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北京航空航天大学“研究生英文教材”系列丛书

Finite Element Analysis in Engineering

工程有限元分析

Cui Deyu　　Xu Yuanming

崔德渝　　徐元铭

北京航空航天大学出版社
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Abstract

This textbook presents the necessary concepts, principles and general procedure of Finite Element Method (FEM) which are primarily applied for linearly elastic structural analysis including plane problems, axisymmetric problems, space problems, plates and shells and fracture mechanics. The FEM of heat transfer in brief is also incorporated. Some straightforward examples are introduced to demonstrate a complete and detailed finite element procedure. The aim of the text is to provide the fundamental theories and numerical methodology in finite element analysis. It focuses on the derivation of key governing equations of the FEM and its engineering application.

This text can be regarded as a text or reference book for the university under- and post-graduate students or engineers whose majors are related to mechanics, aerospace, mechanical and civil engineering, heat transfer and so on.

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Preface

The Finite Element Method (FEM) is a numerical method which has become very powerful and versatile technology to solve various complex problems in engineering. The method was first developed in 1956 for the analysis of aircraft structural problems. Thereafter, as a general engineering approach of analysis, the potentialities of the method for the solution of different types of applied science and engineering problems were recognized. In the past few decades, the finite element technique has been so well established that by now it is considered to be a key indispensable technology for solving a wide variety of practical problems efficiently. Today, the finite element method represents a most general analysis tool and is used in practically all fields of engineering analysis. Specially, in building advanced engineering systems, the techniques related to modelling and simulation in a rapid and effective way play an important role and accordingly the application of the FEM has increased evidently.

In this text the authors have attempted to provide unified and detailed description on the FEM for both under- and post-graduate university students as well as engineers to understand the fundamentals of the FE approach without too much difficulty. The contents of the text primarily cover the solution of linear problems governed by the field equations, with the main focus on structural mechanics and heat transfer. Although sophisticated commercial finite element software packages are available, the theory of finite elements and the practical (modelling and computation) aspects of the technique must be clearly understood by users. The purpose of this text is to present readers the necessary concepts, fundamental principles and effective techniques of the FEM, so that they can use any well known commercial FEM software package comfortably to solve engineering problems in a systematic manner. It is endeavored to give the details of development of each of the techniques and ideas from basic principles. New concepts are illustrated with simple examples wherever possible. The authors of this text try to introduce the finite element method in insightful but simple, informative but concise, and theoretical but applicable manner.

A brief description of each chapter is presented below to allow the reader to obtain an overview of the whole text:

Chapter 1 describes the basic concept of finite element method, discusses the finite element technique for solving truss and beam problems, lists engineering applications of the FEM, and introduces the basic principles behind the finite element method—principle of virtual displacements and variational formulation.

Chapter 2 introduces 2D linear triangular and rectangular elements, discusses the formulation of general element matrices for elasticity problem, describes general procedure of finite element method, details the assembly of global finite element system equations, and shows area (natural) coordinates as well as higher-order triangular elements.

Chapter 3 introduces "coordinate mapping" and isoparametric concepts, presents the isoparametric formulation for 2D solids with an emphasis on constructing the interpolation of field variables and discusses their properties, formulates the finite element matrices for 2D quadrilateral elements, introduces numerical integration scheme to solve the FEM equations, and describes the formulation of isoparametric triangular elements in brief.

Chapter 4 details the procedure used to obtain finite element matrices for axisymmetric ring elements, and uses very straightforward example to demonstrate the finite element analysis for the stress evaluation of an axisymmetric flywheel.

Chapter 5 studies the accuracy and convergence of finite element method, describes local and global coordinate systems, formulates the finite element matrices for 3D tetrahedron and hexahedron elements, introduces numerical integration scheme to solve the three-dimensional FEM equations, and uses 3D elements for modelling a gear wheel system as example to demonstrate the application of the FEM in 3D cases.

Chapter 6 describes the basic concepts in investigating plates and shells, introduces thin plate elements, formulates the finite element matrices for plate and shell elements based on the Reissner – Mindlin theory, presents more practical Mindlin laminated plate element, and describes the examples of analyzing laminated structures.

Chapter 7 describes the displacement and stress fields in the vicinity of crack tip and stress intensity factors, introduces the finite element analysis with conventional elements, and derives the special purpose elements (crack tip elements or singular elements) for use in many fracture mechanics problems.

