

英语数学

王亨时 编

MATHEMATICS
IN ENGLISH

上海交通大学出版社

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MATHEMATICS IN ENGLISH

A PREPARATORY COURSE IN CALCULUS

**By
Wang Hengshi**

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Preface

High school graduates in China nowadays are not familiar with English mathematical terminology. This book is written to prepare them for a course of college calculus offered in English.

Selections are made from high school and college mathematics textbooks compiled abroad. Emphasis has been laid on inequalities, the remainder theorem, partial fractions, the binomial theorem, trigonometric functions and transformations, the straight line, conic sections, parametric equations, polar coordinates, etc., as fundamental to a higher course.

The illustrative material is designed to suit the varying mental types to be found in even a small class. Copious and varied exercises are provided at the end of each chapter in the text and answers to problems are seen in the appendix.

Wang Hengshi

November 1984

Contents

I. Elementary Knowledge	1
1. Some mathematical symbols and notations	1
2. Fractions.....	6
3. Decimals	6
4. Some geometric figures and their terminology... ..	7
II. Inequalities	16
1. Intervals; Absolute value	16
2. Definition.....	17
3. Inequalities in two variables	22
4. Absolute inequalities	27
5. Applications of inequalities—Maxima and minima	32
III. The Remainder Theorem; Synthetic Division	40
1. The remainder theorem	40
2. Synthetic division	42
IV. Indeterminate Coefficients; Partial Fractions	48
1. Indeterminate coefficients	48
2. Partial fractions	53
V. Mathematical Induction; Binomial Theorem	61
1. Mathematical induction	61
2. Binomial theorem.....	64
VI. Trigonometric Functions of Compound and Multiple	

Angles	69
1. Functions of compound angles	69
2. Functions of multiple angles	72
VII. Transformations of Sums and Products	79
1. Transformation of sums or differences into products	79
2. Transformation of products into sums or differences	83
VIII. The Straight Line	83
1. Coordinates	88
2. Directed line segments	89
3. Intercepts	90
4. Symmetry	90
5. Distance formula	91
6. Division of a line at a given ratio	93
7. Area of a triangle	97
8. Inclination and slope	100
9. Necessary and sufficient condition for two straight lines to be parallel or perpendicular	101
10. Equation of a straight line	102
11. To transform the equation of a straight line from general form into normal form	109
12. Distance from a point to a straight line	109
13. Angular bisector	112
14. Angle between two straight lines	114
15. Concurrent lines	116
IX. Conic Sections	121

1. The circle	122
2. The ellipse	129
3. The hyperbola	132
4. The parabola	137
5. Directrices of an ellipse and a hyperbola	140
6. Tangents to conic sections	141
X. Parametric Equations	152
1. To transform parametric equations into equations involving x and y	152
2. Parametric equations of curves	153
3. Graphs of parametric equations	160
XI. Polar Coordinates	163
1. Polar coordinates	163
2. Connection between Cartesian and polar coordinate systems	164
3. Polar equations of curves	164
4. Sketching graphs of curves	172
Appendix I	179
Answers to Problems	179
Appendix II	188
Vocabulary	188

Chapter I

Elementary Knowledge

1. Some mathematical symbols and notations

(1) Four fundamental operations

i) Addition

The expression " $a + b$ " is read " a plus b ".

Here " a " is called the summand and " b " the addend.

ii) Subtraction

The expression " $a - b$ " is read " a minus b ".

Here " a " is called the minuend and " b " the subtrahend.

iii) Multiplication

The expression " $a \times b$ " is read " a times b " or " a multiplied by b ".

Here " a " is called the multiplicand and " b " the multiplier.

iv) Division

The expression " $a \div b$ " is read " a divided by b ".

Here " a " is called the dividend and " b " the divisor.

The results of addition, subtraction, multiplication and division are called the sum, the difference, the product and the quotient respectively.

(2) Signs of equality and inequality

" $a = b$ " is read " a is equal to b " or " a equals b ".

" $a \equiv b$ " is read " a is identically equal to b ".

" $a \approx b$ " or " $a \doteq b$ " is read " a is approximately equal to b ".

" $a \neq b$ " is read " a is not equal to b ".

" $a > b$ " is read " a is greater(larger) than b ".

" $a < b$ " is read " a is smaller(less) than b ".

" $a \geq b$ " is read " a is larger than or equal to b ".

