机械工程学科研究生教学用书

最优化理论与方法

Optimal Theories and Methods

黄 平 主 编
孟永钢 副主编

清华大学出版社

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内容简介

本书系统地介绍了在机械工程学科中常用的最优化理论与方法，分为线性规划与整数规划、非线性规划、智能优化方法、变分法与动态规划4个篇次，共15章。第1篇包含最优化基本要素、线性规划和整数规划。在介绍优化变量、目标函数、约束条件和数学建模等最优化的基本内容后，讨论了线性规划求解基本原理和最常用的单纯形方法，然后给出了两种用于整数线性规划的求解方法。在第2篇的非线性规划中，包含了非线性规划数学分析基础、一维最优化方法、无约束多维最优化方法、约束非线性规划方法等。第3篇的智能优化方法包括启发式搜索方法、Hopfield神经网络优化方法、模拟退火法与均场退火法、遗传算法等内容。在第4篇中，介绍了变分法、最大(小)值原理和动态规划等内容。各章都配备了习题。

本书可作为高等院校机械工程一级学科各专业的最优化理论与方法课程的研究生教材和教师的教学和科研参考书，也可作为其他相关专业的教学用书，以及从事生产规划、优化设计和最优控制方面工作的工程技术与科研人员的参考用书。

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前言

PREFACE

最优化理论与方法是工科研究生学习的一门主干课。该课程主要教授研究生一些实用的最优化理论和方法，使其在今后的研究中能够运用这些理论和方法，在设计、制造和选材等方面获得结构、电路和过程的最优解。

以往大多数的最优化方法课程和书籍专业性较强，常被分为生产规划类的线性规划、机械类的优化设计、计算机类的智能优化和电子类的最优控制等不同课程。随着科学技术的发展，各学科间的交叉与融合越来越紧密，一项科学研究需要应用不同学科的理论与方法已经是极为普遍的，因此这也对最优化理论与方法的研究生教学提出了新的要求。为了适应学科发展现状，我们在多年实践的基础上，编写了本书，以介绍成熟的最优化理论与方法为主，适当介绍最优化理论的新的研究成果和发展趋势，为研究生将来开展的论文研究提供最优化方面的理论基础与实用方法。

本书较系统地介绍了在工科中常用的最优化理论与方法，分为线性规划与整数规划、非线性规划、智能优化方法、变分法和动态规划 4 个篇次，共 15 章。第 1 篇包含最优化基本要素、线性规划和整数规划 3 章。线性规划在工业、农业、商业、交通运输、军事和科学研究的各个领域有广泛应用。例如，在资源有限的情况下，如何合理使用人力、物力和资金等资源，以获取最大效益；如何组织生产、合理安排工艺流程或调整产品成分等，使所消耗的资源(人力、设备台时、资金、原材料等)为最少等。在介绍了最优化的基本内容后，讨论了线性规划求解的基本原理和最常用的单纯形方法，并给出了用于整数线性规划的求解方法。第 2 篇所述内容是 20 世纪中期形成的一个方向，随着计算机技术的发展，出现了许多有效的算法，并得到了快速发展。非线性规划广泛应用于机械设计、工程管理、经济生产、科学研究和军事等方面。这一篇的主要内容包含非线性规划数学基础、一维最优化方法、无约束多维非线性规划方法、约束问题的非线性规划方法和多目标最优化 5 章，这些内容是非线性规划中最基本也是最重要的，可以为优化设计等提供有力的工具。第 3 篇是智能优化方法。智能优化算法有别于一般的按照图灵机进行精确计算的程序，是对计算机模型的一种新的诠释，它模拟自然过程、生物或人类思维等方式来求解最优化问题。例如，模拟退火法源于物

质的退火过程，遗传算法借鉴了生物进化思想，神经网络模拟了人脑的思维等。其中一些方法可以解决组合优化或较有效处理“局部极值”和“全局极值”等问题。智能优化方法很多，本书选取了启发式搜索方法、Hopfield 神经网络优化方法、模拟退火法与均场退火法、遗传算法 4 章内容。第 4 篇包括变分法及其在最优控制中的应用、最大(小)值原理和动态规划共 3 章，这些内容是解决最优控制问题的主要方法。最优控制广泛应用于控制系统、燃料控制系统、能耗控制系统、线性调节器等最优综合和设计场合。

本书介绍的最优化理论与方法范围较宽，包括了目前各工程类专业在科学研究与应用时常用的和主要的方法与手段，这些是作为一名工科研究生需要学习和掌握的。另外，为了兼顾不同学科的特点，在某些内容上具有一定的理论深度。但是本书的重点是让学生掌握这些内容的基本理论和基本方法。考虑到教学时数的限制，书中给出了适当的算例，而具体的工程应用实例有待于学生在今后的研究中进一步学习和领会。本书各章均配备了习题，可作为高等院校机械工程一级学科各专业的最优化理论与方法课程的研究生教材和教师的教学和科研参考书，也可作为其他相关专业的教学用书，以及作为从事生产规划、优化设计和最优控制方面工作的工程技术与科研人员的参考用书。

