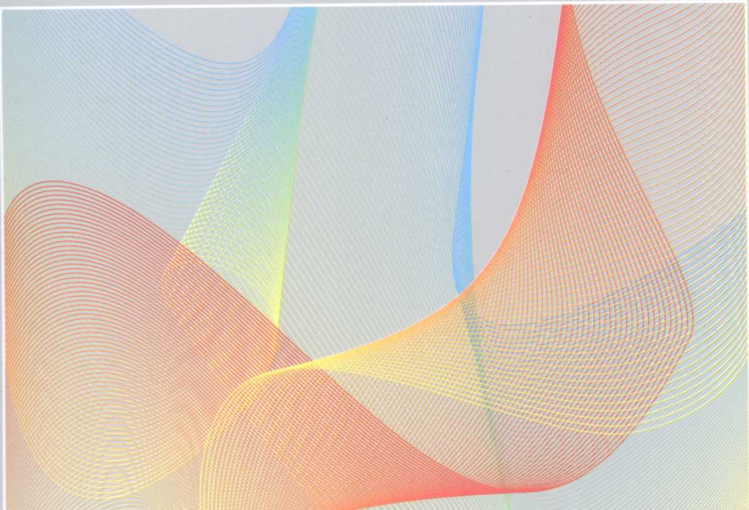
An Introduction to the Finite Element Method

有限单元法导论



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内 容 简 介

本书是一本讲述有限单元法的入门教材.全书共有8章.第1章以简单的力学问题为例介绍了有限单元法的基本原理.第2章至第6章分别讲述了杆、梁、板、壳以及一般三维物体结构问题的有限单元方法，其中讨论了应用有限单元法的各种模拟技术问题，例如，如何评估有限单元法计算的结果.第7章是对结构振动以及动力学分析问题的有限单元法的简介.第8章介绍了有限单元法对结构进行热力学分析的基础知识.在各章的最后还提供了利用有限单元法软件的练习题.本书内容密切结合工程实际问题，简明易懂，特点鲜明.

读者对象为高等院校机械、土木、水利、航空等工程专业的本科生以及相应专业领域的工程设计技术人员.

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出版前言

基于计算力学的各种数字模拟方法在工程中正发挥着日益重要的作用.一个现代企业的重要标志之一即体现在该企业是否实现了从产品设计、分析到生产的计算机一体化(CAD/CAE/CAM-Computer-Aided Design/Computer-Aided Engineering/Computer-Aided Manufacturing).因此,熟练掌握多种工程设计分析的计算机软件的基本原理和使用方法已成为对一个现代工程技术人员的基本要求.

有限单元法(FEM)是计算机辅助工程(CAE)中最重要和使用最广泛的一种数字模拟方法.随着各种英文版有限元软件的引进和在工程中的广泛应用,迅速了解有限单元法的基本原理、使用方法和使用范围,熟悉相关英文术语以便更好地使用这些软件已成为许多工科大学生和工程技术人员的迫切需要.虽然有关有限单元法的中文和英文教材已有很多,但这些教材大多偏重于理论推导和程序编写,因而不便于多数偏重于工程应用的工科大学生和工程技术人员作为入门教材.本教材试图填补这一空白以满足工科大学生和工程技术人员使用英文教材学习有限单元法和相关软件的需求.

本教材作者已在美国辛辛那提大学(The University of Cincinnati)使用了 12 年,经过多次修改和补充,并被世界上多所大学采用.本教材侧重于通过分析简单力学问题讲述有限单元法的基本原理和适用范围,及使用有限元软件时应注意的问题.教材中增添了许多有限元软件在工程分析中的应用实例.教师在使用本教材时可适当增加一些使用有限元软件的小型课题以增加练习材料的深度和广度.作者网页上列有一些辛辛那提大学本科生所做的有限元应用课题可供参考(http://urbana. mie. uc. edu/yliu/Showcase_FEA/showcase_fea. htm).

本教材作者从事计算力学的研究、教学和工程应用已有二十余年.作者在中国西北工业大学飞机结构强度专业获得本科和硕士学位,在美国

伊里诺伊大学香滨分校(UIUC)理论和应用力学系(TAM)获得博士学位. 毕业后，曾在美国依阿华大学(ISU)做博士后研究和福特汽车公司做CAE工程师. 在2003年—2005年，曾获日本国家科学促进委员会(JSPS)高级学者奖励在京都大学做访问教授和中国教育部春晖计划奖励在清华大学做访问教授. 现为多个国际会议和学术分会组织人，两个国际杂志编委，在国际专业刊物和会议论文集上共发表有关计算力学方面的论文七十余篇，并撰写有两本专著.

