大学计算机教育丛书（影印版）

THIRD EDITION

DISCRETE MATHEMATICAL STRUCTURES

离散数学结构

(第三版)

BERNARD KOLMAN
ROBERT C. BUSBY
SHARON ROSS

清华大学出版社 · PRENTICE HALL

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出 版 前 言

我们的大学生、研究生毕业后，面临的将是一个国际化的信息时代。他们将需要随时查阅大量的外文资料；会有更多的机会参加国际性学术交流活动；接待外国学者；走上国际会议的讲坛。作为科技工作者，他们不仅应有与国外同行进行口头和书面交流的能力，更为重要的是，他们必须具备极强的查阅外文资料获取信息的能力。有鉴于此，在国家教委所颁布的"大学英语教学大纲"中有一条规定：专业阅读应作为必修课程开设。同时，在大纲中还规定了这门课程的学时和教学要求。有些高校除开设"专业阅读"课之外，还在某些专业课拟进行英语授课。但教、学双方都苦于没有一定数量的合适的英文原版教材作为教学参考书。为满足这方面的需要，我们挑选了 7 本计算机科学方面最新版本的教材，进行影印出版。首批影印出版的 6 本书受到广大读者的热情欢迎，我们深受鼓舞，今后还将陆续推出新书。希望读者继续给予大力支持。Prentice Hall 公司和清华大学出版社这次合作将国际先进水平的教材引入我国高等学校，为师生们提供了教学用书，相信会对高校教材改革产生积极的影响。

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List of Frequently Used Symbols

Chapter 1

a, b, c, x, y, z, t	elements of a set, p. 2
$\in$	belongs to, p. 2
Z^+	the set of all positive integers, p. 2
N	the set of all nonnegative integers, p. 2
Z	the set of all integers, p. 2
$\mathbf{R}$	the set of all real numbers, p. 2
$\varnothing$	the empty set, p. 2
$\subseteq$	is contained in, p. 3
U	the universal set, p. 3
$\|A\|$	the cardinality of A, p. 4
$P(A)$	the set of all subsets of A, p. 4
$A \cup B$	the union of sets A and B, p. 6
$A \cap B$	the intersection of sets A and B, p. 6
$A - B$	the complement of B with respect to A, p. 7
$\overline{A}$	the complement of A, p. 7
$A \oplus B$	the symmetric difference of sets A and B, p. 9
A^*	the set of all finite sequences of elements of A, p. 19
Λ	the empty sequence or string, p. 19
GCD(a, b)	the greatest common divisor of a and b, p. 24
LCM(a, b)	the least common multiple of a and b, p. 26
$\equiv r \pmod a$	congruent to r mod a, p. 27
$\mathbf{A}^T$	the transpose of the matrix $\mathbf{A}$, p. 34
$\mathbf{A} \vee \mathbf{B}$	the meet of $\mathbf{A}$ and $\mathbf{B}$, p. 35
$\mathbf{A} \wedge \mathbf{B}$	the join of $\mathbf{A}$ and $\mathbf{B}$, p. 35
$\mathbf{A} \odot \mathbf{B}$	the Boolean product of $\mathbf{A}$ and $\mathbf{B}$, p. 36

Chapter 2

$\sim p$	not p, p. 47
$p \wedge q$	p and q, p. 48
$p \vee q$	p or q, p. 48
$\forall$	for all, p. 50
$\exists$	there exists, p. 50
$p \rightarrow q$	p implies q, p. 52
$p \leftrightarrow q$	p is equivalent to q, p. 53

Chapter 3

${}_nP_r$	the number of permutations of n objects taken r at a time, p. 75
$n!$	n-factorial, p. 75
${}_nC_r$	the number of combinations of n objects taken r at a time, p. 78
$p(E)$	the probability of the event E, p. 87
f_E	the frequency of occurrence of event E, p. 88

Chapter 4

$A \times B$	the Cartesian product of A and B, p. 102
$R(x)$	the R-relative set of x, p. 109
$R(A)$	the R-relative set of A, p. 109
$\mathbf{M}_R$	the matrix of R, p. 111
R^∞	the connectivity relation of R, the transitivity closure of R, p. 117
R^*	the reachability relation of R, p. 121
Δ	the relation of equality, p. 124
$[a]$	the equivalence class of a, p. 134
A/R	a partition of set A determined by the equivalence relation R on A, p. 134
$S \circ R$	the composition of R and S, p. 152