本书主编为黄平，副主编为孟永钢。具体参加本书各章内容编写工作的是：李旻(第 1～5 章)、孟永钢(第 6,7 章)，黄平(第 8,9,13 章)、胡广华(第 10,11 章)、邱志成(第 12 章)、刘旺玉(第 14 章)、孙建芳(第 15 章)。在本书编写工作中，我们参考和引用了许多国内外的书籍和文献等材料，为此我们向这些作者表示衷心的感谢，这些参考文献都列在本书各章的后面。另外，由于作者的水平所限，难免存在不足和错误，希望读者给予批评指正。

编者

2008 年 10 月 30 日

Preface

Theories and methods of optimization is one of the main subjects for engineering graduate students. The purpose of this subject is to teach graduate students some common and useful theories and methods of optimization so that they can use the knowledge in design, manufacturing and material selection to obtain the optimal solution of a structure, a circuit or a process.

Most of the traditional text books on optimization are discipline oriented, usually dividing into different courses of operation research, optimal machine design, intelligent optimization and optimal control. Along with the development of science and technology, interdisciplinary merging and fusion become closer and tighter. Nowadays it is quite often to apply theories and methods in different fields to solve problems in scientific researches. To meet the demands of scientific development, the education of theories and methods of optimization for graduate students should be improved. Based on the teaching practice in the past several years, the authors have compiled this new text book, which mainly covers the well-developed theories and methods of optimization, adding a few topics of advances in optimization theory. The book provides the knowledge of basic theories and practical methods of optimization for graduate students to carry out their research work.

The subject matters of this book are grouped into 4 parts in total 15 chapters, linear programming and integer linear programming, nonlinear programming, intelligent optimization methods, calculus of variations and dynamic programming, collecting most of the theories and methods of optimization commonly used in engineering. In the first part of linear programming, 3 chapters of basic elements of optimization, linear programming and integer linear programming are included. Linear programming is of wide applications in industry, agriculture, business, transportation, military operations and scientific researches. For an example, under the condition of finite resources, linear programming can be used to make a plan of the distribution of human, material and financial resources for getting the maximum gain. Meanwhile, consumption of resources can be reduced to minimum by production planning, process rationalization and/or ingredient modification with the optimization method. At first fundamentals of optimization theories are introduced.

Then the principles of linear programming and the simplex method are discussed. In addition, the scheme of integer linear programming is described in this part. The second part is on the nonlinear programming which is a branch formed in the middle of the 20th century. Accompanying with the development of digital computers, nonlinear programming has been growing rapidly, and many effective algorithms has appeared. Nowadays, nonlinear programming has been widely used in machine design, project management, production, scientific research activities and military affairs. In this part there are 5 chapters, including basic mathematics of nonlinear optimization, optimization methods for single argument problems, unconstrained multivariate problems and constrained nonlinear programming, which are the most fundamental and important contents of nonlinear programming, and powerful tools for optimal design. The third part is on intelligent optimization methods, which provide new interpretation of computing, differing from the precise calculation programs of Turing machines. Intelligent optimization methods are inspired from nature, and mimic of natural evolution and biological thinking processes to find optimal solutions. Simulated annealing method, for instance, mimics the annealing process of substances, while genetic algorithm refers to the evolution of organisms, and neural network is a model of human brain. Some of the intelligent optimization methods can effectively solve the problem of "local maxima" or "whole maxima". Among many intelligent optimization methods, the heuristic search method, Hopfield neural network optimization method, the simulated annealing method and mean field annealing method, and the genetic algorithm are selected and included in the book. The last part of the book consists of 3 chapters of calculus of variations and its applications in optimal control, maximum principle and dynamic programming, which are the major methods for solving optimal control problems. Optimal control is widely applied in the fields of system control, fuel consumption control, energy consumption control and linear adjustors.

The major feature of this book is the broadness of its contents, covering most of the common optimization methods used in different engineering disciplines, which are necessary knowledge for engineering graduate students to be mastered. Considering the different requirements for the students in different fields, the book puts emphasis on the fundamentals of theories and methods of optimization although a part of them more theoretical are also included. Because of the limited course time in one semester, practical engineering problems are not discussed much in the book, leaving them for students to study in the future, while giving some

relative simple examples. In the end of each chapter, exercises are prepared for students to do. This book can be used as a textbook for post-graduates majoring in mechanical engineering. It can also serve as a reference book for university teachers and students in their teaching and research work as well as for the researchers and engineers who work on operation research, optimal design or optimal control.

The author in chief of this book is Huang Ping, and the associate author is Meng Yonggang. The following authors took part in the following compilations of the book, Li Min (Chapter 1-5), Meng Yonggang (Chapter 6 and 7), Huang Ping (Chapter 8, 9 and 13), Hu Guanghua (Chapter 10 and 11), Qiu Zhicheng (Chapter 12), Liu Wangyu (Chapter 14), and Sun Jianfang (Chapter 15). During the compilation of the book, we have referred and cited many publications which are listed in the references. To all of the authors of the references, we extend our most sincere thanks. The authors welcome hearing from readers about any errors of fact or omission that may undoubtedly existed in the book.

Authors

October 30, 2008

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1

第 1 篇

线性规划与整数规划