希望通过出版本教材能帮助更多的工科大学生和工程技术人员尽快了解有限单元法的基本原理和使用范围，并熟练掌握合理使用各种有限元软件的技能.

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Preface

This is a basic introduction to the finite element method (FEM) for undergraduate students in engineering or other readers who have no previous experience with the FEM. The textbook cover the basic concepts in the FEM using the simplest mechanics problems as examples, and lead to the discussions and applications of the one-dimensional (1-D) bar and beam, 2-D plane stress and plane strain, plate and shell, and 3-D solid elements in the analyses of structural stresses, vibrations and dynamics. The understanding of the FEM principles and procedures, and correct usage of the FEM software are emphasized throughout the book.

The textbook have been developed by the author for the undergraduate courses on the FEM in the Mechanical Engineering Department at the University of Cincinnati since 1997. The materials in these notes are aimed for students in mechanical, civil, and aerospace engineering, who need a general background in the FEM so that they can apply the FEM to the design and analysis of components, structures or systems using available commercial FEM software. For students who will conduct research on the FEM, the textbook should only serve as an introduction, and they should consult to the references listed at the end of the book for more rigorous treatment of the subject in order to have the necessary theoretical background and programming skills in developing new capabilities in the FEM.

The textbook include eight chapters and can be used for an undergraduate FEM course in one semester (15 weeks with three one-hour lectures each week) or in two quarters (20 weeks with three one-hour lectures each week, with four or five accompanying computer laboratory sessions each quarter). Chapter 1 gives a basic introduction to the concepts of the FEM using the spring system as examples. It also reviews the matrix algebra that is essential for the FEM. Chapter 2 introduces the bar and beam elements and outlines the general procedures in the formulations and application of the FEM. Chapter 3 covers 2-D problems in elasticity, that is, plane stress and plane strain elements. Chapter 4 discusses various modeling techniques in the FEM and related topics, such as error indicators and how to evaluate the FEM results. Chapter 5 gives an introduction to the plate and shell elements, emphasizing the correct use of these types of elements. Chapter 6 provides the formulations and applications of the FEM in general 3-D elasticity problems. Chapter 7 is an introduction of the FEM in structural vibration and dynamics analysis, covering normal modes, harmonic and transient responses of structures using the FEM. Chapter 8 covers the basics in thermal analysis of structures using the FEM. Exercise problems and/or projects using FEM software packages are provided at the end of each chapter. Further readings are provided in References to conclude the textbook.

The author thanks many of his former undergraduate and graduate students at the University of Cincinnati for their suggestions on the earlier versions of the textbook and for their contributions to many of the examples used in the textbook.

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Contents

Chapter 1 Introduction

1.1 Some Basic Concepts

1. A Simple Idea

The *finite element method* (FEM), or *finite element analysis* (FEA), is based on the idea of building a complicated object with simple blocks, or, dividing a complicated object into small and manageable pieces. Application of this simple idea can be found everywhere in everyday life (see, e.g., Fig. 1.1), as well as in engineering. For example, children play the toy Lego by using many small pieces of simple geometries to build various objects such as trains, ships or buildings. With more and smaller pieces, these objects often look more realistic. As another example, a digital image, which looks smooth and colorful, is in fact composed of millions of dots that just have one simple color.

(a) A fire engine built with Lego

(b) A house built with many elements–bricks, beams, columns, panels and so on

Figure 1.1 Objects built with simple and small pieces

In mathematical terms, this is simply the use of the limit concept, that is, to approach or represent a smooth object with finite number of simple pieces and increasing the number of such pieces will increase the accuracy of this representation. For example, ancient people used this concept to estimate the area of a circle well before the formula $A=\pi R^2$ was established (where R is the radius of the circle). In doing so, a circle is approximated by a polygon or divided into many triangles (Fig. 1. 2). The area of one triangle is given by

