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COLLECTED WORKS OF H.S.TSIEN

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Contents

Boundary Layer in Compressible Fluids

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Presented at the Aerodynamics Session, Sixth Annual Meeting, I. Ae. S. January 26, 1938

Summary

The first part of the paper is concerned with the theory of the laminar boundary layer in compressible fluids. The known solution for incompressible fluids is extended to large Mach's numbers by successive approximation. The compressibility effect on surface friction is discussed, and the results applied to estimate the ratio between wave resistance and frictional drag of projectiles and rockets. In the second part the heat transfer between a hot fluid and a cool surface, then between a hot body and a cool fluid is discussed. A general relation between drag and heat transfer as function of Mach's number is given. The limits where cooling becomes illusory because of the heat produced by friction are determined.

THE solution of flow problems in which the density is variable is in general very difficult; hence, every case in which an exact or even an approximate solution of the equations of the motion of compressible fluids can be obtained has considerable theoretical interest. Several authors noticed that the theory of the laminar boundary layer can be extended to the case of compressible fluids moving with arbitrarily high velocities without encountering insurmountable mathematical difficulties. Busemann[1] established the equations and calculated the velocity profile for one speed ratio. (By speed ratio is understood the ratio of the airspeed to the velocity of sound.) Frankl[2] also made an analysis of the same problem, however, it is complicated and depends on several arbitrary approximations. The senior author[3] obtained a first approximation by a simple but apparently not sufficiently exact calculation. Hence, in the first part of the present paper, a better method for the solution of the problem is developed.

The boundary layer theory for very high velocities is not without practical interest. First, the statement can be found often in technical and semi-technical literature on rockets and similar high-speed devices that the skin friction becomes more and more insignificant at high speeds. Of course, it is known that with increasing Reynolds Number, the skin friction coefficient is decreasing, *i.e.*, the skin friction becomes relatively small in comparison with the drag produced by wave formation or direct shock. Since high-speed flight will be performed mostly at high altitude where the air is of very low density, so that the kinematic viscosity is large, the resulting Reynolds Number will be relatively small in spite of the high speed.

Another interesting point in the theory of the boundary layer in compressible fluids is the thermodynamic aspect of the problem. In the case of low speeds the influence of the heat produced in the boundary layer can be neglected both in the calculation of the drag and of the heat transferred to the wall. In the case of high speeds, however, the heat produced in the boundary layer is not negligible, but determines the direction of heat flow. In the second part of the paper a few simple examples of heat flow through the boundary layer are discussed.

It has been found necessary in most parts of this analysis to make the assumption of laminar flow. This assumption was found necessary because of the lamentable state of knowledge concerning the laws of turbulent flow of compressible fluids at high speeds. This assumption is somewhat justified by the fact that—as mentioned above—in many problems where the results of this paper can be applied, the Reynolds Number is relatively small, so that a considerable portion of the boundary layer is probably, *de facto*, laminar. Ackeret[4] called attention to the possibility that the stability conditions in supersonic flow might be quite different from those occurring in flow with low velocities. The authors also believe that the stability criteria as developed by Tollmien and others, cannot be applied without modification. Finally, some conclusions of the paper, as will be pointed out, are also applicable to turbulent flow. In other cases, as in the calculation of drag, the assumption of laminar flow surely gives at least the lower limit of its value.

I

If the x-axis is taken along the plate in the direction of the free stream, the y-axis perpendicular to the plate, and u and v indicate the x and y components of the velocity at any point, then the simplified equation of motion in the boundary layer is

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \qquad (1)$$

where both the density ρ and the viscosity μ are variables.

The equation of continuity in this case is

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad (2)$$

A third equation determines the energy balance between the heat produced by viscous dissipation and the heat transferred by conduction and convection. With the same simplification as used in Eqs. (1) and (2), one can write

$$\rho u \frac{\partial}{\partial x}(c_p T) + \rho v \frac{\partial}{\partial y}(c_p T) = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \mu \left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where c_p is the specific heat at constant pressure, and λ is the coefficient of heat conduction. If Prandtl's number, $c_p\mu/\lambda$ is assumed to be equal to 1, then it can be easily shown that both Eqs. (1) and (3) can be satisfied by equating the temperature T to a certain parabolic function of the velocity u only. This relation between T and u is

$$\frac{T}{T_0} = \frac{T_w}{T_0} - \left(\frac{T_w}{T_0} - 1\right)\frac{u}{U} + \frac{\kappa - 1}{2} M^2 \frac{u}{U}\left(1 - \frac{u}{U}\right) \tag{4}$$

where

U = free stream velocity.
M = speed ratio, or Mach's number of the free stream.
T_0 = temperature of the free stream.
T_w = temperature at the wall of the plate.