Chapter 8 describes the FEM procedure for solving heat transfer problems based on the variational approach, formulates the general finite element matrices and the FEM equations

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for steady state heat transfer in 1D, 2D and 3D cases caused by conduction, convection, radiation, incident heat flux and internal heat sources.

Most of the materials in the text are selected from lecture notes prepared for classes conducted by the first author since 2004 for the foreign graduate students in Beihang University. Those lecture notes were written using the materials in many excellent existing books on the FEM (partially listed in the References). The authors would like to express their sincere appreciation to all the scientists and engineers who has made great contribution for the development of the FEM. Since finite element analysis has been well introduced and described in detail in various existing books, the authors have tried their best to make this text useful for those applying finite element method in practice.

We are much indebted to Prof. Wang Shoumei and Prof. Gu Zhifen for reading the manuscript and for making the valuable suggestions and modification to improve the presentation, for which we are very grateful. We also would like to extend very special thanks to Prof. Zhang Xing, for his active support of our research work and his helpful advice to make the arrangement for Chapter 7. The authors would like to also thank the students for their help in the past few years in developing these courses.

Cui Deyu, Xu Yuanming
1 March, 2012

Contents

Chapter 1
Introduction to Finite Element Method

1.1 Basic Concept of Finite Element Method

The Finite Element Method (FEM) by now has become the most powerful and versatile tool of analysis and is applicable to wide range problems. In structural analysis, FEM provides the feasibility of analyzing various types of the engineering structures with complex geometry and loading conditions. The basic idea of the finite element method can be described mainly in the following two aspects:

① A structure is divided (hypothetically) into many small, interconnected subregions called *finite elements* -which are so small that the *shape of a field* distribution (such as displacement, stress or temperature distribution) can then be approximated with various types of modelling functions, leaving only the *magnitude* to be found. The "shapes" may be polynomials, trigonometric functions and so on, as we shall see.

② All the individual elements have to be assembled together in such a way that the field quantities are tried to keep continuous in some fashion across element interfaces, and the global equilibrium in the entire problem domain as well as the prescribed boundary conditions are to be satisfied.

The first part of above finite element process is fundamental and involves choosing the correct and appropriate types of elements and describing and evaluating their properties. Although commercial FEM software packages do keeps users from having to evaluate element properties, it does help to be able to understand and exercise judgment when modelling the structure, and spotting wrong answers due to using inadequate elements. It also helps to understand the pedigree of elements when deciding whose finite element software packages to try! The second part of the process is the assembly of the elements and then solving the complete structure and so on. The commercial finite element software

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packages will do this of course, but again it pays to know what is involved since sometimes this process breaks down, or it simply becomes an inefficient process because the structure has been described inadequately. It can be concluded that the finite element method is essentially a process through which a continuum with infinite degrees of freedom can be approximated by an assemblage of elements (or subregions) each with a specified but now finite number of unknowns. The suitable structure discretization into elements, including the type, size and arrangement of elements, governs the accuracy of results (see References [1]~[4]).

1.2 General Description of Finite Element Method

1.2.1 Finite Element Technique in Structure Analysis

In structure analysis, the equations governing engineering phenomena are usually derived from equilibrium equations and constitutive laws with strain - displacement relations and prescribed boundary conditions, and they can be characterized by a set of differential equations. The finite element method represents mathematically an approximate solution of a boundary-value problem described by a differential equation, and all the fundamental concepts required for the numerical solution of differential equations are to be considered. The finite element discretization techniques are usually arranged by one of four approaches:

➢ The displacement method (the displacements are those variables which are solved first).

➢ The force method (the forces or stresses are those variables which are solved first).

➢ The hybrid and mixed method (see References [5] and [6]).

Structural analysis can be summarized in the schematics of Fig. 1.1, in which

① Assume the field functions of displacements or/and stresses;

② Establish the relationship between applied forces and stresses;

③ Establish the relationship between displacements and strains;

④ Establish the relationship between stresses and strains.

The *displacement method* is normally used in engineering application, that is, determine the displacement field firstly then find strain and stress fields. The route followed in the displacement method is ①→③→④→②. Note that the stress-strain law, however complicated, has nothing whatsoever to do with equilibrium ② or compatibility ③ conditions, with which we shall be most concerned (see Reference [21]).