$$S_i = \frac{1}{2}R_iL_i,$$

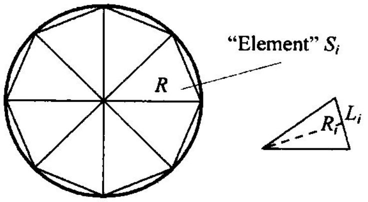


Figure 1. 2 Approximation of the area of a circle using a finite number of triangles

where R_i is the height and L_i the base length of the triangle. The area A of the circle can be therefore obtained in the following manner:

$$S_N = \sum_{i=1}^{N} S_i = N\left(\frac{1}{2}R_iL_i\right) \rightarrow \frac{1}{2}RL_{\text{total}} = \pi R^2 = A, \quad as\ N \rightarrow \infty,$$

where N is the total number of triangles (or *elements*) and $L_{\text{total}}=NL_i$ is, in the limit, the circumference of the circle, which is $2\pi R$ as it is known now.

From the above mentioned examples, one can conclude that objects with complicated geometries can be represented by many small pieces (or *elements*) with simple geometries. As the number of such pieces increases, the representation becomes more accurate. This is exactly the same concept used in the development of the FEM as one will see in later chapters.

2. Why To Adopt the Finite Element Method?

Computers have revolutionized the practice of engineering. Design of a product that used to be done by tedious hand drawings has been replaced by computer-aided design (CAD) using computer graphics. Analysis of a design used to be done by hand calculations and many of the testing have been replaced by computer simulations using computer-aided engineering (CAE) software. Together, CAD, CAE, and CAM (computer-aided manufacturing) have dramatically changed the landscape of engineering. For example, a car, that used to take five to six years from design to product, can now be produced starting from the concept design to the manufacturing within about 18 months using the CAD/CAE/CAM technologies. A company without adopting the CAD/CAE technologies is deemed to lose ground in the competitive market. FEM is the most widely applied simulation tool in CAE or one of the most powerful calculators available for engineering students.

3. Applications of the FEM in Engineering

There are numerous applications of the FEM in industries today and below is only a very short list:

- Mechanical/Aerospace/Automobile/Civil/Electrical Engineering
- Structure stress and dynamic analysis (Fig. 1.3)
- Thermal/Fluid flows
- Electrostatics/Electromagnetics
- Geomechanics
- Biomechanics

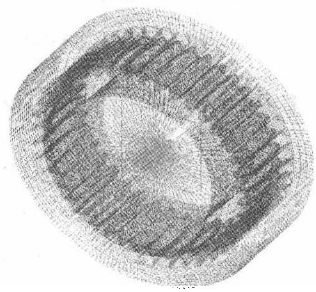
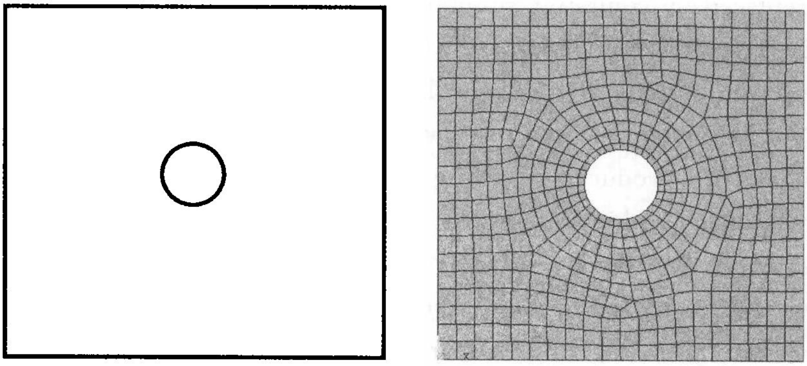


Figure 1.3 Modeling of gear coupling using the FEM

4. FEM in Structural Analysis (The Procedure in Using the FEM)

- Divide structure into pieces (elements with nodes, Fig. 1.4)
- Describe the behavior of the physical quantities on each element
- Connect (assemble) the elements at the nodes to form an approximate system of equations for the whole structure
- Solve the system of equations involving unknown quantities at the nodes (for example, the displacements)



(a) A plate with a hole (b) An FEM discretization (mesh)

Figure 1.4 Divide structure into pieces

- Calculate desired quantities (for example, strains and stresses) at selected elements

5. Computer Implementations

A typical FEM software has the following three key components:

- Preprocessor (used to build FE models, apply loads and constraints)
- FEA solver (assemble and solve the FEM system of equations)
- Postprocessor (sort and display the results)

The computer graphical-user interface (GUI) of the ANSYS software is shown in Fig. 1.5. Other FEM packages have similar interfaces.

