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Fuzzy Sets:

An Overview of Fundamentals, Applications, and Personal Views

George Klir

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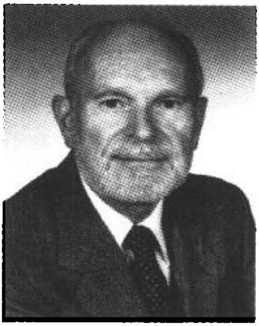
BIOGRAPHICAL SKETCH



GEORGE J. KLIR is currently a Distinguished Professor of Systems Science at the Department of Systems Science and Industrial Engineering, and Director of the Center for Intelligent Systems, Thomas J. Watson School of Engineering and Applied Science, State University of New York at Binghamton. He was born on April 22, 1932, in Prague, Czechoslovakia, immigrated to the USA in 1966, and became naturalized in 1972. He received the M. S. Degree (Summa Cum Laude) in Electrical Engineering from the Czech Technical University in Prague in 1957, and the Ph. D. degree in Computer Science from the Czechoslovak Academy of Sciences in 1964. He is also a graduate of the IBM System Research Institute in New York.

He began his professional career at the Computer Research Institute and Charles University in Prague. After immigrating to the United States, he held academic positions at the UCLA (1966—68) and Fairleigh Dickinson University in New Jersey (1968—69). Since 1969, he has been with the State University of New York at Binghamton, where he served as Chairman of the Department of Systems Science (1978—94) and Director of the Center for Intelligent Systems (since 1994). He has also worked part time for IBM, Sandia Laboratories, and Bell Laboratories, and taught summer courses at the University of Colorado, Portland State University in Oregon, and Rutgers University in New Jersey. During the academic years 1975—1976 and 1982—1983, he was a Fellow at the Netherlands Institute for Advanced Studies in Wassenaar, Netherlands, and in 1980 he was a Fellow of the Japan Society for the Promotion of Science. He is a Fellow of IEEE and IFSA.

During the earlier stages of his professional career, Dr. Klir conducted research in the areas of systems modeling and simulation, logic design, computer architecture, and discrete mathematics. His current research interests include the areas of intelligent systems, information theory, knowledge acquisition and discovery, fuzzy set theory, fuzzy measure theory, systems modeling, and soft computing, as well as some aspects of the philosophy of science. He has supervised 26 doctoral dissertations and has taught graduate courses on Fuzzy Systems, Generalized Information Theory, Systems Problem Solving, Discrete Mathematics, Logic Design and Computer Architecture, Fault—Tolerant Computing, Automata Theory, and Combinatorial Analysis. He is the author of over three hundred articles, holds a number of patents, and is an author or co—author of 16 books, among them Cybernetic Modelling (Van Nostrand, 1967), Methodology of Switching Circuits (Van Nostrand, 1972), Architecture of Systems Problem Solving (Plenum Press, 1985), Fuzzy Sets, Uncertainty, and Information (Prentice Hall, 1988), Facets of Systems Science (Plenum Press, 1991), Fuzzy Measure Theory (Plenum Press, 1992), Fuzzy Sets and Fuzzy Logic: Theory and Applications (Prentice Hall, 1995), and Uncertainty—Based Information (Springer—Verlag, 1998). He is also an editor of 9 books.

Dr. Klir has been Editor—in—Chief of the International Journal of General Systems since 1974 and Editor of the IFSR Book Series on Systems Science and Engineering since 1985. He is a member of Editorial Boards of 18 journals. He was President of the Society for General Systems Research in 1981—82, the first President of the International Federation for Systems Research (IFSR) in 1980—84, President of the North American Information

Processing Society (NAFIPS) in 1988 – 1991, and President of the International Fuzzy Systems Association (IFSA) in 1993 – 1995. He has received numerous awards and honors, including 3 honorary doctoral degrees, the Gold Medal of Bernard Bolzano in mathematical sciences from the Czech Academy of Sciences, Lotfi A. Zadeh Best Paper Award, Distinguished Leadership Award from the International Society for the Systems Sciences, Award for the Highest Achievement in Scholarship from the Simon Bolivar University in Caracas, Venezuela, and SUNY University Award for Excellence in Research. His biography is included in many biographical sources, including Who's Who in America, Who's Who in the World, American Men and Women of Science, Outstanding Educators of America, Contemporary Authors, etc. His research has been supported by grants from NSF, NATO, NASA and Sandia Laboratories.

