数理语言学

A Course in Mathematical Linguistics

方 立 著

北京语言文化大学出版社

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Preface 1
(前　言)

乔姆斯基把语言学与自然科学等量齐观的结果，使语言学与现代数学、现代逻辑等学科发生了多重的交叉。这一方面大大促进了信息论、计算机科学和人工智能的发展；另一方面也为我们学习和研究美国理论语言学带来了不少的困难。正如莱福德(A. Radford)在他的《转换句法学》前言中说明他编写该书的理由时所说："除了聪明过人的博士研究生外，一般人是无法读懂《论约束》和《比萨讲演集》并领会它们的思想的。"而盖茨达(G. Gazdar)等人在介绍他们的《广义短语结构语法》时直言不讳地说："这本书不太好读。"蒙太古写的"恰当描写英语中的量词"一文不过几页，多蒂(D. Dowty)等人写了一本313页的导论书说明蒙太古在该文中提出的语法系统的数学性质，但在前言中又说："作为这本书的背景知识，我们预设你们已经熟悉了逻辑、集合论的基础知识。……"对于一个只有人文科学背景知识的人来说，研究当代美国理论语言学的难度也就可想而知了。本课题研究的主要目的也就是要弄清生成语法学家在理论建设中所使用的主要的数学和逻辑手段。

笔者在着手做这个课题时，曾征求过香港城市大学徐烈炯教授的意见，他认为这本书的起点要低，使得大家能读得懂。根据他的意见，笔者在编写过程中遵守循序渐进的原则，并假定读者没有多少数学、逻辑基础，对每一个概念尽

量做到用通俗的语言定义并用最简单的例子作说明，所配练习也主要用于巩固每一章节所讲的内容并提供了全部答案。本书的对象是攻读美国理论语言学的英文研究生，旨在为其提供一本可选择的基础理论教材，但实际上本书也适用于有一定英语水平并对形式语言学感兴趣的其他读者。

经过试用表明，如果每周上四学时，本书的内容足够使用一学期；以此类推，如果每周授课二小时，则可使用一学年。

全书共分为十四章。其内容如下：

第 1 章：集合论

第 2 章：关系

第 3 章：函数

从数学的角度考察语言，语言就是句子集。这就有必要了解集合论以及不同集合成员之间的各种关系。这三章讲述的就是这些内容。

第 4 章：代数中的基本概念

第 5 章：群和整环

第 6 章：格和布尔代数

语言是由若干个系统组成的，用数学的方法描写语言自然需要了解代数系统是什么样子的。群、整环、格和布尔代数都是代数系统。

第 7 章：逻辑中的基本概念

第 8 章：命题逻辑

第 9 章：谓词逻辑

建立严密的系统和理论都离不开逻辑作为手段，这就

是为什么我们需要懂点逻辑。

第 10 章：高阶逻辑

第 11 章：时态逻辑和模态逻辑

第 12 章：内涵逻辑

许多形式语义学家认为，语言的复杂性不仅要求我们借用一般的逻辑系统，而且还要借用表达力丰富的、发达的逻辑系统。

本书至此主要论述形式语义学。

第 13 章：形式语言理论

生成语法学家建立的形式句法学理论主要跟若干种形式语言与形式语法相关，因此有必要了解一些基本的形式语言与形式语法。由于形式语言理论（常被认为是数学的一个分支）的基础是数学，因此如要达到真正理解并能建立形式句法学，只懂得本章的内容是不够的，也需要懂得 1－9 章提供的内容。显然，必备的形式语言理论知识对攻读计算机学科的学生也是不可缺少的。

第 14 章：自动机理论

本书介绍几种自动机的目的是因为这些自动机跟第 13 章介绍的几种形式语法存在着一一对应的关系，这就使得我们有可能从一个不同的角度考察这几种形式语法的数学性质，进而也可考察不同生成语法学家建立的句法学的生成能力。

在本课题完成之际，首先感谢香港城市大学徐烈炯教授提出的各种宝贵意见。

为了保证本课题的研究有一个较好的质量，笔者聘请

了几名学者作为顾问。他们是：北京师范大学数学系沈复兴教授；湖州师范专科学校数学系朱玉锴教授；中国社会科学院哲学所逻辑室邹崇理博士。他们帮助我澄清了某些概念并对文稿提出了一些修改意见。其中，邹崇理博士通读了第一稿并检查了大部分练习，朱玉锴教授通读了第二稿并检查了全部练习，沈复兴教授通读了第三稿。通读文稿的还有我的同事尤爱莉。在此，我对他们表示衷心的感谢。

