钱学森力学手稿

9

钱学森

西安交通大学出版社
XI'AN JIAOTONG UNIVERSITY PRESS

钱学森力学手稿

9

钱学森

西安交通大学出版社
XI'AN JIAOTONG UNIVERSITY PRESS

图书在版编目(CIP)数据
钱学森力学手稿.9:英文/钱学森著. —西安:西安交通大学出版社,2013.2
ISBN 978-7-5605-4547-9

Ⅰ.①钱… Ⅱ.①钱… Ⅲ.①钱学森(1911~2009)-力学-手稿-英文 Ⅳ.①O3-53

中国版本图书馆CIP数据核字(2012)第209959号

书　　名 钱学森力学手稿9
著　　者 钱学森
责任编辑 杨　璠

出版发行 西安交通大学出版社
(西安市兴庆南路10号　邮政编码710049)
网　　址 http://www.xjtupress.com
电　　话 (029)82668357　82667874(发行中心)
(029)82668315　82669096(总编办)
传　　真 (029)82668280
印　　刷 中煤地西安地图制印有限公司

开　　本 787mm×1092mm　1/16　**印张** 12.5　**字数** 300千字
版次印次 2013年2月第1版　2013年2月第1次印刷
书　　号 ISBN 978-7-5605-4547-9/O·409
定　　价 70.00元

读者购书、书店添货、如发现印装质量问题,请与本社发行中心联系、调换。
订购热线:(029)82665248　(029)82665249
投稿热线:(029)82664954
读者信箱:jdlgy@yahoo.cn

出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长，学习他的科学思想和卓越风范，展示其丰功伟绩和人格魅力，西安交通大学举办了“纪念钱学森诞辰100周年”系列活动：作为制片方之一，参与西部电影集团摄制传记故事片《钱学森》；与中央电视台合作，出品纪录片《实验班的故事——沿着钱学森走过的路》；扩建钱学森生平业绩展馆，向校内外开放；举办钱学森科学与教育思想研讨会；出版发行《钱学森力学手稿》、《钱学森年谱（初编）》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿，这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风，是钱老勇攀科学高峰和严谨治学的集中体现。这里，我们将部分原稿整理汇集成册，出版《钱学森力学手稿》，作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷，包含两部分内容。第一部分是草稿，包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时，为了求得圆柱壳体的临界压力，在有关的五百多页草稿中，对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算，最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表，每个数字有效位数都长达八位，在手摇机械式计算机作为主要计算工具的年代，这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿，以航空航天工程为核心，涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下，由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助，在此深表感谢。

Contents

Section 1

Wind-Tunnel Testing Problem in Superaerodynamics

1 orig. 4 carbons
double space

Wind-Tunnel Testing Problems in Superaerodynamics

Hsue-shen Tsien
Guggenheim Aeronautical Laboratory
Massachusetts Institute of Technology

Wind-tunnels are, perhaps, the most useful tool in aerodynamic investigations and certainly have contributed much in the modern development of fluid mechanics. It is thus natural, when one turns to the new field of aerodynamics, the aerodynamics of rarefied gases or superaerodynamics, that one should think of using the wind tunnel again. Only here ~~the old faithful tool~~ it has to be adapted to entirely new circumstances and many new problems, both in its design and in its operation, appear. It is the purpose of this paper to discuss some of these problems, so as to gain an orientation in the new field of experimentation.

1. Tunnel Design

To test models in the wind tunnel at its test section, it is of primary importance to obtain a uniform stream at the desired temperature, pressure and velocity. For subsonic wind tunnels with ordinary pressures, this can be achieved without much difficulty. For supersonic wind tunnels at ordinary pressures, the expansion part of the tunnel, or the nozzle, before the test section, is first designed to obtain a uniform stream at its exit without considering the effects of the viscosity of air. Then the boundary layer along the wall of the nozzle is calculated with the pressure gradient thus determined. Finally the thickness of the boundary layer, or rather the ~~enlarged~~ needed space for the boundary layer flow at lower velocity is provided by making the nozzle larger than the dimensions first determined by the calculated amount. (Ref 1) This design procedure is found to give satisfactory nozzles for supersonic wind tunnels.