PREFACE

Fuzzy set theory has lately become a subject of considerable attention. The growing visibility of the theory is a result of its highly successful practical applications, developed primarily in Japan since the late1980s. Among these applications, fuzzy controllers have been by far the most visible, particularly in connection with consumer products, and have overshadowed the role of fuzzy set theory in many other areas. This is somewhat unfortunate. In spite of their considerable success, fuzzy controllers represent, after all, only a very narrow application area of fuzzy set theory. Most applications of the theory, some of them quite profound, are in other areas. Included are areas such as decision making, clustering and pattern recognition, processing and understanding of images, diagnosis (medical, engineering, cognitive), risk analysis and reliability theory, engineering design, database and information retrieval systems, expert systems, business and management, economic theory, processing of natural language, and numerous other application areas.

Although known applications of fuzzy set theory in the mentioned areas are already quite impressive, the significance of the theory transcends these or any other specific applications. By opening a radically new way of thinking, thinking in terms of degrees of membership and truth rather than absolute membership and truth, fuzzy sets and fuzzy logic will eventually affect virtually all aspects of human affairs. This new way of thinking is substantially more attuned to our interaction with the real world than the traditional thinking restricted to sets with sharp boundaries and logic that recognizes only absolute truths and falsities. As Bernard Russell once concluded, classical logic "is not applicable to this terrestrial life, but only to an imagined celestial existence." In science, this change of thinking amounts to a major paradigm shift, too radical at first sight, which can only be initiated by someone who has tremendous courage, in addition to the requisite insight.

It was Lofti Zadeh, a Professor at the University of California at Berkeley, who had the insight as well as the courage to initiate in 1965 this radically new paradigm by introducing the concept of a fuzzy set, a set in which the membership is a matter of degree rather than a matter of either affirmation or denial. The reaction to this proposal was fairly typical of a paradigm shift. A few scholars greeted it with enthusiasm, but many expressed skepticism, and some were even openly hostile towards it. Nevertheless, the new idea persevered, matured significantly over the years by the work of growing number of its supporters, and began eventually to demonstrate its very impressive pragmatic utility.

At this time, fuzzy set theory is still quite young and rapidly developing.However, the established body of knowledge regarding the theory and its many applications is already quite large. Unless one has taken an active part in the development of this knowledge, it is rather difficult to comprehend it without any guidance. To provide such a guidance is the primarily purpose of this book.

As indicated by its subtitle, the book consists of three parts: I. Fundamentals; II. Applications; and III. Personal Views. Part I is a simple introduction to the most fundamental ideas pertaining to fuzzy set theory, fuzzy logic, and fuzzy systems. Omitted are many details, secondary concepts, methods, and special mathematical topics, which are not considered essential for comprehending the overall nature of this emerging field. Yet, the coverage is fairly comprehensive as far as the basic ideas are concerned. The presentation is rigorous, but informal. Part II is an overview of established applications of fuzzy set theory and fuzzy logic.

Together, Parts I and II are intended to provide the reader with a quick and easy introduction to fuzzy set theory and its applications. For further study of various special topics, the reader is guided through relevant literature via *Bibliographical Comments* and *Bibliographical Index*. For a comprehensive deeper study of the field, the graduate text *Fuzzy Set and Fuzzy Logic: Theory and Applications* (Prentice Hall, 1995), coauthored by G.J.Klir and B.Yuan, offers the most natural continuation of this small introductory book. The primarily advantage is that both books use the same terminology and notation.

Part III is very different from Parts I and II. It consists of seven chapters, six of which (Chapter 14-19) are self-contained papers with their own references. These are my previously published papers, which are appropriately adapted for their use in this book. The only chapter in Part III that is not based on previously published papers is Chapter 13.

The purpose of Part III is twofold: (i) to express my personal views on some issues regarding fuzzy set theory and fuzzy logic; and (ii) to provide the reader with additional information regarding some historical and other aspects of fuzzy set theory, which are important for a deeper understanding of the theory, but are too specific or too technical to be covered in Parts I and II.

