



Greiner Maruhn

NUCLEAR MODELS

核模型

Springer

世界图书出版公司

Walter Greiner · Joachim A. Maruhn

NUCLEAR MODELS

With a Foreword by
D. A. Bromley

With 50 Figures,
and 39 Worked Examples and Problems

Springer

世界图书出版公司

书 名: Nuclear Models
作 者: W. Greiner, J. A. Maruhn
中译名: 核模型
出 版 者: 世界图书出版公司北京公司
印 刷 者: 北京世图印刷厂
发 行: 世界图书出版公司北京公司 (北京朝内大街 137 号 100010)
联系电话: 010-64015659, 64038347
电子信箱: kjsk@vip.sina.com
开 本: 16 开 印 张: 25
出版年代: 2004 年 11 月
书 号: 7-5062-6586-9 / O · 440
版权登记: 图字:01-2004-2322
定 价: 74.00 元

世界图书出版公司北京公司已获得 Springer-Verlag 授权在中国大陆
独家重印发行。

Professor Dr. Walter Greiner
Professor Dr. Joachim A. Maruhn
Institut für Theoretische Physik der
Johann Wolfgang Goethe-Universität Frankfurt
Postfach 11 1932
D-60054 Frankfurt am Main
Germany

Street address:

Robert-Mayer-Strasse 8-10
D-60325 Frankfurt am Main
Germany

email:

greiner@th.physik.uni-frankfurt.de (W. Greiner)
maruhn@th.physik.uni-frankfurt.de (J. A. Maruhn)

Title of the original German edition: *Theoretische Physik, Ein Lehr- und Übungsbuch*,
Band 11: Kernmodelle © Verlag Harri Deutsch, Thun 1995

Die Deutsche Bibliothek – CIP-Einheitsaufnahme

Greiner, Walter:

Nuclear models : with 39 worked examples and problems / Walter Greiner ; Joachim A. Maruhn. With a foreword by
D. A. Bromley. – Berlin ; Heidelberg ; New York ; Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ;
Santa Clara ; Singapore ; Tokyo : Springer, 1996

Einheitssacht.: Kernmodelle <engl.>

ISBN 3-540-59180-X

NE: Maruhn, Joachim:

ISBN 3-540-59180-X Springer-Verlag Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1996

Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

This reprint has been authorized by Springer-Verlag (Berlin/Heidelberg/New York) for sale in the People's Republic of China only and not for export therefrom.

Foreword to Earlier Series Editions

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics – with mathematics, the most fundamental of sciences – using Walter Greiner's textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicists remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck and by Landau and Lifshitz. From the students' viewpoint, there are a great many obvious advantages to be gained through use of consistent notation, logical ordering of topics and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

The present five-volume set, *Theoretical Physics*, is in fact only that part of the complete set of textbooks developed by Greiner and his students that presents the quantum theory. I have long urged him to make the remaining volumes on classical mechanics and dynamics, on electromagnetism, on nuclear and particle physics, and on special topics available to an English-speaking audience as well, and we can hope for these companion volumes covering all of theoretical physics some time in the future.

What makes Greiner's volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common "it follows that . . ." which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools under study are applied to actual problems of interest to a working physicist. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1798–1857) in his *Positive Philosophy* who noted, "To understand a science it is necessary to know its history". This is all too often forgotten in modern physics teaching and the

bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner's lectures, which underlie these volumes, are internationally noted for their clarity, their completeness and for the effort that he has devoted to making physics an integral whole; his enthusiasm for his science is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again the reader will find that, after dipping into a particular volume to review a specific topic, he will end up browsing, caught up by often fascinating new insights and developments with which he had not previously been familiar.