However when one tries to use the same design procedure to the superaerodynamics windtunnel, one is immediately confronted with the difficulty of extremely large viscous effect. In other words, the boundary layer will be so thick as to occupy the main portion of the nozzle passage. To demonstrate this effect, let us consider that the length of the test section is L and the width of the square test section be b. Then the Reynolds number based upon the conditions in the test section is $Re = \frac{UL}{\nu}$ where U is the velocity in the test section. If, as a rough estimate, we take the thickness of the boundary layer to be zero at the beginning of the test section and equal to a value δ calculated by the well-known Blasius formula for a flat plate at the end of the test section, then

$$\delta = 3.65 L \frac{1}{\sqrt{Re}} \qquad (1)$$

Now if this boundary layer actually occupies half the tunnel width b/2, then

$$\delta = \frac{b}{2} = 3.65 L \frac{1}{\sqrt{Re}} \qquad (2)$$

On the other hand, the ratio of the mean free path l and the boundary layer thickness δ is known (Ref. 2) to be equal to

$$\frac{l}{\delta} = \frac{1.255\sqrt{\gamma}}{3.65} \frac{M}{\sqrt{Re}} \qquad (3)$$

where γ is the ratio of specific heats and can be taken as 1.4, M is the Mach number in the test section. By combining (2) and (3), we have

$$\left(\frac{l}{\delta}\right)\left(\frac{L}{b}\right) = 0.0557 M \qquad (4)$$

This relation is shown in Fig. 1. Thus for a Mach number M equal to 2, and $L/b = 2$, the boundary layer will completely fill up the test section, if the mean free path is equal to 5.6% of the boundary layer thickness or 2.8% of the tunnel width. This means that the extremely strong viscous effect at low densities, makes the ordinary concept of designing a wind tunnel totally inapplicable.

The same fact can be also demonstrated by calculating the ratio of frictional loss on the walls of the test section and the shock loss in the diffuser after the test section. Consider the diffuser to be a straight tube of approximately the same cross-section area as the test section, then the pressure loss due to friction, Δp_1 is

$$\Delta p_1 = \frac{\text{Frictional Force}}{b^2}$$

$$= \frac{\rho U^2}{2} 4bL \cdot C_f \frac{1}{b^2}$$

Taking C_f to be Blasius value again or $C_f = \frac{1.328}{\sqrt{Re}}$, we have

$$\Delta p_1 = 2\rho U^2 \left(\frac{L}{b}\right) \frac{1.328}{\sqrt{Re}} \qquad (5)$$

Now the shock loss can be estimated as that due to a normal shock with no recovery after the shock. Then if p is the pressure in the test section, the pressure loss due to shock Δp_2 is

$$\Delta p_2 = \left[\left\{1 + \frac{\gamma-1}{2} M^2\right\}^{\frac{\gamma}{\gamma-1}} - \left\{\frac{2\gamma}{\gamma+1} M^2 - \frac{\gamma-1}{\gamma+1}\right\}\right] p \qquad (6)$$

By combining (5) and (6), the ratio of these two pressure losses is

$$\frac{\Delta p_1}{\Delta p_2} = \frac{2\gamma M^2 \left(\frac{L}{b}\right) \frac{1.328}{\sqrt{Re}}}{\left[\left\{1 + \frac{\gamma-1}{2} M^2\right\}^{\frac{\gamma}{\gamma-1}} - \left\{\frac{2\gamma}{\gamma+1} M^2 - \frac{\gamma-1}{\gamma+1}\right\}\right]} \qquad (7)$$

Introducing the mean free path ratio given by (3), we have

$$\left(\frac{\Delta p_1}{\Delta p_2}\right) = \left(\frac{L}{b}\right)\left(\frac{l}{\delta}\right) \frac{3.264 \times 2\gamma M}{\left[\left\{1+\frac{\gamma-1}{2}M^2\right\}^{\frac{\gamma}{\gamma-1}} - \left\{\frac{2\gamma}{\gamma+1}M^2 - \frac{\gamma-1}{\gamma+1}\right\}\right]} \qquad (8)$$

This relation is plotted in Fig. 2. Therefore if the Mach number M is 2, and $L/b = 2$ as before, then when the ratio l/δ is 0.056, the ratio of frictional loss to shock loss is 0.628. Therefore the frictional loss and the shock loss is of the same order of magnitude.