(3) Ratio and proportion

" $a:b$ " is read "the ratio of a to b ".

When two ratios are equal, e. g., " $a:b = c:d$ ", we say that a, b, c, d are in proportion. The expression " $a:b = c:d$ " is then read "the ratio of a to b is equal to the ratio of c to d ", or " a is to b as c is to d ".

(4) Involution and evolution

" a^2 " is read " a square (squared)" or " a (raised) to the second power" or " a (raised) to the power of two".

" a^3 " is read " a cube (cubed)" or " a (raised) to the third power" or " a (raised) to the power of three".

" a^n " is read " a (raised) to the n^{th} power" or " a (raised) to the power of n ".

Here " a " is called the base and " n " the index or power or exponent.

" \sqrt{a} " is read "the square root of a ".

" $\sqrt[3]{a}$ " is read "the cube root of a ".

" $\sqrt[n]{a}$ " is read "the n^{th} root of a ".

The symbol " $\sqrt{\quad}$ " is called the radical sign, and the expression under the radical sign is called the radicand.

(5) Logarithm and limit

" $\log a$ " is read "the (common) logarithm of a (to the base 10)".

" $\ln a$ " is read "the natural or Napierian logarithm of a (to the base e)", or "ell-en a ".

" $\lim f(x)$ " is read "the limit of f of x ".

" $\lim_{x \rightarrow a} f(x)$ " is read "the limit of f of x as x tends to a (or as x approaches a)".

" $\lim_{x \rightarrow a^+} f(x)$ " is read "the limit of f of x as x tends to a from the right".

" $\lim_{x \rightarrow a^-} f(x)$ " is read "the limit of f of x as x tends to a from the left".

(6) Set notation

" $a \in A$ " is read "the element a belongs to the Set A ", or " a is an element of A ".

" $A \subseteq B$ " is read " A is a subset of B ".

" $A \subset B$ " is read " A is a proper subset of B ".

" $A \cup B$ " is read " A union B ", or "the union of A and B ".

" $A \cap B$ " is read " A intersection B ", or "the intersection of A and B ".

" ϕ " is read "the empty set". This is a set which is devoid of members.

" \bar{A} " is read "the complement of A ".

This denotes a set of those elements which are not in the set A relative to a universal set I .

(7) Differentiation and integration

" $\frac{dy}{dx}$ " is read " dy by dx " or "the first derivative of y

wrt x ". (wrt = with respect to)

" $f'(x)$ " is read " f prime of x ".

This notation was introduced by the famous mathematician Lagrange (1736—1813 A.D.).

It is equivalent to the notation " $\frac{dy}{dx}$ ", which

was introduced by another great mathematician Leibniz (1646—1716 A.D.).

" $\frac{d^2y}{dx^2}$ " is read " d second y by dx second" or "the second

derivative of y wrt x ".

" $\int f(x)dx$ " is read "the integral of f of x , dx ". (indefinite integral)

" $\int_a^b f(x)dx$ " is read "the integral from a to b of f of x , dx ". (definite integral)

(8) Trigonometric functions

$\sin A$ sine of A

In a right triangle ABC , as shown in Fig. I-1,

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}.$$

$\cos A$ cosine of A

In Fig. I-1,

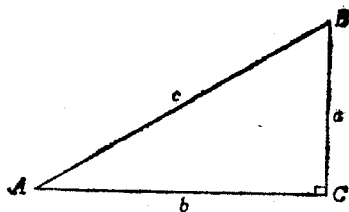


Fig. I-1

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}.$$

$\operatorname{tg} A$; $\tan A$ tangent of A

In Fig. I-1,

$$\operatorname{tg} A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}.$$

$\operatorname{ctg} A$; $\cot A$ cotangent of A

This is the reciprocal of $\operatorname{tg} A$.

$\operatorname{csc} A$; $\operatorname{cosec} A$ cosecant of A

This is the reciprocal of $\sin A$.

$\sec A$ secant of A

This is the reciprocal of $\cos A$.

(9) Some other symbols

a_1 a sub one.

a_n a sub n .

a' a prime.

a'' a double prime.

$|a|$ the absolute value of a .

$n!$ factorial n or n factorial.

() parentheses, open interval.

[] brackets, closed interval.

{ } braces; set notation; sequence notation.

Σ summation notation, which is read "sigma".

$\sum_{i=1}^n x_i$ sigma of x_i as i runs from 1 to n .

