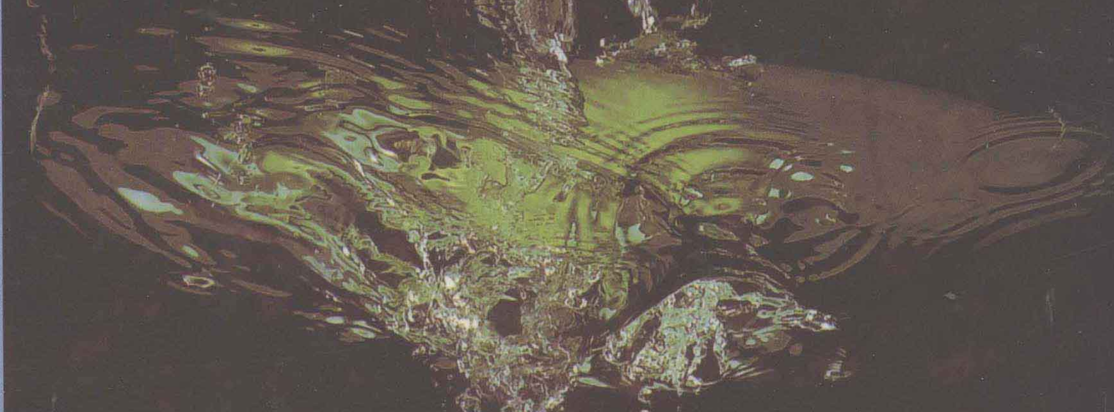
Walter Appel

Mathematics
for Physics and Physicists

数学物理



世界图书出版公司
www.wpcbj.com.cn

PRINCETON UNIVERSITY PRESS

Mathematics for Physics and Physicists

Walter APPEL

Translated by Emmanuel Kowalski

Princeton University Press
Princeton and Oxford

图书在版编目（CIP）数据

数学物理 = Mathematics for Physics and Physicists：英文/（法）阿培（Appel，W.）著．—影印本．—北京：世界图书出版公司北京公司，2012.8
ISBN 978－7－5100－5063－3

Ⅰ．①数… Ⅱ．①阿… Ⅲ．①数学物理方法—英文 Ⅳ．①O411.1

中国版本图书馆 CIP 数据核字（2012）第 186143 号

书　　名： Mathematics for Physics and Physicists
作　　者： Walter Appel
中 译 名： 数学物理
责任编辑： 高蓉　刘慧

出 版 者： 世界图书出版公司北京公司
印 刷 者： 三河市国英印务有限公司
发　　行： 世界图书出版公司北京公司（北京朝内大街 137 号 100010）
联系电话： 010－64021602，010－64015659
电子信箱： kjb@ wpcbj. com. cn

开　　本： 16 开
印　　张： 42
版　　次： 2013 年 1 月
版权登记： 图字：01－2012－5581

书　　号： 978－7－5100－5063－3　　**定　　价：** 129.00 元

To Kirone MALLICK, Angel ALASTUEY and Michael KIESSLING,
for their friendship and our discussions – scientific or otherwise.

Thanks

Many are those who I wish to thank for this book: Jean-François Colombeau who was in charge of the course of Mathematics for the "Magistère des sciences de la matière" of the ENS Lyon before me; Michel Peyrard who then asked me to replace him; Véronique Terras and Jean Farago who were in charge of the exercise sessions during the three years when I taught this course.

Many friends and colleagues were kind enough to read and re-read one or more chapters: Julien Barré, Maxime Clusel, Thierry Dauxois, Kirone Mallick, Julien Michel, Antoine Naert, Catherine Pépin, Magali Ribaut, Erwan Saint-Loubert Bié; and especially Paul Pichaureau, TEX-guru, who gave many (very) sharp and (always) pertinent comments concerning typography and presentation as well as contents. I also wish to thank the Éditions H&K and in particular Sébastien Desreux, for his hard work and stimulating demands; Bérangère Condomines for her attentive reading.

I am also indebted to Jean-François Quint for many long and fascinating mathematical discussions.

This edition owes a lot to Emmanuel Kowalski, friend of many years, Professor at the University of Bordeaux, who led me to refine certain statements, correct others, and more generally helped to clarify some delicate points.