Having used a number of Greiner's volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

Yale University
New Haven, CT, USA
1989

D. Allan Bromley
Henry Ford II Professor of Physics

Preface

Theoretical physics has become a many-faceted science. For the young student it is difficult enough to cope with the overwhelming amount of new scientific material that has to be learned, let alone to obtain an overview of the entire field, which ranges from mechanics through electrodynamics, quantum mechanics, field theory, nuclear and heavy-ion science, statistical mechanics, thermodynamics, and solid-state theory to elementary-particle physics. And this knowledge should be acquired in just 8–10 semesters during which, in addition, a Diploma or Master's thesis has to be worked on or examinations prepared for. All this can be achieved only if the university teachers help to introduce the student to the new disciplines as early on as possible, in order to create interest and excitement that in turn set free essential new energy. Naturally, all inessential material must simply be eliminated.

At the Johann Wolfgang Goethe University in Frankfurt we therefore confront the student with theoretical physics immediately in the first semester. Theoretical Mechanics I and II, Electrodynamics, and Quantum Mechanics I – an Introduction are the basic courses during the first two years. These lectures are supplemented with many mathematical explanations and much support material. After the fourth semester of studies, graduate work begins and Quantum Mechanics II – Symmetries, Statistical Mechanics and Thermodynamics, Relativistic Quantum Mechanics, Quantum Electrodynamics, the Gauge Theory of Weak Interactions, and Quantum Chromodynamics are obligatory. Apart from these, a number of supplementary courses on special topics are offered, such as Hydrodynamics, Classical Field Theory, Special and General Relativity, Many-Body Theories, Nuclear Models, Models of Elementary Particles, and Solid-State Theory. Some of them, for example the two-semester courses on Theoretical Nuclear Physics and Theoretical Solid-State Physics, are obligatory.

This volume is devoted to the Theory of Nuclear Models, which forms a two-semester cycle together with a course on Nuclear Reactions. For this field it appeared to be especially important to present a relatively short textbook actually suitable for accompanying a lecture, since while there are excellent and comprehensive treatises on nuclear models, the wealth of material presented in those tends to overwhelm the students initially. In this connection we mention preferentially the three-volume work by Eisenberg and Greiner,¹ on which the present treatment

¹ J. M. Eisenberg and W. Greiner, *Nuclear Theory*, 3 Volumes, Third Edition (North Holland, Amsterdam 1973–1987).

is based in many respects, and the textbook by Ring and Schuck,² which puts more emphasis on many-body approaches.

A textbook for direct use with a lecture has to concentrate on the most essential points, emphasize the explanation of ideas and methods, and forego the presentation of a wealth of individual results, which cannot be shown in a lecture anyway if it is not to degenerate into a slide show. Another characteristic that makes the theory of nuclear models different from the classical fields of theoretical physics is the scarcity of examples that can be calculated from start to finish without the use of computers.

For all of these reasons the focus is on the discussion of the most important types of models and the requisite mathematical methods. Since experience shows that most students have not really mastered the crucial methods of angular-momentum coupling and second quantization, we have not relegated these topics to an appendix but treated them at the start of the book. Of course these chapters can be ignored if desired. Even in these chapters the material was carefully restricted to what is actually used in the rest of the book. Following this there is a short discussion of group-theoretical methods, which are essential, for example, for the IBA model.

The fifth chapter treats the theory of the radiation field up to the definition of multipole transition probabilities. Again with a view to brevity the magnetic transitions are only discussed in general terms. The sixth chapter presents the classical collective models, which because of their didactic value and their fundamental importance for introducing concepts form the centerpiece of the book. A short overview of the phenomenological properties of nuclear matter is followed by a treatment of the geometric collective model (surface vibrations, the rotation-vibration model, etc.) in the various limiting cases, the IBA model, and the collective theory of giant resonances.

Only a little less space is devoted to microscopic models in Chapter 7. The most important concepts, from Hartree-Fock theory via phenomenological single-particle models to the relativistic mean-field model, are introduced successively. The next chapter treats the coupling of single-particle and collective motion both with respect to the particle-plus-core model and to the microscopic description of collective vibrations.