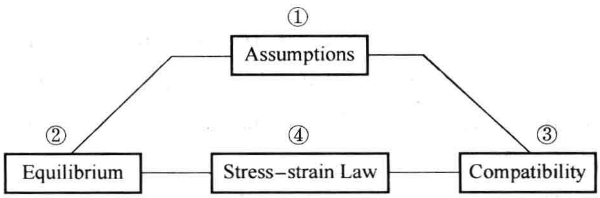

Fig. 1.1 The route of structure analysis

For the numerical solution of a structural or continuum problem it is basically necessary to establish and solve algebraic equations that govern the response of the system. Using the finite element method, it becomes possible to establish and solve the governing equations for complex problems in a very effective way. When the solution region is considered as built up of many small, interconnected elements, in each element, a convenient approximate pattern of solution is then assumed and the conditions of overall equilibrium of the structure are derived. The satisfaction of these conditions with applied loads and boundary conditions will yield an approximate solution for the displacements and stresses.

The displacement-based finite element method can be regarded as an extension of the displacement method of analysis, which have been used for many years in the analysis of beam and truss structures. The basic steps in the analysis of a beam and truss structure using the displacement method are:

① Idealize the total structure as an assemblage of beam and truss elements that are interconnected at the structural joints (which are often called as nodes).

② Identify the unknown joint displacements to completely define the displacement response of the structural idealization.

③ Construct the stiffness matrices of elements and establish force balance equations corresponding to the unknown joint displacements and solve these equations.

④ Calculate the internal element stress distributions with the element end-displacements obtained.

⑤ Interpret the displacements and stresses predicted by the solution of the structural idealization when considering the assumptions used.

In practical analysis and design the most important steps of the complete analysis are the proper idealization of the actual problem, as performed in step ①, and the correct interpretation of the results in step ⑤.

1. Stiffness Matrix of Truss and Beam Elements

As mentioned above, the construction of the stiffness matrices for truss and beam elements is a necessary step in the analysis of a beam and truss structure. According to the mechanics of materials approach, the stiffness matrices of truss and beam elements can be obtained (see Reference [23]).

(1) Truss Element Formed by Two Nodes (Fig. 1.2)

Truss elements are primarily used in truss structure, and, by definition, they are assumed to be acted upon only by axial forces. The stiffness coefficients of a truss element can be obtained directly from Hooke's law. Referring to Fig. 1.2(a), it is seen that if node 2 is fixed and node 1 is allowed to move, then

$$\left.\begin{aligned} P_{x1} &= \frac{EA}{l}u_1 \\ P_{x2} &= -\frac{EA}{l}u_1 \end{aligned}\right\} \tag{1.1}$$

in which P_{x1} and P_{x2} are nodal forces applied to nodes 1 and 2 respectively, E is the Young's modulus of material, A and l are the cross sectional area and the length of a truss element, respectively. Similarly, it can be derived from Fig. 1.2(b) that if end 1 is now fixed but end 2 allowed to move, then

$$\left.\begin{aligned} P_{x1} &= -\frac{EA}{l}u_2 \\ P_{x2} &= \frac{EA}{l}u_2 \end{aligned}\right\} \tag{1.2}$$

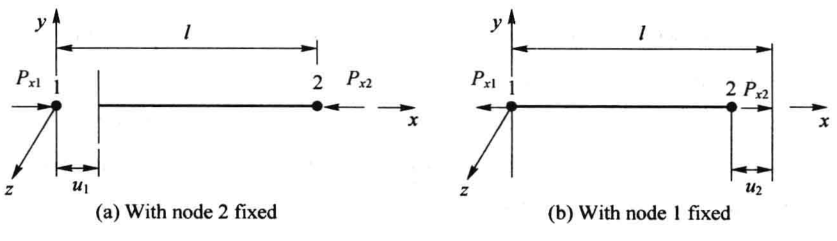

Fig. 1.2 A truss element

Combining Eqs. (1.1) and (1.2), the complete stiffness relationship can be expressed as

$$\boldsymbol{P}_e = \boldsymbol{K}_e \boldsymbol{\delta}_e \tag{1.3}$$

where $\boldsymbol{P}_e = \begin{bmatrix} P_{x1} \\ P_{x2} \end{bmatrix}$ and $\boldsymbol{\delta}_e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ are the nodal force and displacement vectors respectively in the local x coordinate, and $\boldsymbol{K}_e$ is the stiffness matrix of a truss element given by

$$\boldsymbol{K}_e=\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}=\frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{1.4}$$

in which $k_{ij}\ (i,j=1,2)$ denotes the stiffness coefficients. It is clear that k_{ij} is the axial force developed at joint i when joint j is displaced through a unit distance.