2. Fractions

(1) Proper fractions

$\frac{1}{2}$ one half, a half, one over two.

$\frac{1}{3}$ one third, a third, one over three.

$\frac{1}{4}$ one fourth; one quarter; one over four.

$\frac{2}{3}$ two-thirds; two over three.

$\frac{4}{5}$ four-fifths; four over five.

(2) Improper fractions

$\frac{10}{7}$ ten over seven.

(3) Mixed fractions

$4\frac{2}{3}$ four and two-thirds.

$2\frac{1}{2}$ two and one half.

3. Decimals

0.1 O point one; zero point one, nought (naught)

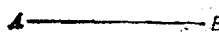
point one; point one.

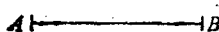
0.01 O point O one, zero point zero one, nought
point nought one, point O one.

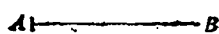
1.25 one point two five.

21.37 twenty-one point three seven.

4. Some geometric figures and their terminology

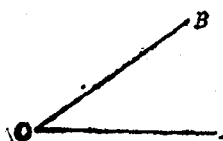
 the straight line AB .

 the line segment AB .

 the ray AB .

$AB \parallel CD$ AB is parallel to CD .

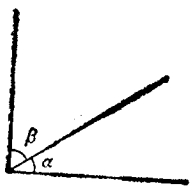
$AB \perp CD$ AB is perpendicular to CD .

 Angle AOB is formed by two rays OA and OB proceeding from the same point O . Angle AOB can also be formed by first taking a radius vector OA and then revolving it until it takes up the position OB .

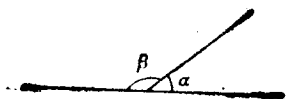
When OA is revolved in an anticlockwise direction, the angle traced is positive.

When OA is revolved in a clockwise direction, the angle traced is negative.

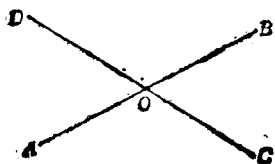
OA and OB are the two arms of $\angle AOB$, and OA is called the initial side and OB the terminate side.



When the sum of two adjacent angles α and β is a right angle, they are said to be complementary or one is the complement of the other.

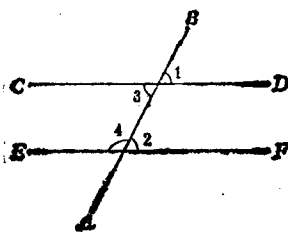


When the sum of two adjacent angles α and β (in the left figure, α is acute and β is obtuse) is a straight angle, they are said to be supplementary or one is the supplement of the other.

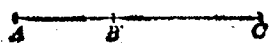


Two straight lines AB and CD intersect with each other at O , or we can say, AB intersects CD at O .

In this case, $\angle AOD$ and $\angle BOC$ are called vertical (opposite) angles and they are equal in magnitude. So are angles AOC and BOD .



AB intersects two parallel lines CD and EF . $\angle 1$ and $\angle 2$ are corresponding angles (equal in magnitude); $\angle 2$ and $\angle 3$ are alternate interior angles; $\angle 3$ and $\angle 4$ are interior angles on the same side (they are supplementary).



Three points A, B, C lie on the same straight line. They are said to be collinear. Three points are collinear if and only

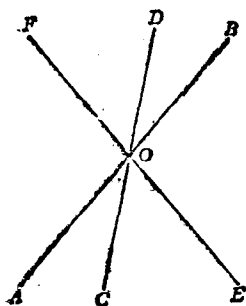
if the determinant

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0,$$

where (x_1, y_1) , (x_2, y_2) , (x_3, y_3) are the coordinates of the three points.

The term "necessary and sufficient" is frequently used as a synonym for "if and only if".

The term "if and only if" is sometimes abbreviated to "iff".



Three straight lines AB, CD, EF intersect at one point O. They are said to be concurrent.

If the equations of the three straight lines are

$$AB: A_1x + B_1y + C_1 = 0,$$

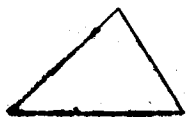
$$CD: A_2x + B_2y + C_2 = 0,$$

$$EF: A_3x + B_3y + C_3 = 0,$$

they are concurrent if and only if the determinant

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0,$$

provided no two of them are parallel.



When the three angles of a triangle are all acute, the triangle is called an acute triangle.