上述几位学者没有通读第四稿，亦即最后一稿，因此本书如尚有不妥之处，纯系笔者自己的责任。

北京语言文化大学图书馆马新副研究员为了保证本课题研究的顺利进行及时地从国外订购或从外校借调了有关的书籍，对他我同样表示感谢。

北京语言文化大学教务处、人事处、语言教学研究所的领导和工作人员为本项目的完成提供了各种保障和支援。值此机会，我对他们表达谢意。

最后，我特别要感谢国家社会科学基金为该项目提供了所需的资助。

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1995 年 11 月 8 日于北京

Preface 2

Almost four decades have gone by since Noam Chomsky launched a revolution in linguistics in 1957, a revolution that has profoundly shaken the field. Unfortunately, this revolution does not seem to have generated much interest among the majority of linguists in China and, therefore, has made little impact on their approaches to the study of language. Of course, many factors have contributed to this. Among them is the incomprehensibility of the works written by Chomsky and many other generative grammarians. To many of us, the incomprehensibility of these works resides in the unfathomable nature of many of the mathematical notions exploited in representing the intuitive linguistic knowledge of human beings. Technical devices such as 'c-command' and 'bind', which are useful in handling co-referential relations, and notions such as 'lambda-abstraction', 'lambda-conversion' and 'type-raising', which are used for the purpose of maintaining the principle of compositionality, certainly, are not understandable without a sufficient background knowledge of mathematical logic. This book, therefore, aims at helping the Chinese students to acquire a good understanding of these and related notions.

The book is intended for use at the graduate level and contains enough material to be used for a semester course.

Since the book provides all the answers to the exercises, which are designed to reinforce the mathematical notions discussed in the text, and is written in plain and simple language, it can also be used for home study.

I am grateful to Professor Xu Liejiong for his suggestions and encouragement.

I would also like to thank Professor Shen Fuxing, Professor Zhu Yujie and Dr. Zou Chongli for their helpful discussions and comments. For any deficiencies remaining, I myself is to blame.

My thanks also go to Ma Xin for having made available a large number of books that are relevant to the subject matter discussed in this book, and to You Aili for her kind assistance.

I am grateful to the BLCU Department of Personnel whose support enabled me to spend the first term of the academic year from September 1, 1993 to August 31, 1994 at the BLCU Institute of Language Teaching and Research writing the first draft of this book.

Finally, I wish to thank the National Social Sciences Foundation for funding this project.

Symbols

$a \in A$	a is a member of set A	2
$A \notin a$ or $a \overline{\in} A$	a is not a member of set A	2
$\{0,1\ldots\}$	set with members $0,1\ldots$	3
$\{x \mid x\ldots\}$	set of all x such that x is ...	3
$\varnothing$ or $\{\ \}$	empty set	6
U, E, or I	universal set	7
$A \subseteq B$ or $B \supseteq A$	A is a subset of B	7
$A \nsubseteq B$ or $B \nsupseteq A$	A is not a subset of B	7
$A \subset B$ or $B \supset A$	A is a proper subset of B	8
$P(A)$ or 2^A	power set of A	8
$A \neq B$	A is not equal to B	8
$\#(A)$ or $\lvert A \rvert$	cardinality of set A	9
$A \sim B$	A is equivalent to B	9
$\aleph_0$	aleph zero	10
$A < B$	A is smaller than B	11
$A \cup B$	union of sets A and B	14
$\bigcup \{A,B,C\}$	union of sets A,B and C	15
$A \cap B$	intersection of sets A and B	15
$\bigcap \{A,B,C\}$	intersection of sets A,B and C	16
$A - B$	difference of sets A and B	16
A'	complement of set A (This symbol could have different meanings depending on the context.)	17
$x + y$	arithmetic addition	19

$x \times y$	arithmetic multiplication	19
$\bigcup_{i=1}^{n} A_i$	abbreviation for $A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$	23
$\bigcap_{i=1}^{n} A_i$	abbreviation for $A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$	23
$\langle a, b \rangle$	ordered pair	27
$A_1 \times A_2$	Cartesian product of sets A_1 and A_2	28
N	noun	28
V	verb	28
aRb or Rab	binary relation between a and b	30
R^{-1}	inverse of R	32
$R_1 \cdot R_2$	composite of R_1 and R_2	32
I_x	identity relation	33
U_x	universal relation in X	35
$[a]_R$ or $[a]$	equivalence class containing a	39
$x \sim a$	x is equivalent to a	40
π_R^A	partition which R induces on set A	40
$\leqslant$	partial ordering relation	42
$x \mid y$	'x divides y with no remainder' relation	42
$x \parallel y$	x and y are incomparable members	43
$a > b$	a covers b	46
$\bigwedge S = a$ or glb $S = a$	a is the greatest lower bound of S	47
$\bigvee S = a$ or lub $S = a$	a is the least upper bound of S	47
$F: X \to Y$	F is a function that maps X into	