Chapter 5

1_A	the identity function on A, p. 170
f^{-1}	the inverse of the function f, p. 173
f_A	the characteristic function of a set A, p. 177
$\lfloor x \rfloor$	the largest integer less than or equal to x, p. 178
$\lceil x \rceil$	the smallest integer greater than or equal to x, p. 178
$\begin{pmatrix} a_1 & a_2 & \dots & a_n \\ p(a_1) & p(a_2) & \dots & p(a_n) \end{pmatrix}$	permutation of the set $A = \{a_1, a_2, \dots, a_n\}$, p. 181
$O(f)$	the order of a function f, p. 190
$\Theta(f)$	the θ-class of a function f, p. 192

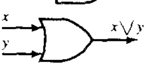
Chapter 6

(V, E, γ)	the graph with vertices in V and edges in E, p. 197
D_n	the discrete graph on n vertices, p. 200
K_n	the complete graph on n vertices, p. 200
L_n	the linear graph on n vertices, p. 200
G_e	the subgraph obtained by omitting e from G, p. 202
G^R	the quotient graph with respect to R, p. 202
$\chi(G)$	the chromatic number of G, p. 218
P_G	the chromatic polynomial of G, p. 220

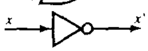
Chapter 7

$\leq$	a partial order relation, p. 226
$a \vee b$, LUB(a, b)	the least upper bound of a and b, p. 242
$a \wedge b$, GLB(a, b)	the greatest lower bound of a and b, p. 242
a'	the complement of a, p. 254

and gate, p. 269



or gate, p. 269



inverter (NOT), p. 269

Chapter 8

(T, v_0)	the tree with root v_0, p. 287
$T(v)$	the subtree of T with root v, p. 290

Chapter 9

S^S	the set of all functions from S to S, p. 335
S/R	the quotient semigroup of a semigroup S, p. 343
Z_n	the quotient set $Z/\equiv$ (mod n), p. 344
f_R	natural homomorphism of S onto S/R, p. 345
aH	a left coset of H in G, p. 363

Chapter 10

$(V, S, v_0, \mapsto)$	phrase structure grammar, p. 370
$\Rightarrow$	direct derivability, p. 370
$\langle\ \rangle ::=$	BNF specification of a grammar, p. 378
$L(G)$	the language of G, p. 371
$\mathcal{F}$	the set of transition functions of a finite-state machine, p. 391
$(S, I, \mathcal{F})$	a finite-state machine, p. 391
f_x	the transition function corresponding to input x, p. 391
$(S, I, \mathcal{F}, s_0, T)$	Moore machine, p. 393
M/R	quotient machine of machine M, p. 394
$l(w)$	the length of a string w, p. 376

Chapter 11

e	an (m, n) encoding function, p. 422
$\delta(x, y)$	the distance between the words x and y, p. 424
$\mathbf{A} \oplus \mathbf{B}$	the mod 2 sum of **A** and **B**, p. 426
$\mathbf{A} * \mathbf{B}$	the mod 2 Boolean product of **A** and **B**, p. 427
d	an (n, m) decoding function, p. 432
ϵ_j	a coset leader, p. 436
$x * \mathbf{H}$	the syndrome of x, p. 439

About the Authors

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Bernard Kolman received his B.S. (summa cum laude with honors in mathematics and physics) from Brooklyn College in 1954, his Sc.M. from Brown University in 1956, and his Ph.D. from the University of Pennsylvania in 1965, all in mathematics. During the summers of 1955 and 1956 he worked as a mathematician for the U.S. Navy, and IBM, respectively, in areas of numerical analysis and simulation. From 1957–1964, he was employed as a mathematician by the UNIVAC Division of Sperry Rand Corporation, working in the areas of operations research, numerical analysis, and discrete mathematics. He also had extensive experience as a consultant to industry in operations research. Since 1964, he has been a member of the Mathematics Department at Drexel University, where he also served as Acting Head of this department. Since 1964, his research activities have been in the areas of Lie algebras and operations research.