The final chapter presents large-amplitude collective motion, concentrating on ways to describe nuclear fission and similar processes. This includes two-center models, the general problem of collective mass parameters, time-dependent Hartree-Fock, the generator-coordinate method, and an elementary overview of high-spin states.

In addition to the classical syllabus of nuclear models that still form the basic equipment of the nuclear theorist, short discussions of topics of present-day interest are interspersed in many places – such as superheavy elements, high-spin states, and the relativistic mean-field model. These should give young physicists an impression of the continuing vitality of this science. The reader will also note in various places that the book is based on repeated practical experience with such a course and offers many explanations and illustrations motivated by typical student questions.

² P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York 1980).

We express our sincere thanks to Dr. Dirk Troltenier, Michael Bender, Christian Spieles, and Klemens Rutz for their efficient help in the formulation, formatting, and editing of the text as well as to Ms. Astrid Steidl for support in producing some of the graphics.

Finally we wish to thank Springer-Verag; in particular Dr. H. J. Kölsch for his encouragement and patience, and Petra Treiber for the production and Dr. Victoria Wicks for expertly copy-editing the English edition.

Frankfurt am Main,
November 1995

Walter Greiner
Joachim Maruhn

Contents

1. Introduction	1
1.1 Nuclear Structure Physics	1
1.2 The Basic Equation	2
1.3 Microscopic versus Collective Models	3
1.4 The Role of Symmetries	5
2. Symmetries	7
2.1 General Remarks	7
2.2 Translation	8
2.2.1 The Operator for Translation	8
2.2.2 Translational Invariance	9
2.2.3 Many-Particle Systems	10
2.3 Rotation	11
2.3.1 The Angular Momentum Operators	11
2.3.2 Representations of the Rotation Group	16
2.3.3 The Rotation Matrices	20
2.3.4 SU(2) and Spin	21
2.3.5 Coupling of Angular Momenta	25
2.3.6 Intrinsic Angular Momentum	27
2.3.7 Tensor Operators	30
2.3.8 The Wigner–Eckart Theorem	35
2.3.9 6j and 9j Symbols	37
2.4 Isospin	39
2.5 Parity	42
2.5.1 Definition	42
2.5.2 Vector Fields	42
2.6 Time Reversal	43
3. Second Quantization	47
3.1 General Formalism	47
3.1.1 Motivation	47
3.1.2 Second Quantization for Bosons	50
3.1.3 Second Quantization for Fermions	52
3.2 Representation of Operators	53
3.2.1 One-Particle Operators	53
3.2.2 Two-Particle Operators	56
3.3 Evaluation of Matrix Elements for Fermions	58
3.4 The Particle-Hole Picture	60

4. Group Theory in Nuclear Physics	65
4.1 Lie Groups and Lie Algebras	65
4.2 Group Chains	72
4.3 Lie Algebras in Second Quantization	73
5. Electromagnetic Moments and Transitions	75
5.1 Introduction	75
5.2 The Quantized Electromagnetic Field	75
5.3 Radiation Fields of Good Angular Momentum	77
5.3.1 Solutions of the Scalar Helmholtz Equation	77
5.3.2 Solutions of the Vector Helmholtz Equation	78
5.3.3 Properties of the Multipole Fields	81
5.3.4 Multipole Expansion of Plane Waves	82
5.4 Coupling of Radiation and Matter	85
5.4.1 Basic Matrix Elements	85
5.4.2 Multipole Expansion of the Matrix Elements and Selection Rules	88
5.4.3 Siegert's Theorem	90
5.4.4 Matrix Elements for Emission in the Long-Wavelength Limit	91
5.4.5 Relative Importance of Transitions and Weisskopf Estimates	95
5.4.6 Electric Multipole Moments	97
5.4.7 Effective Charges	97
6. Collective Models	99
6.1 Nuclear Matter	99
6.1.1 Mass Formulas	99
6.1.2 The Fermi-Gas Model	101
6.1.3 Density-Functional Models	103
6.2 Nuclear Surface Deformations	106
6.2.1 General Parametrization	106
6.2.2 Types of Multipole Deformations	108
6.2.3 Quadrupole Deformations	110
6.2.4 Symmetries in Collective Space	115
6.3 Surface Vibrations	117
6.3.1 Vibrations of a Classical Liquid Drop	117
6.3.2 The Harmonic Quadrupole Oscillator	124
6.3.3 The Collective Angular-Momentum Operator	128
6.3.4 The Collective Quadrupole Operator	130
6.3.5 The Quadrupole Vibrational Spectrum	132
6.4 Rotating Nuclei	138
6.4.1 The Rigid Rotor	138
6.4.2 The Symmetric Rotor	143
6.4.3 The Asymmetric Rotor	145