	Y	51
S	sentence	51
NP	noun phrase	51
/	forward slash	51
S/NP	syntactic functor which says that the function maps NP to S	51
e	entity	51
t	truth value	51
$\langle e,t \rangle$	semantic functor which says that the function maps e to t	51
$G \cdot F$	composite of F and G	57
VP	verb phrase	58
Ix	identity function	59
F^{-1}	inverse of F	60
Fc	characteristic function	62
$[0,1]$	set of real numbers between 0 and 1	63
PP	prepositional phrase	64
AP	adjective phrase	64
$*$	abstract operator	66
e_l	left identity element	69
e_r	right identity element	69
e	(two-sided) identity element	69
z_l	left zero	70
z_r	right zero	70
z	(two-sided) zero	70
a_l^{-1}	left inverse of a	70
a_r^{-1}	right inverse of a	70
a^{-1}	(two-sided) inverse of a	70

V^*	set of all strings on V	77
$\frown$	concatenation	77
$+_4$	addition modulo 4	81
$\times_3$	multiplication modulo 3	91
$a \vee b$	least upper bound for $\{a,b\}$	96
$a \wedge b$	greatest lower bound for $\{a,b\}$	96
$\vee$	join	96
$\wedge$	meet	96
$'$	operation of complementation	109
$\therefore$	therefore	116
$\forall$	universal quantifier	116
$\neg$ (or $\sim$)	negation symbol	131
$\neg p$	negation of p	131
$\wedge$ (or &)	conjunction symbol	131
$p \wedge q$	conjunction	131
$\vee$	disjunction symbol	131
$p \vee q$	disjunction	131
$\rightarrow$	implication symbol	131
$p \rightarrow q$	implication or conditional	131
$\leftrightarrow$ (or $\equiv$)	equivalence symbol	131
$p \leftrightarrow q$	equivalence or biconditional	131
$f^{\neg}$	truth function for the negation	137
$f^{\wedge}$	truth function for the conjunction	138
$f^{\vee}$	truth function for the disjunction	139
$\textcircled{\vee}$	exclusive *or*	140
$f^{\rightarrow}$	truth function for the implication	141

$f^{\leftrightarrow}$	truth function for the biconditional	143
$M \Rightarrow N$	M logically implies N	153
$\exists$	existential quantifier	168
$[\alpha]^M$	value of α relative to model M	177
$[\alpha]^{M,g}$	value of α relative to model M and assignment g	179
$g[e/v]$	assignment of values to variables where $g(v)=e$	180
$\langle a,b \rangle$	derived type or complex type	208
$D_b{}^{D_a}$	set of all functions from D_a to D_b	112
λ	lambda operator	214
V_t	transitive verb	228
Det	determiner	238
$<$	earlier than	244
$\not<$	not earlier than	244
T	set of moments of time	244
$[\alpha]^{M,t_j,g}$	value of α relative to model M, moment of time t_j, and assignment g	244
P	past tense operator	247
F	future tense operator	247
W	set of possible worlds	253
$[\alpha]^{M,w_j,g}$	value of α relative to model M, possible world w_j, and assignment g	253
$\Diamond$	possibility operator	253

$\Box$	necessity operator	253
$[\alpha]^{M,w_i,t_j,g}$	value of α relative to model M, possible world w_i, moment of time t_j, and assignment g	258
$\delta^{\beta/\alpha}$	result of substituting β for all the occurrences of α in δ	265
$\hat{}$	intension	270
$\check{}$	intension operator	272
$\langle s,a\rangle$	extension operator	273
$\| \|$	intensional type	273
e	length of a string	289
A^*	empty string	290
V_T	set of all strings on A	290
V_N	terminal alphabet	291
P	nonterminal alphabet	291
$A\rightarrow\omega/\varphi\_\psi$	set of inference rules	291
	A is rewritten as ω in the context of φ_ψ	300
A^*	closure or Kleene star of A	305
Q	set of states	322
Σ	input alphabet	322
q_0	initial state	322
F	set of final states	322
G	transition function	322
# (or B)	blank symbol	322
$\widetilde{x}$	reversal of x	333

Contents