Actually a truss element may not be aligned with the axes in a global coordinate system (Fig. 1.3). In this case, the element stiffness matrix derived above has to be transformed into the one corresponding to the global coordinates. Referring to Fig. 1.3, the transformation relation of nodal displacements in the local and global coordinate systems can be expressed as

$$\left.\begin{aligned} u_1&=\underline{u}_1\cos(x,\underline{x})+\underline{v}_1\cos(x,\underline{y})=\underline{u}_1\cos\alpha+\underline{v}_1\sin\alpha \\ u_2&=\underline{u}_2\cos(x,\underline{x})+\underline{v}_2\cos(x,\underline{y})=\underline{u}_2\cos\alpha+\underline{v}_2\sin\alpha \end{aligned}\right\} \tag{1.5}$$

or in matrix form

$$\boldsymbol{\delta}_e=\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}=\begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \underline{u}_1 \\ \underline{v}_1 \\ \underline{u}_2 \\ \underline{v}_2 \end{bmatrix}=\boldsymbol{T}\,\underline{\boldsymbol{\delta}}_e \tag{1.6}$$

in which $\underline{\boldsymbol{\delta}}_e=\begin{bmatrix} \underline{u}_1 & \underline{v}_1 & \underline{u}_2 & \underline{v}_2 \end{bmatrix}^{\mathrm{T}}$ denotes the nodal displacement vector in the global coordinate system $(\underline{x},\underline{y})$ and $\boldsymbol{T}$ is the coordinate transformation matrix given by

$$\boldsymbol{T}=\begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix} \tag{1.7}$$

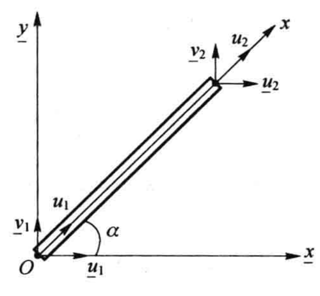

Fig. 1.3 Nodal displacements in local and global coordinates

Thus, in the global coordinate system, the element stiffness matrix and nodal force vector can be derived as follows

$$\underline{P}_e = T^T P_e \quad \text{and} \quad \underline{K}_e = T^T K_e T \tag{1.8}$$

where $\underline{P}_e = [\underline{P}_{x1} \quad \underline{P}_{y1} \quad \underline{P}_{x2} \quad \underline{P}_{y2}]^T$.

(2) Beam Element Formed by Two Nodes (Fig. 1.4)

A beam element is concerned with bending action only and its stiffness relationship is just a simple extension of the well-known slope-deflection equations. Referring to Fig. 1.4, the slope-deflection equations are of the form

$$\left.\begin{aligned} M_1 &= \frac{6EI}{l^2}v_1 + \frac{4EI}{l}\theta_1 - \frac{6EI}{l^2}v_2 + \frac{2EI}{l}\theta_2 \\ M_2 &= \frac{6EI}{l^2}v_1 + \frac{2EI}{l}\theta_1 - \frac{6EI}{l^2}v_2 + \frac{4EI}{l}\theta_2 \end{aligned}\right\} \tag{1.9}$$

in which EI is the flexural stiffness of the beam element.