Differentiating Eq. (4) one obtains

$$\frac{1}{T_0}\left(\frac{\partial T}{\partial y}\right)_w = \frac{1}{U}\left[\frac{\kappa - 1}{2} M^2 - \left(\frac{T_w}{T_0} - 1\right)\right]\left(\frac{\partial u}{\partial y}\right)_w \tag{5}$$

where the subscript w refers to conditions existing at the surface of the plate. Now $(\partial u/\partial y)_w$ is always positive; therefore, if $[(\kappa - 1)/2]M^2 > (T_w/T_0) - 1$ heat is transferred from the fluid to the wall, if $[(\kappa - 1)/2]M^2 = (T_w/T_0) - 1$ there is no heat transfer between the fluid and the wall, and if $[(\kappa - 1)/2]M^2 < (T_w/T_0) - 1$ heat is transferred from the wall to the fluid. If there is no heat transfer, the energy content per unit mass $(u^2/2) + c_pT$ is constant in the whole region of the boundary layer.[5,6]

The pressure being constant the relation between ρ and T is,

$$\rho = \rho_0 \frac{T_0}{T} \tag{6}$$

The expression for the viscosity based on the kinetic theory of gases is

$$\mu = \mu_0(T/T_0)^{1} \tag{7}$$

However, the following formula is in closer agreement with experimental data

$$\mu = \mu_0(T/T_0)^{0.76} \tag{7a}$$

Busemann[1] calculated the limiting case for which $[(\kappa - 1)/2]M^2 = (T_w/T_0) - 1$ using Eq. (7) and found

TABLE 1

M	0	1	2	5	10	∞
Θ	1.16	1.20	1.25	1.39	1.50	1.57

that for a high Mach's number, the velocity profile is approximately linear. The senior author,[3] using this linear velocity profile, the integral relation between the friction and the momentum, and Eq. (7) found that

$$C_f = \frac{\textit{Frictional force per unit width of plate}}{(\rho_0 U^2/2) \times \text{length of plate}}$$

$$= \Theta \sqrt{\frac{\mu_r}{\rho_0 U x}}\left\{1 + \frac{\kappa - 1}{2} M^2\right\}^{-1/4} \tag{8}$$

The dimensionless quantity Θ shown in Table 1 is a function of Mach's number only.

However, if Eq. (7a) is used, then

$$C_f = \Theta \sqrt{\frac{\mu_0}{\rho_0 U x}}\left\{1 + \frac{\kappa - 1}{2} M^2\right\}^{-0.12} \tag{8a}$$

It is evident that this linear approximation is not satisfactory for small values of Mach's number. For $M = 0$, the case is the same as the Blasius solution[7] for incompressible fluids for which Θ is 1.328.

To solve the problem more rigorously, one has to resort to Eqs. (1) and (2). By introducing the stream function ψ which is defined by

$$\frac{\rho}{\rho_0} u = \frac{\partial \psi}{\partial y}, \quad -\frac{\rho}{\rho_0} v = \frac{\partial \psi}{\partial x}$$

the equation of continuity, Eq. (2), is satisfied automatically. Now, if in Eq. (1) ψ is introduced as the independent variable as was done by von Mises[8] in his simplification of the boundary layer equation for incompressible fluids, and all terms are redused to non-dimensional form then

$$\frac{\partial u^*}{\partial n^*} = \frac{\partial}{\partial \psi^*}\left(u^*\rho^*\mu^* \frac{\partial u^*}{\partial \psi^*}\right) \tag{9}$$

where

$$\begin{aligned} u^* &= u/U \\ n^* &= n/L \\ \psi^* &= (\psi/UL)\sqrt{\rho_0 UL/\mu_0} \\ \rho^* &= \rho/\rho_0 \\ \mu^* &= \mu/\mu_0 \end{aligned} \tag{9a}$$

and L is a convenient length, say length of the plate.

