Harvey and Liu 1967). Genealogical space is the structural frame upon which all kinship systems depend to exist, but up to the present its basic properties have been regarded as self-evident and it has been rarely discussed (e. g., Fisher 1960). 'Kinship category' is a new concept concerned with one part of the genealogical space, wherein all kin relationships are reduced to the two basic units 'parent' and 'child', and are expressed by the products of the two units as generators. The kinship categories represented by the numerical notation system are computable, and in fact the 'generation transition' of the kin relationship itself is a kind of typical binary operation. Thus we can point out that a set of kin groups composed of the kinship categories possesses the following properties of algebraic group theory:

(1) Identity: 00 is the identity unit or unit element.

(2) Inverses: Each element has its inverse in the set.

(3) Associativity: This property is satisfied by the binary operation.

(4) Group equations $a \cdot x = b$ and $x \cdot a = b$ are solvable.

But this group is not commutative and its elements are productive without any limitation, so it may be called an unfinite non-Abelian group.

However, kinship category is not the only kinship structure derivable from genealogical space, there are also many other different forms. For example, 'kinship type' is a unit well-known to anthropologists, and is customarily expresed by the combination of language-oriented notation system such as F, M, B, Z, S, D, etc. If the operators or generators producing kinship systems were to be explored exhaustively, then we could compose all sorts of structures within the genealogical space for the analyses of any complicated structural problems.

The kinship structures of the Australian aborigines are well known for their unique and complicated genealogical space characterized by socalled 'double descent' or 'section system'. Owing to the basic structural differences from other societies none of the previously employed devices are effective. To establish the spacial forms of genealogical space inherent in prescriptive marriage systems, to explore its mathematical properties and to compose the mathematical models of kinship structures are the main aims of this paper. For this purpose I propose here two basic units: *father-child link* and *mother-child link*. The former is represented by *m* and the latter by *f*, which are used as generators for the analysis of the section systems.

No society in Australia is more controversial than that of the Murngin, which has been studied and discussed by social anthropologists for almost forty years and has been considered nearly an insolvable problem (Barnes 1967). There may be many reasons for this failure. But the main cause is not due to the lack of data or the missing of some crucial facts as Barnes thought. In fact no society has ever been so intensively investigated by trained social anthropologists accumulating a bewildering wealth of materials as the Murngin. What then is the real cause of the failure? It can be no other than the insufficiency of the kinship theory applied. Seeing the conceptual confusion caused by Lévi-Strauss and Leach in treating uncle/niece marriage as a form of matrilateral cross-cousin marriage (Liu 1968), we can not help but admit that it is not the aborigines but the anthropologists who are to be blamed for the theoretical impasse.

In the following discussion, some characteristics of prescriptive marriage systems are shown by structural models of several typical and conjectual systems depicted in Figure 1. Here the mathematical Cayley diagram is adopted. The solid line represents generator *m* and the dotted line represents generator *f*. The arrow-head indicates the transition of generation from ascending to descending generation. If the generation transition is reversible, no sign is attached to the line.

Figure 1*a*, *b* and *c* represent three well-known societies, Arunta, Ambrym and Kariera, characterized by bilateral cross-cousin marriage systems. The Arunta practice second cross-cousin marriage, Kariera first cross-cousin marriage, and the Ambrym are in the middle, with first cross-cousin once removed, that is, an oblique marriage. Here the Korean 'inch system' is adopted to show kinship distance. Applying the Harvey-Liu system, ***kinship distance Kd*** is expressible in following formula: $Kd=Sxy$. The kinship distance between spouses decreasses 'one inch' from society to adjacent society, say, from Arunta to Ambrym and Ambrym to Kariera. If the decrease of kinship distance continues with the same spacing, following first cross-cousin marriage we could get 'uncle/niece marriage' and 'sibling marriage' in sequence. Supposing sibling marriage

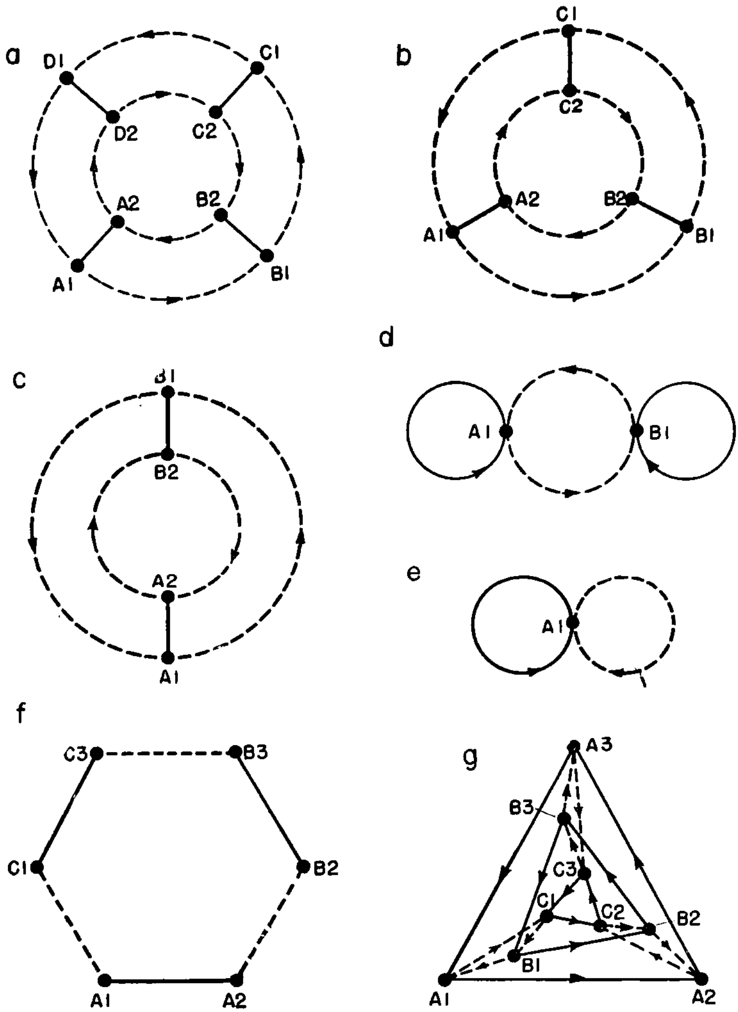

FIG. 1. Structural models of prescriptive marriage systems. *a.* Arunta *b.* Ambrym *c.* Kariera *d.* Uncle/niece marriage *e.* Sibling marriage *f.* Patrilateral cross-cousin marriage *g.* Matrilateral cross-cousin marriage