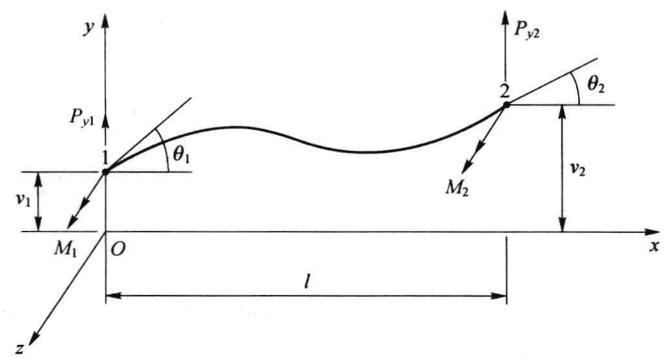

Fig. 1.4 A beam element

The end shear forces P_{y1} and P_{y2}, which are equal and opposite, form a couple equal to the sum of the end moments, as shown in Fig. 1.4. Therefore, from Eq. (1.9), the shear forces are

$$P_{y1} = -P_{y2} = \frac{M_1 + M_2}{l} = \frac{12EI}{l^3}v_1 + \frac{6EI}{l^2}\theta_1 - \frac{12EI}{l^3}v_2 + \frac{6EI}{l^2}\theta_2 \tag{1.10}$$

Expressing Eqs. (1.9) and (1.10) in matrix form, the stiffness relationship of a beam element becomes

$$\boldsymbol{P}_e = \boldsymbol{K}_e \boldsymbol{\delta}_e \tag{1.11}$$

where $\boldsymbol{P}_e = [P_{y1} \quad M_1 \quad P_{y2} \quad M_2]^T$ and $\boldsymbol{\delta}_e = [v_1 \quad \theta_1 \quad v_2 \quad \theta_2]^T$ are the nodal force and displacement vectors, respectively, and $\boldsymbol{K}_e$ denotes the stiffness matrix of the beam element and is given by

$$\mathbf{K}_e = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} = \frac{EI}{l} \begin{bmatrix} \frac{12}{l^2} & \frac{6}{l} & -\frac{12}{l^2} & \frac{6}{l} \\ \frac{6}{l} & 4 & -\frac{6}{l} & 2 \\ -\frac{12}{l^2} & -\frac{6}{l} & \frac{12}{l^2} & -\frac{6}{l} \\ \frac{6}{l} & 2 & -\frac{6}{l} & 4 \end{bmatrix} \tag{1.12}$$

where k_{ij} $(i,j=1,4)$ denotes the coefficients in stiffness matrix. In fact, k_{ij} is the force (or couple) developed along the ith degree of freedom of the beam when its jth degree of freedom is displaced (or rotated) through a unit distance (or angle).

2. A Numerical Example for Truss and Beam Structures

A very straightforward example is used to demonstrate a complete and detailed finite element procedure to compute displacements in a truss and beam structure. Above mentioned analysis steps and element stiffness matrices are applied here (see Reference [25]).

Example 1.1: A cantilever beam ab supporting a concentrated load F_b at the free end. There is a small gap Δ between the beam and a vertical column cd, as shown in Fig. 1.5. Determine the system global displacements and angles of rotation if using the idealization in Fig. 1.5 (b) (provided the load is applied slowly enough, the stresses are small enough not to cause yielding) $F_b=1\times10^5$ N, $\Delta=0.01$ m, $l=1$ m, and

➢ $E=2\times10^{11}$ Pa, $I=1\times10^{-5}$ m^4 for the beam elements ① and ② ;

➢ $E=2\times10^{11}$ Pa, $A=1\times10^{-3}$ m^2 for the truss element ③.

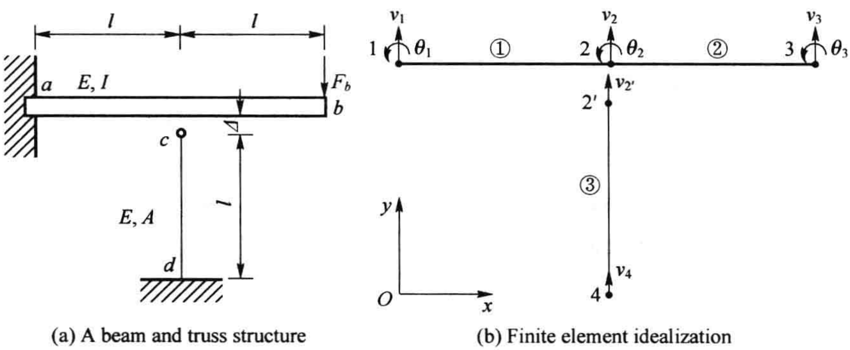

Fig. 1.5 Contact problem of a cantilever beam

Assume that in the first analysis we primarily want to calculate the deflection and the

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