These elementary calculations makes it clear that for the design of the nozzle and test section for a superaerodynamics windtunnel, it is no longer possible to separate the compressibility effects and the viscous effect. In fact, the concept of boundary layer is also of doubtful value due to the extremely small Reynolds number encountered. Therefore to actually design such a nozzle to obtain the nearest approximation to the ideal uniform flow, it will be necessary to use the exact Navier-Stokes equations instead of the approximate boundary layer equations. Of course, it may be argued that for superaerodynamics, the Navier-Stokes equations are no more exact and additional corrections must be added (Ref. 4). However, recent investigations by R. Schamberg (Ref. 3) have shown that these additional corrections are small in case of slip-flows concerned here and will not essentially alter the flow pattern. Hence for a first approximation, just like the non-viscous isentropic flow as a first approximation for ordinary supersonic nozzles, we can use the Navier-Stokes equations. The simplest case to be considered is certainly the axially symmetric nozzles. If x is the coördinate in the axial direction, r the coördinate in the radial direction and u and v are the corresponding velocity components (Fig. 3), the equations are:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x} = 0 \qquad (9)$$

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} - \text{Grad}(\tau)_x \qquad (9)$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial r} - \text{Grad}(\tau)_r \qquad (10)$$

$$\rho \frac{D}{Dt}\left(\frac{u^2+v^2}{2} + c_p T\right) = \Phi - \left\{u\,\text{Grad}(\tau)_x + v\,\text{Grad}(\tau)_r\right\}$$

$$+ \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r \frac{\partial T}{\partial r}\right) \qquad (11)$$

where
$$\frac{D}{Dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial r}$$

Φ = dissipation function

τ = stresses tensor

(9), (10), (11) + (12) together with the equation of state

$$\frac{p}{\rho} = RT \qquad (13)$$

then determines the five unknowns u, v, p, ρ, and T. Of course, the actual process of making this calculation will be extremely tedious and some approximation method of solution may have to be developed. One possibility would be to adopt the Kármán-Pohlhausen method for boundary layer to this case: We integrate the differential equations once with respect to r and thus only try to satisfy the equations "on the average" over the cross-section of the nozzle. The "distribution" of u, v over the cross-section will then be set in the form of a polynomial in r. Critical study in this procedure is already made by S. A. Schaaf (Ref. 4) at the suggestion of the author.

For ordinary supersonic diffuser, high efficiency of pressure recovery can be generally achieved by using a long diffuser. However, for superaerodynamics wind tunnel, due to the extremely large loss through friction, long diffusers are undesirable. In fact, the pressure loss can be reduced by using a shortest possible diffuser.

2. Flow Measurement

The quantities which determine the flow field are two out of the three variables p, ρ, T and the velocity components. The quantities p, ρ, T are related by the equation of states and therefore only two is necessary for the determination of all three. Generally for windtunnel work, the quantities actually measured are p, ρ and q, the magnitude of the velocity.

For the measurement of pressure, a manometer is used. For ordinary pressure, one uses a fluid manometer filled with water, alcohol or mercury. However for the extremely low pressure encountered in the superaerodynamic flow, some other form of manometer is necessary. One of the most used type is the Pirani gauge. The conventional form of Pirani gauges has a pressure sensitivity of about 10^{-2} micron.* It utilizes the change of temperature of a wire heated with constant energy caused by a change in the pressure of the gas surrounding it. The temperature change is measured by the change in the resistance of the wire. The wire is located in a small chamber which is connected to the point of measurement by a hole, flush with the gas stream, if static pressure is to be measured. The question of best design of the connecting tube for quick response is studied by S. A. Schaaf. (Ref. 5)

To measure ~~the~~ density ρ, the conventional method ~~is through~~ utilizes the ~~the use of the different~~ the difference in the velocity ~~velocity~~ of light rays in mediums of different density with different optical arrangements, we have the shadowgraph method, the schlieren method and the interferometer method. However, if the density of the medium is very low as the case of superaerodynamic flow, the sensitivity of these methods become extremely poor. For instance, in case of schlieren method, the percentage change in illumination I by passing through

---

* 1000 micron = 1 mm. Hg.

a region of thickness b in air is given by

$$\frac{\Delta I}{I} = k\frac{f}{\varepsilon}\ 0.000294\left(\frac{\rho}{\rho_0}\right)\left[\frac{b}{\rho}\frac{\Delta\rho}{\Delta n}\right] \tag{14}$$

where ρ_0 is the air density at 32°F and 1 atmosphere pressure, and $\Delta\rho/\Delta n$ is the density gradient normal to the light ray, f and ε are the focal length and the normal, unobscured width of the light source image perpendicular to the knife edge. k is factor of order 1, determined by the particular optical path used. Therefore the sensitivity of the schlieren method decreases with the factor (ρ/ρ_0). Some improvement can be made by altering the quantities f and ε, but practical limitations and diffraction difficulties do not allow the increase of sensitivity to satisfactory values.