6.5	The Rotation–Vibration Model	147
6.5.1	Classical Energy	147
6.5.2	Quantal Hamiltonian	151
6.5.3	Spectrum and Eigenfunctions	155
6.5.4	Moments and Transition Probabilities	159
6.6	γ -Unstable Nuclei	168
6.7	More General Collective Models for Surface Vibrations	170
6.7.1	The Generalized Collective Model	170
6.7.2	Proton–Neutron Vibrations	177
6.7.3	Higher Multipoles	177
6.8	The Interacting Boson Model	178
6.8.1	Introduction	178
6.8.2	The Hamiltonian	180
6.8.3	Group Chains	182
6.8.4	The Casimir Operators	184
6.8.5	The Dynamical Symmetries	187
6.8.6	Transition Operators	192
6.8.7	Extended Versions of the IBA	193
6.8.8	Comparison to the Geometric Model	196
6.9	Giant Resonances	197
6.9.1	Introduction	197
6.9.2	The Goldhaber–Teller Model	200
6.9.3	The Steinwedel–Jensen Model	202
6.9.4	Applications	205
7.	Microscopic Models	207
7.1	The Nucleon–Nucleon Interaction	207
7.1.1	General Properties	207
7.1.2	Functional Form	210
7.1.3	Interactions from Nucleon–Nucleon Scattering	211
7.1.4	Effective Interactions	214
7.2	The Hartree–Fock Approximation	217
7.2.1	Introduction	217
7.2.2	The Variational Principle	218
7.2.3	The Slater–Determinant Approximation	219
7.2.4	The Hartree–Fock Equations	220
7.2.5	Applications	225
7.2.6	The Density Matrix Formulation	227
7.2.7	Constrained Hartree–Fock	229
7.2.8	Alternative Formulations and Three-Body Forces	230
7.2.9	Hartree–Fock with Skyrme Forces	231
7.3	Phenomenological Single-Particle Models	237
7.3.1	The Spherical-Shell Model	237
7.3.2	The Deformed-Shell Model	248
7.4	The Relativistic Mean-Field Model	261
7.4.1	Introduction	261
7.4.2	Formulation of the Model	261
7.4.3	Applications	266