Eq. (9) can be further simplified by introducing a new independent variable $\zeta = \psi^*/\sqrt{n^*}$,

then

$$-\frac{\zeta}{2}\frac{du^*}{d\zeta} = \frac{d}{d\zeta}\left(u^*\rho^*\mu^* \frac{du^*}{d\zeta}\right) \tag{10}$$

This can be solved by the method of successive approximations. As ρ^* and μ^* are functions of tempera-

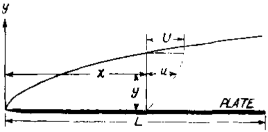

Fig. 1

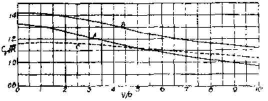

Fig. 2. Skin friction coefficients. (a) No heat transferred to wall. (b) Wall temperature $^1/_4$ of free stream temperature. (c) von Kármán's first approximation.

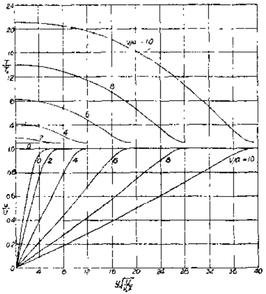

Fig. 3. Velocity and temperature distribution when no heat is transferred to wall

ture only as shown in Eqs. (6) and (7) or (7a) and the temperature is a function of u^* then by starting with the known Blasius' solution[6] the right-hand side of Eq. (10) can be expressed in terms of ζ. Therefore, one can write

$$u^*\rho^*\mu^* = f(\zeta)$$

Consequently, the solution of Eq. (10) is

$$u^* = C\int_0^\zeta \frac{F}{f}\,d\zeta \qquad (11)$$

where

$$F = exp\left(-\int_0^\zeta \frac{\zeta d\zeta}{f}\right)$$

and C is determined by the boundary condition,

$$\frac{1}{C} = \int_0^\infty \frac{F}{f}\,d\zeta \qquad (11a)$$

A second approximation can be made based upon the value of u^* obtained from Eq. (11). It has been found in the cases investigated that the third or fourth approximation gives sufficient accuracy.

Having computed the final u^*, the y corresponding to u^* can be calculated from

$$y\sqrt{U\rho_0/\mu_0 x} = \int_0^\zeta d\zeta/\rho^* u^* \qquad (12)$$

Also the skin friction can be computed by the momentum law,

$$C_f = \frac{F}{\frac{\rho_0}{2}U^2L} = \frac{2\int_0^\infty (1-u^*)d\zeta}{\sqrt{R}} \qquad (13)$$

The velocity profile, the temperature distribution, and the frictional drag coefficient are calculated for different values of the Mach's number of the free stream, for the case $[(\kappa-1)/2]M^2 = (T_w/T_0) - 1$ using the approximate viscosity relation of Eq. (7a). The results are shown in Figs. 2 and 3. The velocity profiles for high speeds are very nearly linear, but it can be seen that the wall temperature for greater Mach's numbers is very high. If the free stream temperature is 40°F., then the wall temperature will be 1600°F., 3620°F., 6540°F., and 10,170°F. for Mach's numbers of 4, 6, 8, and 10, respectively. Therefore, there is no doubt that the law of viscosity as expressed by Eq. (7a) will not hold. Also at such high temperatures, the heat transfer due to radiation cannot be neglected. Therefore, the results for extreme Mach's numbers are qualitative only.

The change in the constant $C_f\sqrt{R}$ is appreciable, but not great. It decreases from 1.328 for $M = 0$ to 0.975 for $M = 10$, or about 30 percent. However, for $0 < M < 3$ the change of the constant is very small.

Fig. 2 also shows that Eq. (8a) which was obtained by using the linear approximation is fairly accurate for very high Mach's numbers

As examples, consider first a projectile and second, a wingless sounding rocket. Taking the diameter of the projectile to be 6 in., the length 24 in., the velocity 1500 ft./sec. and the altitude 32,800 ft. (10 km.), then the Reynolds Number based on the total length is 7.86×10^6 and the speed ratio is 1.52. From Fig. 2 the skin friction coefficient is

$$C_f = (1.286 \times 10^{-3})/\sqrt{7.86} = 0.000459.$$

Changing the skin friction coefficient (based on the skin area) to the drag coefficient (based on the maximum cross-section), one obtains

$$C_{D_f} = 0.0055.$$

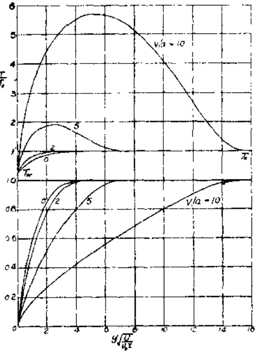

FIG. 4. Velocity and temperature distribution when the wall temperature is 1/4 of the free stream temperature.