A new approach to this problem density measurement is the method of absorption. It is found for instance, that oxygen at low pressures shows a strong absorption band at wave lengths around 1350 Å or ultra-violet light. The percentage absorption is proportional to the number of molecules that meet the light ray and is, therefore, proportional to the density of the gas. The measurement is then similar to that of the interferometer method where the density of gas is determined. A similar method is the utilization of the after-glow of nitrogen. These method are now being studied by R. A. Evans (Ref 6)

The conventional method for the measurement of velocity is through the use of dynamic pressure rise in a Pitot-tube. A straight forward application of this method is, however, difficult for rarefied gases. The formula used is, however, based upon the neglection of viscosity effects. But for rarefied gases, the viscosity effect is of great importance as pointed out in the previous section. Then the dynamic pressure would be quite different then that given by the usual formula. To estimate this effect, let us consider the case of low Mach number so that compressibility effects can be neglected. Then as a first approximation, take the flow field around the

Pitot-tube as that of a source of strength S in non-viscous flow of uniform velocity U. (Fig. 4) The "radius" of the tube a is

$$a = \sqrt{\frac{S}{\pi U}}$$

and the stagnation point is located at

$$r_s = \sqrt{\frac{3S}{4\pi U}} = \frac{\sqrt{3}}{2} a \qquad (15)$$

The velocity introduced by the source is then

$$- U \cdot \frac{3}{4} \frac{a^2}{r^2}$$

By calculating the viscous stress from this approximate disturbance velocity, we have for flow along the axis

$$u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = +\nu U a^2 \cdot \frac{3}{2r^4} \qquad (16)$$

Hence if p_0 is the stagnation pressure and p° the ~~free stream~~ static pressure,

$$p_0 - p^\circ = \frac{1}{2}\rho U^2 + \mu U a^2 \frac{3}{2} \int_{+\infty}^{r_s} \frac{dr}{r^4}$$

$$= \frac{1}{2}\rho U^2 - \frac{1}{2}\mu U a^2 \frac{1}{r_s^3}$$

Or

$$p_0 - p^\circ \cong \frac{1}{2}\rho U^2 \left[1 - \frac{8}{3\sqrt{3}} \frac{\nu}{aU} \right] \qquad (17)$$

For rarefied gases, the value of ν/aU or the reciprocal of the Reynolds' number of the Pitot-tube could be of the order of unity. Then the dynamic pressure rise $p_0 - p'$ is not the usual value $\frac{1}{2}\rho U^2$ but a value much less than that. We shall be seriously in error if we use the ordinary formula to calculate the velocity U.

When the velocity of flow is high, we have the added complication due to the shock. The conventional Rayleigh formula for Pitot tubes in supersonic flow is based upon the assumption of very thin shock wave ahead of the Pitot tube. Now the thickness of the shock is proportional to the mean free path. Hence in rarefied flows, the thickness of the shock will be so increased as to cause interference with flow in the neighborhood of the Pitot-tube. This together with the viscous effect mentioned in previous paragraph definitely show the inapplicability of the Rayleigh formula for supersonic velocity of rarefied gases.

With these great complications in applying the conventional velocity measuring device to superaerodynamic flows, one is naturally lead to the thought of other avenues of approach. One possibility is the use of hot-wire. If the wire diameter is of the order of 0.0001 inches, and if the pressure of the gas stream is approximately 100 microns, the ratio of the mean free path to the wire diameter will be approximately 180. Therefore the flow around the wire is definitely the free molecular flow. (Ref. 2) We have thus a simple physical situation. If θ is the inclination of the solid surface to a gas stream of velocity U, and if the molecular velocity distribution is assumed to be Maxwellian, then the translational energy of molecules $E_{\theta_{tr.}}$ incident upon the unit area is

$$E_{i_{tr.}} = \rho \frac{c}{2\sqrt{\pi}} \left\{ e^{-\frac{U^2}{c^2}\sin^2\theta} \left(c^2 + \frac{1}{2}U^2\right) + \sqrt{\pi}\,\frac{U}{c}\sin\theta \left(\frac{5}{4}c^2 + \frac{1}{2}U^2\right)\left[1 + \mathrm{erf}\left(\frac{U}{c}\sin\theta\right)\right]\right\} \qquad (18)$$

where $c^2 = 2RT$, T temperature of the gas stream. For the sake of simplicity let us consider a surface in the direction of flow, $\theta = 0$, then

$$E_{i_{tr.}} = \rho \frac{c}{2\sqrt{\pi}}\left(c^2 + \frac{1}{2}U^2\right), \quad \theta = 0$$

The total incident energy per unit area is then

$$E_i = \rho \frac{c}{2\sqrt{\pi}}\left[\frac{1}{2}U^2 + \left(\frac{1}{2}R + c_v\right)T\right] \qquad (19)$$