For sundry diverse reasons, no less important, I want to thank Craig Thompson, author of *Good-bye Chunky Rice* [89]and *Blankets* [90], who created the illustrations for Chapter 1; Jean-Paul Marchal, master typographer and maker of popular images in Épinal; Angel Alastuey who taught me so much in physics and mathematics; Frédéric, Laëtitia, Samuel, Koupaïa and Alanis Vivien for their support and friendship; Claude Garcia, for his *joie de vivre*, his *fideos*, his fine bottles, his knowledge of topology, his advice on Life, the Universe... and the Rest! without forgetting Anita, Alice and Hugo for their affection.

I must not forget the many readers who, since the first French edition, have communicated their remarks and corrections; in particular Jean-Julien Fleck, Marc Rezzouk, Françoise Cornu, Céline Chevalier and professors Jean Cousteix, Andreas de Vries, and François Thirioux; without forgetting all the students of the "Magistère des sciences de la matière" of ENS Lyon, promotions 1993 to 1999, who followed my classes and exercise sessions and who contributed, through their remarks and questions, to the elaboration and maturation of this book.

Et, bien sûr, les plus importants, Anne-Julia, Solveig et Anton – avec tout mon amour.

Although the text has been very consciously written, read, and proofread, errors, omissions or imprecisions may still remain. The author welcomes any remark, criticism, or correction that a reader may wish to communicate, care of the publisher, for instance, by means of the email address *Errare.humanum.est@h-k.fr*.

A list of *errata* will be kept updated on the web site

http://press.princeton.edu

A book's apology

W HY SHOULD A PHYSICIST STUDY MATHEMATICS? *There is a fairly fashionable current of thought that holds that the use of advanced mathematics is of little real use in physics, and goes sometimes as far as to say that knowing mathematics is already by itself harmful to true physics. However, I am and remain convinced that mathematics is still a precious source of insight, not only for students of physics, but also for researchers.*

Many only see mathematics as a tool – and of course, it is in part a tool, but they should be reminded that, as Galileo said, the book of Nature is written in the mathematician's language.[1] *Since Galileo and Newton, the greatest physicists give examples that knowing mathematics provides the means to understand precise physical notions, to use them more easily, to establish them on a sure foundation, and even more importantly, to discover new ones.*[2] *In addition to ensuring a certain rigor in reasoning which is indispensable in any scientific study, mathematics belongs to the natural lexicon of physicists. Even if the rules of proportion and the fundamentals of calculus are sufficient for most purposes, it is clear that a richer vocabulary can lead to much deeper insights. Imagine if Shakespeare or Joyce had only had a few hundred words to chose from!*

Is is therefore discouraging to sometimes hear physicists dismiss certain theories because "this is only mathematics." In fact, the two disciplines are so closely related that the most prestigious mathematical award, the Fields medal, was given to the physicist Edward WITTEN *in* 1990*, rewarding him for the remarkable mathematical discoveries that his ideas led to.*

How should you read this book? Or rather how should you not read *some parts?*

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[1] *"Philosophy is written is this vast book which is forever open before our eyes – I mean, the Universe – but it can not be read until you have learned the tongue and understood the character in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric pictures, without which it is not humanly possible to understand a single word [...]."*

[2] I will only mention NEWTON (gravitation, differential and integral calculus), GAUSS (optics, magnetism, *all* the mathematics of his time, and quite a bit that was only understood much later), HAMILTON (mechanics, differential equations, algebra), HEAVISIDE (symbolic calculus, signal theory), GIBBS (thermodynamics, vector analysis) and of course EINSTEIN. One could write a much longer list. If Richard FEYNMAN presents a very "physical" description of his art in his excellent physics course [35], appealing to remarkably little formalism, it is nevertheless the fact that he was the master of the elaborate mathematics involved, as his research works show.

Since the reader may want to learn first certain specific topics among those present, here is a short description of the contents of this text:

- *The first chapter serves as a reminder of some simple facts concerning the fundamental notion of* convergence*. There shouldn't be much in the way of new mathematics there for anyone who has followed a "rigorous calculus" course, but there are many counter-examples to be aware of. Most importantly, a long section describes the traps and difficulties inherent in the process of exchanging two limits* in the setting of physical models. *It is not always obvious where, in physical reasoning, one has to exchange two* mathematical *limits, and many apparent paradoxes follow from this fact.*
- *The real beginning concerns the theory of integration, which is briefly presented in the form of the Lebesgue integral based on measure theory (Chapter 2). For many readers, this may be omitted in a first reading. Chapter 3 discusses the basic results and techniques of integral calculus.*
- *Chapters 4 to 6 present the theory of functions of a complex variable, with a number of applications:*
  - *the "residue method," which is an amazing tool for integral calculus;*
  - *some physical notions, such as* causality, *are very closely related to analyticity of functions on the complex plane (Section 13.4);*
  - *harmonic functions (such that* $\triangle f = 0$*) in two dimensions are linked to the real parts of holomorphic (analytic) functions (Chapter 5);*
  - *conformal maps (those which preserve angles) can be used to simplify boundary conditions in problems of hydrodynamics or electromagnetism (Chapter 6);*
- *Chapters 7 and 8 concern the theory of distributions ("generalized functions") and their applications to physics. These form a relatively self-contained subset of the book.*
- *Chapters 9 to 12 deal with Hilbert spaces, Fourier series, Fourier and Laplace transforms, which have too many physical applications to attempt a list. Chapter 13 develops some of those applications, and this chapter also requires complex analysis.*
- *Chapter 14 is a short (probably too short) introduction to the Dirac notation used in quantum mechanics:* kets $|\psi\rangle$ *and* bras $\langle\psi|$. *The notions of generalized eigenbasis and self-adjoint operators on Hilbert space are also discussed.*
- *Several precise physical problems are considered and solved in Chapter 15 by the method of Green functions. This method is usually omitted from textbooks on electromagnetism (where a solution is taken straight out of a magician's hat) or of field theory (where it is assumed that the method is known). I hope to fill a gap for students by presenting the necessary (and fairly simple) computations from beginning to end, using physicist's notation.*

- *Chapters 16 and 17 about tensor calculus and differential forms are also somewhat independent from the rest of the text. Those two chapters are only brief introductions to their respective topics.*
- *Chapter 18 has the modest goal of relating some notions of topology and group theory to the idea of* spin *in quantum mechanics.*
- *Probability theory, discussed in Chapters 19 to 21, is almost entirely absent from the standard physics curriculum, although the basic vocabulary and results of probability seem necessary to any physicist interested in theory (stochastic equations, Brownian motion, quantum mechanics and statistical mechanics all require probability theory) or experiments (Gaussian white noise, measurement errors, standard deviation of a data set...)*
- *Finally, a few appendices contain further reminders of elementary mathematical notions and the proofs of some interesting results, the length of which made their insertion in the main text problematic.*

Many physical applications, using mathematical tools with the usual notation of physics, are included in the text. They can be found by looking in the index at the items under "Physical applications."

Remarks on the biographical summaries

The short biographies of mathematicians which are interspersed in the text are taken from many sources:

- Bertrand HAUCHECORNE and Daniel SURATTEAU, *Des mathématiciens de A à Z*, Ellipses, 1996 (French).
- The web site of Saint-Andrews University (Scotland)

  www-history.mcs.st-andrews.ac.uk/history/Mathematicians

  and the web site of the University of Colorado at Boulder

  www.colorado.edu/education/DMP

  from which certain pictures are also taken.
- *The literary structure of scientific argument*, edited by Peter DEAR, University of Pennsylvania Press, 1991.
- Simon GINDIKIN, *Histoires de mathématiciens et de physiciens*, Cassini, 2000.
- *Encyclopædia Universalis*, Paris, 1990.

Translator's foreword

I am a mathematician and have now forgotten most of the little physics I learned in school (although I've probably picked up a little bit again by translating this book). I would like to mention here two more reasons to learn mathematics, and why this type of book is therefore very important.

First, physicists benefit from knowing mathematics (in addition to the reasons Walter mentioned) because, provided they immerse themselves in mathematics sufficiently to become fluent in its language, they will gain access to new *intuitions*. Intuitions are very different from any set of techniques, or tools, or methods, but they are just as indispensable for a researcher, and they are the hardest to come by.[3] A mathematician's intuitions are very different from those of a physicist, and to have both available is an enormous advantage.

The second argument is different, and may be subjective: physics is *hard*, much harder in some sense than mathematics. A very simple and fundamental physical problem may be all but impossible to solve because of the complexity (apparent or real) of Nature. But mathematicians *know* that a simple, well-formulated, natural mathematical problem (in some sense that is impossible to quantify!) *has a "simple" solution*. This solution may require inventing entirely new concepts, and may have to wait for a few hundred years before the idea comes to a brilliant mind, but it *is* there. What this means is that if you manage to put the physical problem in a very natural mathematical form, the guiding principles of mathematics may lead you to the solution. Dirac was certainly a physicist with a strong sense of such possibilities; this led him to discover antiparticles, for instance.

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[3] Often, nothing will let you understand the intuition behind some important idea except, essentially, rediscovering by yourself the most crucial part of it.

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