Professor Kolman is the author of numerous papers, primarily in Lie algebras, and has organized several conferences on Lie algebras. He is also well known as the author of many mathematics textbooks that are used worldwide and have been translated into several other languages. He belongs to a number of professional associations and is a member of Phi Beta Kappa, Pi Mu Epsilon, and Sigma Xi.

Robert C. Busby received his B.S. in Physics from Drexel University in 1963 and his A.M. in 1964 and Ph.D. in 1966, both in mathematics from the University of Pennsylvania. From September 1967 to May 1969 he was a member of the mathematics department at Oakland University in Rochester, Michigan. Since 1969 he has been a faculty member at Drexel University, in what is now the Department of Mathematics and Computer Science. He has consulted in applied mathematics in industry and government. This includes a period of three years as a consultant to the Office of Emergency Preparedness, Executive Office of the President, specializing in applications of mathematics to economic problems. He has had extensive experience developing computer implementations of a variety of mathematical applications.

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PREFACE

Discrete mathematics for computer science is a difficult course to teach and to study at the freshman and sophomore level for several reasons. It is a hybrid course. Its content is mathematics, but many of its applications, and more than half of its students, are from computer science. Thus careful motivation of topics and previews of applications are important and necessary strategies. Moreover, the number of substantive and diverse topics covered in the course is high, so the student must absorb these rather quickly.

Approach

First, we have limited both the areas covered and the depth of coverage to what we deemed prudent in a *first* course taught at the freshman and sophomore level. We have identified a set of topics that we feel are of genuine use in computer science and that can be presented in a logically coherent fashion. We have presented an introduction to these topics along with an indication of how they can be pursued in greater depth.

For example, we cover the simpler finite-state machines, not Turing machines. We have limited the coverage of abstract algebra to a discussion of semigroups and groups and have given applications of these to the important topics of finite-state machines and error-detecting and error-correcting codes. Error-correcting codes, in turn, have been primarily restricted to simple linear codes.

Second, the material has been organized and interrelated to minimize the mass of definitions and the abstraction of some of the theory. Relations and digraphs are treated as two aspects of the same fundamental mathematical idea, with a directed graph being a pictorial representation of a relation. This fundamental idea is then used as the basis of virtually all the concepts introduced in the book, including functions, partial orders, graphs, and algebraic structures. Whenever possible, each new idea introduced in the text uses previously encountered material and, in turn, is developed in such a way that it simplifies the more complex ideas that follow. Thus partial orders, lattices, and Boolean algebras develop from general relations. This material in turn leads naturally to other algebraic structures.

What Is New in the Third Edition

We have been very pleased by the warm reception given to the first two editions of this book. We have repeatedly been told that the book works well in the classroom because of the unifying role played by two key concepts: relations and digraphs. Thus we have not drastically interfered with the organization or flow of the material. We have added some more flexibility and modularity while continuing the centrality of relations and digraphs. In preparing this edition, we have incorporated many faculty and student suggestions. Although many changes have been made in this edition, our goal continues to be that of *maximizing the clarity of presentation.* To achieve this goal, the following features have been developed in this edition:

New Sections Have Been Added on

- Mathematical Structures (showing similarities and differences in the structure of sets and set operations, integers and integer arithmetic, and matrices and matrix operations).
- The predicate calculus.
- Recurrence relations.
- Functions for computer science.
- Growth of functions.
- Minimal spanning trees.

- A new chapter has been added on Graph Theory.
- Appendix B, Experiments in Discrete Mathematics, has been added.
- Coding exercises have been included in each chapter.
- More material on recursion has been included.
- More material on logic and methods of proof has been presented.

- The presentation on permutations and combinations has been expanded.
- More figures and illustrative examples have been prepared.
- The Exercise Sets have been revised. Many of the routine exercises have been kept, others of this type have been created, and more emphasis has been placed on exercises asking the student to explain and describe.

Exercises

The exercises form an integral part of the book. Many are computational in nature, whereas others are of a theoretical type. Many of the latter and the experiments, to be further described below, require verbal solutions. Answers to all odd-numbered exercises appear in the back of the book. Solutions to all exercises appear in the **Instructor's Manual**, which is available (to instructors only) gratis from the publisher. The Instructor's Manual also includes notes on the pedagogical ideas underlying each chapter, goals and grading guidelines for the experiments further described below, and a test bank.