July 1999 George J. Klir

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Six papers that are contained in Part III of this book are adapted from their original publications. The following is the list of copyright owners of the original publications, whose permissions to use adapted versions of the papers in this book are gratefully acknowledged:

Paul P.Wang, Duke University – Chapters 14 and 15
World Scientific – Chapter 16
Elsevier Science B. V. – Chapter 17
Kluwer Academic Publishers – Chapter 18
Gordon and Breach Science Publishers – Chapter 19

Among the six papers included in Part III, five are my own and one is coauthored. I gratefully acknowledge the consent of my coauthor, David Harmanec (Chapter 16), to include the paper in the book.

Last, but not least, I would like to express my gratitude to Monika Fridrich, a close friend, for her excellent work on preparing a rather difficult camera-ready copy of this book.

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PART I: FUNDAMENTALS

CHAPTER 1. CLASSICAL SETS VERSUS FUZZY SETS

1.1. Introduction

The classical concept of a *set* is fundamental to virtually all branches of classical mathematics, pure as well as applied. Intuitively, a set is any collection of definite and distinct objects that are conceived as a whole. Objects that are included in a set are called its *members*. Sets whose members are themselves sets are usually referred to as *families of sets*.

Classical sets must satisfy two basic requirements. First, members of each set must be distinguishable from one another; and second, for any given set and any given object, it must be possible to determine whether the object is, or is not, a member of the set.

Fuzzy sets differ from classical sets by rejecting the second requirement. Contrary to classical sets, fuzzy sets are not required to have sharp boundaries that distinguish their members from other objects. The membership in a fuzzy set is not a matter of affirmation or denial, as it is in a classical set, but a matter of degree.

Due to their sharp boundaries, classical sets are usually referred to in fuzzy set literature as *crisp sets*. This convenient and well-established term is adopted in this book. Also adopted is the usual notation, according to which both crisp and fuzzy sets are denoted by capital letters, while objects by which sets are formed are denoted by lower case letters. Moreover, the standard symbol $x \in A$ is employed to describe that a given object x is a member of crisp set A, while $x \notin A$ describes that x is not a member of A. When a crisp set A consists of members $a_1, a_2, \cdots, a_n$, we write

$$A = \{a_1, a_2, \cdots, a_n\};$$

when A consists of members that satisfy property P, we write

$$A = \{a_i \mid P(a_i)\},$$

where the symbol $\mid$ denotes the phrase ¡ such that ¡ – and $P(a_i)$ designates a proposition ¡ a_i has the property P. ¡ –

Given two crisp sets, A and B, if every member of A is also a member of B, then A is called a *subset* of B and we write $A \subseteq B$. The *empty set*, $\varnothing$, which does not contain any objects, is, by definition, a subset of any set, including itself.

In each particular application of set theory, all the sets of concern are subsets of a fixed set, which consists of all objects relevant to the application. This set is called a *universal set*, and it is always denoted in this book by X. The set of all subsets of X, which is called the *power set* of X, is denoted by $\mathscr{P}(X)$.

A common way of defining an arbitrary crisp subset A of a given universal set X is to assign the number 1 to each member of X that is also a member of A, and to assign the number 0 to the remaining members of X. The reader may recall that this assignment is called in classical set theory a *characteristic function* of A. It is important to realize that the numbers 1 and 0 are employed here only as convenient symbols and have no numerical significance.

The concept of a universal set, which emerges from the context of each application of set theory, is as fundamental for fuzzy sets as it is for crisp sets. Moreover, universal sets are always assumed to be crisp, regardless of whether we deal with their crisp or fuzzy subsets. Each fuzzy set is defined in terms of a relevant crisp universal set by a function analogous to the characteristic function of crisp sets. This function is called a *membership function*. To define a fuzzy set A on a given universal set X, the membership function assigns to each member x of X a real number in the unit interval [0,1]. This number is viewed as the *degree of membership* of x in A.

Contrary to the symbolic role of numbers 1 and 0 in characteristic functions of crisp sets, numbers assigned to objects by membership functions of fuzzy sets have clearly a numerical significance. This