7.5	Pairing	269
7.5.1	Motivation	269
7.5.2	The Seniority Model	272
7.5.3	The Quasispin Model	278
7.5.4	The BCS Model	280
7.5.5	The Bogolyubov Transformation	285
7.5.6	Generalized Density Matrices	290
8.	Interplay of Collective and Single-Particle Motion	293
8.1	The Core-plus-Particle Models	293
8.1.1	Basic Considerations	293
8.1.2	The Weak-Coupling Limit	294
8.1.3	The Strong-Coupling Approximation	296
8.1.4	The Interacting Boson–Fermion Model	302
8.2	Collective Vibrations in Microscopic Models	303
8.2.1	The Tamm–Dancoff Approximation	303
8.2.2	The Random-Phase Approximation (RPA)	309
8.2.3	Time-Dependent Hartree–Fock and Linear Response	312
9.	Large-Amplitude Collective Motion	317
9.1	Introduction	317
9.2	The Macroscopic-Microscopic Method	318
9.2.1	The Liquid-Drop Model	318
9.2.2	The Shell-Correction Method	320
9.2.3	Two-Center Shell Models	324
9.2.4	Fission in Self-Consistent Models	335
9.3	Mass Parameters and the Cranking Model	337
9.3.1	Overview	337
9.3.2	The Irrotational-Flow Model	337
9.3.3	The Cranking Formula	338
9.3.4	Applications of the Cranking Formula	340
9.4	Time-Dependent Hartree–Fock	344
9.5	The Generator-Coordinate Method	346
9.6	High-Spin States	353
9.6.1	Overview	353
9.6.2	The Cranked Nilsson Model	355
	Appendix: Some Formulas from Angular-Momentum Theory	359
	References	363
	Subject Index	369

Contents of Examples and Exercises

2.1	Cartesian Form of the Angular-Momentum Operator \hat{J}_z	12
2.2	Cayley–Klein Representation of the Rotation Matrix	24
2.3	Coupling of Two Vectors to Good Angular Momentum	31
2.4	The Position Operator as an Irreducible Spherical Tensor	33
2.5	Commutation Relations of the Position Operator	34
2.6	The Two-Nucleon System	41
2.7	The Time-Reversal Operator for Spinors with Spin $\frac{1}{2}$	46
3.1	Two-Body Operators in Second Quantization	57
4.1	The Lie Algebra of Angular-Momentum Operators	68
4.2	The Casimir Operator of the Angular-Momentum Algebra	69
4.3	The Lie Algebra of $SO(n)$	70
5.1	The Vector Spherical Harmonics	79
5.2	The Weisskopf Estimates for Electric Transitions	96
6.1	Volume and Center-of-Mass Vector for a Deformed Nucleus	109
6.2	Quadrupole Deformations	113
6.3	Angular-Momentum Operator for Quadrupole Phonons in Second Quantization	129
6.4	^{114}Cd as a Spherical Vibrator	136
6.5	Angular-Momentum Operators in the Intrinsic System	139
6.6	States with Angular Momentum 2 in the Asymmetric Rotor Model	146
6.7	The Time Derivatives of the $\alpha_{2\mu}$	150
6.8	Transformation of the Quadrupole Operator	162
6.9	Quadrupole Moments and Transition Probabilities	162
6.10	^{238}U in the Rotation–Vibration Model	166
6.11	Casimir Operators of $U(N)$	187
6.12	Contributions to the Thomas–Reiche–Kuhn Sum Rule	199
7.1	The Angular Average of the Tensor Force	210
7.2	Matrix Elements in the Variational Equation	224
7.3	The Skyrme Energy Functional	234
7.4	The Eigenfunctions of the Harmonic Oscillator	240
7.5	The Quadrupole Moment of a Nucleus	246
7.6	Single-Particle Energies in the Deformed Oscillator	254
7.7	The Crossing of Energy Levels	259
7.8	Pairing in a $j = \frac{7}{2}$ Shell	278

8.1 The Spectrum of ^{183}W in the Strong-Coupling Model 301

8.2 Tamm–Dancoff Calculation for ^{16}O 306

8.3 Derivation of the Tamm–Dancoff Equation 308

8.4 The Extended Schematic Model 311

9.1 The Cranking Formula for the BCS Model 341

9.2 The Harmonic Oscillator in the Generator–Coordinate Method ... 348

1. Introduction

1.1 Nuclear Structure Physics

The nuclear models discussed in this book belong to the realm of *nuclear structure theory*. In present usage, nuclear structure physics is devoted to the study of the properties of nuclei at low excitation energies, where individual energy levels can be resolved. This means that typically quantum effects are predominant and the states of the nucleus have a very complicated structure that depends on the intricate interrelations of all the many nucleons involved.