Experiments

Appendix B contains a number of assignments that we call experiments. These provide an opportunity for discovery and exploration, or a more-in-depth look at various topics discussed in the text. These are suitable for group work. Content prerequisites for each experiment are given in the Instructor's Manual.

End of Chapter Material

Every chapter contains a summary of Key Ideas for Review and a set of Coding Exercises.

Content

Chapter 1 contains a miscellany of basic material required in the course. This includes sets, subsets, and their operations; sequences; division in the integers; and matrices. New to this edition is a section on Mathematical Structures, showing the similarities and differences among some of the concepts discussed earlier in the chapter. Chapter 2 covers logic and related material, including methods of proof and mathematical induction. It includes two sections that are new to this edition: Conditional Statements and Methods of Proof. Chapter 3, on counting, deals with permutations, combinations, the pigeonhole principle, elements of probability, and a new section on Recurrence Relations.

Chapter 4 presents basic types and properties of relations, along with their representation as directed graphs. Connections with matrices and other data structures are also explored in this chapter. Chapter 5 deals with the notion of a function and gives several important examples of functions, including permutations. New to this edition are sections on Functions for Computer Science and Growth of Functions. Chapter 6, new to this edition, provides an elementary introduction to some of the ideas and applications of graph theory. It gives additional flexibility and modularity to the text.

Chapter 7 covers partially ordered sets, including lattices and Boolean algebras. Chapter 8 introduces directed and undirected trees. New to this edition is a section on Minimal Spanning Trees. In Chapter 9 we give the basic theory of semigroups and groups. These ideas are applied in Chapters 10 and 11. Chapter 10 is devoted to finite-state machines. It complements and makes effective use of ideas developed in previous chapters. Chapter 11 treats the subject of binary coding.

Appendix A discusses Algorithms and Pseudocode. The simplified pseudocode presented here is used in some text examples and exercises; these may be omitted without loss of continuity. Appendix B gives a collection of experiments dealing with extensions or previews of topics in various parts of the course.

Use of This Text

This text can be used by students in mathematics as an introduction to the fundamental ideas of discrete mathematics, and as a foundation for the development of more advanced mathematical concepts. If used in this way, the topics dealing with specific computer science applications can be ignored or selected independently as important examples. The text can also be used in a computer science or computer engineering curriculum to present the foundations of many basic computer-related concepts, and provide a coherent development and common theme for these ideas. The instructor can easily develop a suitable course by referring to the chapter prerequisites, which identify material needed by that chapter.

Acknowledgments

We are pleased to express our thanks to the following reviewers of the first two editions: Harold Fredricksen, Naval Postgraduate School; Thomas E. Gerasch, George Mason University; Samuel J. Wiley, La Salle College; Kenneth B. Reid, Louisiana State University; Ron Sandstrom, Fort Hays State University; Richard H. Austing, University of Maryland; Nina Edelman, Temple University; Paul Gormley, Villanova University; Herman Gollwitzer and Loren N. Argabright, both at Drexel University; and Bill Sands, University of Calgary, who brought to our attention a number of errors in the second edition; and of the third edition: Moshe Dror, University of Arizona, Tucson; Lloyd Gavin, California State University at Sacramento; Robert H. Gilman, Stevens Institute of Technology; Earl E. Kymala, California State University at Sacramento; and Art Lew, University of Hawaii, Honolulu. The suggestions, comments and criticisms of these people greatly improved the manuscript.

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B. K.
R. C. B.
S. C. R.

CONTENTS

CHAPTER 1

FUNDAMENTALS

Prerequisites

There are no formal prerequisites for this chapter; the reader is encouraged to read carefully and work through all examples.

In this chapter we introduce some of the basic tools of discrete mathematics. We begin with sets, subsets, and their operations, notions with which you may already be familiar. Next we deal with sequences, using both explicit and recursive patterns. Then we review some of the basic divisibility properties of the integers. Finally, we introduce matrices and their operations. This gives us the background needed to begin our exploration of mathematical structures.

1.1. Sets and Subsets

Sets

A **set** is any well-defined collection of objects, called the **elements** or **members** of the set. For example, the collection of all wooden chairs, the collection of all one-