The drag coefficient due to wave formation taken from Kent's experiments[4] is

$$C_{D_w} = 0.190.$$

Therefore, the ratio of skin friction to wave resistance is $0.0055/0.190 = 0.029$.

However, the ratio is greatly changed in the case of the rocket. Taking the diameter of the rocket to be 9 in., the length 8 ft., and the altitude of flight 50 km.* (164,000 ft.), the velocity 3400 ft./sec., then the Reynolds Number based on a density ratio at that altitude of 0.00067 and temperature 25°C. (deduced from data on meteors) is 6.14×10^4, and the speed ratio is 3.00. From Fig. 2, the skin friction coefficient is

$$C_f = (1.213 \times 10^{-2})/\sqrt{11.4} = 0.00360.$$

Then

$$C_{D_f} = 0.123$$

* The hydrodynamic equation holds so long as the mean free path of the molecules is small in comparison with the thickness of the boundary layer. For this case the thickness of the boundary layer is zero at the nose, however, at a distance ... of the length of the rocket it already amounts to 3.2 cm., while the calculated mean free path of the air molecule at the altitude considered is about 1.1×10^{-3} cm. Hence it appears that even for this case the theory can be safely applied. This conclusion is substantiated by the experimental results of H. Ebert in "*Darstellung der Strömungsvorgänge von Gasen bei neidrigen Drucken mittels Reynoldsscher Zahlen*," Zeitschrift für Physik, Bd. 85, S. 561-564, 1933.

The drag coefficient due to wave formation from Kent's experiments[4] is

$$C_{D_w} = 0.100$$

Therefore, the ratio of skin friction and wave resistance is now $0.123/0.100 = 1.23$. If the boundary layer is partly turbulent, the ratio will be even greater. This shows clearly the importance of skin friction in the case of a slender body moving with high speed in extremely rarefied air. It also disproves the belief that wave resistance would always be the predominating part in the total drag of a body moving with a velocity higher than that of sound. The reason underlying this fact can be easily understood when one recalls that the wave resistance of a body is approximately directly proportional to the velocity, while the skin friction is proportional to the velocity raised to a power between 1.5 and 2. Therefore, the ratio of skin friction to wave resistance increases with the speed. With very high velocities and high kinematic viscosity, the wave resistance may even be a negligible portion of the total drag of the body.

II

In order to point out the thermodynamic aspect of the problem two cases will be considered: the flow of a hot fluid along a surface which is kept at a constant temperature inferior to that of the fluid, and the case of a hot wall cooled by a fluid of lower temperature. The problems treated in this part have been discussed before in two very interesting papers by L. Crocco.[5,6] He especially gives an elegant treatment of the cooling problem in the case of very high velocities ("Hyperaviation"). The authors feel that their treatment is somewhat more general and extended than Crocco's previous analysis.

An interesting general relation between the heat transferred through the wall and the frictional drag can be obtained using the assumption that Prandtl's number, *i.e.*, the ratio $c_p\mu/\lambda$, is equal to unity. The same assumption was used also in the previous calculations. It is remarkable that the relation holds also as well for laminar as for turbulent flow. The heat flow q per unit time and unit area of the wall surface is

$$q = \lambda_w(\partial T/\partial y)_w$$

and the frictional drag τ per unit area is

$$\tau = \mu_w(\partial u/\partial y)_w$$

Using Eq. (4) the ratio q/τ can be calculated from the relation

$$\frac{q}{\tau} = \frac{\lambda_w}{\mu_w}\frac{T_0}{U}\left[\left(1 - \frac{T_w}{T_0}\right) + \frac{\kappa - 1}{2}M^2\right] \quad (14)$$

where T_0 is the absolute temperature, and U the velocity of the fluid in the free stream, T_w the absolute temperature at the wall, λ_w and μ_w are the heat conduction and