In contrast, at higher energies and especially for heavy-ion reactions, quantum mechanics becomes less important and the preeminent place is instead given to methods of statistical mechanics. Theories then typically employ bulk properties of nuclear matter such as the equation of state or the dissipation coefficients, or are even based on purely classical many-body physics like the cascade models.

Of course it is impossible to give an exact energy boundary between these types of theories. The theories presented here, however, are typically employed for excitation energies up to 2–3 MeV. Usually only the lowest few energy levels can be described well by a theoretical model, and the number of levels increases so rapidly above that energy range that it becomes impossible to make any sensible comparison with experiment (for nuclei with an odd number of neutrons or protons or both this is even more dramatic – most nuclear models prefer even–even nuclei with their relatively simple spectra). Also one should remember that in experimental spectra only a relatively small number of states can be identified as to spin and parity, and that to really test a model transitions, i.e., essentially overlaps between the wave functions, are needed, which again are often not known even for the most interesting states.

It is thus not surprising that the models presented in this book usually explain a relatively small number of low-lying states and to a modest accuracy, and even this is a considerable achievement. To esteem that, remember that we are dealing with a system of particles whose number is neither small enough to allow direct solution nor large enough to make statistical methods highly accurate, and which interact through an interaction that has still not been pinned down to any definite form. It is this extraordinary difficulty and the freedom with which methods and ideas from many other branches are applied here that make nuclear structure physics so fascinating and so much alive.

1.2 The Basic Equation

To find the proper theoretical starting point some more ballpark estimates of the relevant physical quantities have to be introduced. Let us first recall a few numbers from elementary experimental nuclear physics.

The elements known at the time of this writing have nuclei consisting of (at present) $Z = 1, \dots, 111$ protons and $N = 0, \dots, 161$ neutrons, giving a total number of A nucleons. The radii of nuclei follow the empirical law

$$R(A) = r_0 A^{1/3} \quad (1.1)$$

with $r_0 \approx 1.2$ fm. Nuclear radii thus range up to about 7.5 fm. The formula also implies that the nuclear volume is proportional to the number of particles in the nucleus, indicating the near incompressibility of nuclear matter (the true density profile observed by electron scattering is a bit more complicated). The least-bound nucleon has a binding energy of the order of 8 MeV and a kinetic energy close to 40 MeV.

This information is already sufficient to form some rough ideas about what is essential in the theories. Since a nucleon has a mass of $mc^2 \approx 938$ MeV, the kinetic energy is quite negligible by comparison, so that a *nonrelativistic* approach appears quite sufficient, and this assumption is made in the vast majority of nuclear structure models. More recently, however, relativistic approaches have become important – this theme is taken up in Sect. 7.4 in connection with the relativistic mean-field model, and we also explain there why relativistic effects can be important in spite of the simple estimate given above.

The velocity of a nucleon with a kinetic energy of $T = 40$ MeV is given by

$$v = \sqrt{\frac{2T}{m}} = c \sqrt{\frac{2T}{mc^2}} \approx 0.3 c \quad , \quad (1.2)$$

and the associated de Broglie wavelength by

$$\lambda = \frac{2\pi\hbar}{mv} = \frac{2\pi(\hbar c)}{(mc^2)(v/c)} \approx 4.5 \text{ fm} \quad . \quad (1.3)$$

Here the useful constant $\hbar c \approx 197.32$ MeV fm was used. The result shows that quantum effects are certainly not negligible, as λ is by no means small compared to the nuclear radii. This is even more pronounced for the more tightly bound nucleons, which have a smaller kinetic energy.

Taking these considerations into account, the starting point for a theory of nuclear eigenstates should be a stationary Schrödinger equation very generally given by

$$\hat{H} \psi = E \psi \quad . \quad (1.4)$$

The rest of this book is about what to write for \hat{H} and which degrees of freedom to use in the wave functions.