6. Available Commercial FEM Software Packages

There are many commercial FEM packages, or CAD/CAE software in general, available for conducting FEA in nearly all fields of engineering. The following is only a short list:

- ANSYS (General purpose, PCs, and workstations)
- UG/NX (Complete CAD/CAM/CAE package)
- NASTRAN (General purpose FEA on mainframes and PCs)
- ABAQUS (Nonlinear and dynamic analyses)

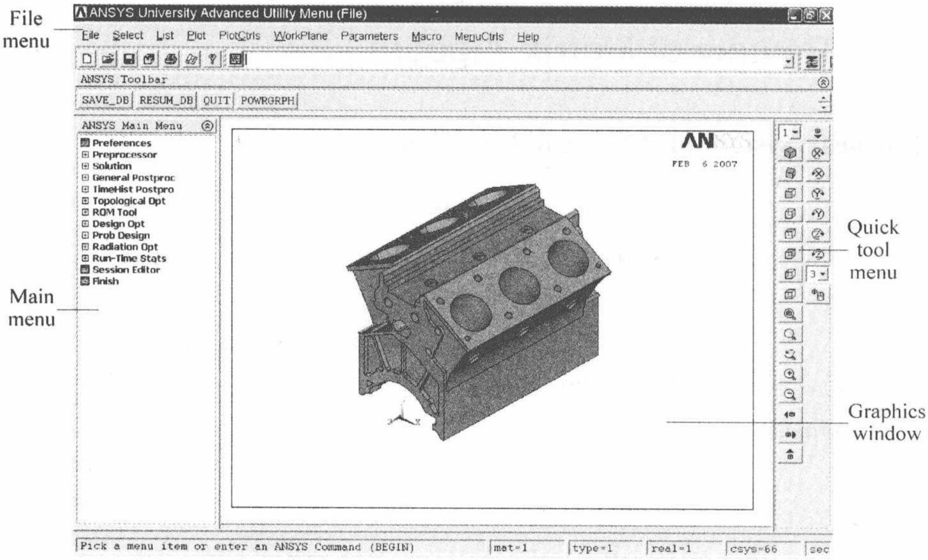


Figure 1.5 GUI of the ANSYS software

- COSMOS (General purpose FEA)
- ALGOR (PCs and workstations)
- PATRAN (Pre/Post processor)
- HyperWorks/HyperMesh (Pre/Post processor)
- Dyna-3D (Crash/Impact analysis)
- Others

7. Objectives of This Course

- Understand the fundamental ideas of the FEM;
- Know the behavior and usage of each type of elements covered in this course;
- Be able to prepare a suitable FE model for a given problem;
- Can interpret and evaluate the quality of the results (know the physics of the problems);
- Be aware of the limitations of the FEM (do not misuse the FEM).

1.2 Review of Matrix Algebra

1. Linear System of Algebraic Equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned} \tag{1.1}$$

where $x_1, x_2, \cdots, x_n$ are the unknowns.

In *matrix form*:

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{1.2}$$

where

$$\mathbf{A} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad \mathbf{x} = \{x_i\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}, \quad \mathbf{b} = \{b_i\} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}. \tag{1.3}$$

$\mathbf{A}$ is called an $n \times n$ (square) matrix, and $\mathbf{x}$ and $\mathbf{b}$ are (column) vectors with dimension n.

2. Row and Column Vectors

$$\mathbf{v} = [v_1 \quad v_2 \quad v_3], \quad \mathbf{w} = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}. \tag{1.4}$$

3. Matrix Addition and Subtraction

For two matrices $\mathbf{A}$ and $\mathbf{B}$, both of the *same size* $(m \times n)$, the addition and subtraction are defined by

$$\begin{aligned} \mathbf{C} &= \mathbf{A} + \mathbf{B}, \quad \text{with} \quad c_{ij} = a_{ij} + b_{ij} \\ \mathbf{D} &= \mathbf{A} - \mathbf{B}, \quad \text{with} \quad d_{ij} = a_{ij} - b_{ij}. \end{aligned} \tag{1.5}$$