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**图书在版编目(CIP)数据**

已实现波动率度量及其建模研究/田金方著.—济南:山东大学出版社,2012.10
ISBN 978-7-5607-4689-0

Ⅰ.①已…
Ⅱ.①田…
Ⅲ.①期货交易—研究②股票交易—研究
Ⅳ.①F713.35②F830.91

中国版本图书馆 CIP 数据核字(2012)第 265388 号

**山东大学出版社出版发行**
(山东省济南市山大南路 20 号　邮政编码:250100)
山东省新华书店经销
济南景升印业有限公司印刷
880 毫米×1230 毫米　1/32　5 印张　106 千字
2012 年 10 月第 1 版　2012 年 10 月第 1 次印刷
定价:20.00 元

# 前　言

自从 Black & Scholes(1973)建立期权定价公式以来，波动率就在风险管理、衍生产品定价以及资产组合等金融领域中扮演着至关重要的角色。在 Black & Scholes(1973)的期权定价公式中，作者假定波动率是一常数，这一假设严重违背了金融数据的实证特征，现实金融市场的波动率不但随着时间的变化而变化，而且波动率还具有一些的典型特征——长记忆性，聚合性、不对称性等(Ghysels，Harvey & Renault，1996；Poon & Granger，2003 等)。因此，在过去的几十年里，众多研究者和实际工作者广泛关注波动率的动态性质，深究资产收益率波动的背后驱动因素。

伴随着 Engle(1982)的 ARCH 模型，众多条件异方差模型如雨后春笋般涌现出来，形成了基于平方收益的 GARCH 族和 SV 族模型。这些目前普遍使用的方法至多使用日频率的金融数据，并将波动率看成是不可观测的潜变量，通过建立模型来估计这些未知的资产收益波动率。众所周知，传统的潜变量模型有很多缺陷，例如，模型的参数估计困难，没有利用高频数据，标准化收益率不是正态分布或近似正态分布，预测不精确，且很难推广到多维情形。

最近，Anderson et al(2000)和其他经济学家提出一种新的非参数波动率估计方法，该方法利用日内高频信息，得到

事后波动率的“可观测”估计量——已实现波动率。

标准形式的已实现波动率仅仅是给定时间区间上高频日内收益平方和的平方根,即高频收益率二阶非中心子样矩的平方根。在有效市场假设下,随着日内收益率的增加,日内收益平方和收敛到相应时间区间上的名义波动率或者可积波动率。因此,原则上,已实现波动率可以为我们提供一个一致的非参数波动率估计量。

已实现波动率估计量不依赖于任何模型,没有估计误差,因此我们可以将波动率看成一个“可观测”的变量,而不是GARCH(1,1)等模型所假设的潜变量。这样,已实现波动率就开创了波动率研究的新时代,它能够使我们直接分析、建模和预测波动率,并且许多复杂的动态模型也能方便地直接估计和最优化,再不用依赖于估计方法复杂的潜变量模型。更重要的是,从预测的观点来看,较好的波动率估计量能够很好地分析资产收益率波动的背后驱动因素,提高波动率的预测精度。

不幸的是,由于市场微观结构效应的存在,随着数据频率的增加,市场微观结构效应的干扰作用变大,对数资产价格也随之偏离扩散过程的假设。在分笔交易数据中,实际数据违背了已实现波动率的常规假设——无摩擦的连续价格过程,高频时间区间上计算的已实现波动率已不再是日波动率的一致估计。

尽管一些经济学家研究了市场微观结构效应影响下的已实现波动率估计和预测问题,但是他们的分析结论具有这样那样的缺陷。

首先,为了解决微观结构效应带来的高频估计偏度,许

多学者认为估计波动率的频率不易过高。Andersen 及其合作者在研究道琼斯 30 只股票的基础上，提出利用 5 分钟频率的数据度量已实现波动率，这样可以最大程度地减少微观结果效应的作用。之后，他们又提出了已实现波动率的"特征图"方法，用其选择最优的数据区间。然而，由于不同的子样区间以及不同的股票市场可能具有不同的最优数据区间，因此 Andersen 方法很难得到同一效度的波动率估计，同时该方法浪费掉很多高频数据。

其次，在波动率的建模中，通常利用已实现波动率的分整可积模型来刻画波动率的长记忆性，例如：ARFIMA、FIGARCH 等模型。这类模型仅仅考虑数学公式的演算和推导，没有考虑模型的经济含义，同时，分整差分算子的利用使得模型构建区间加长，这样会损失很多原始观测数据，更重要的是，这些模型只能刻画证券市场价格过程标定规律的单重分形特点，而不能描述许多研究工作最近发现的多重分形特征。为了克服分整可积模型的缺陷，Corsi (2004)基于异质市场假说构建了已实现波动率模型，他将市场参与者的投资行为划分为三类，分别对应波动率的三个异质市场驱动因素，提出了不同波动率驱动成分的简单叠加模型，命名为 HAR。虽然 HAR 模型简单，但模拟分析和实证分析都显示 HAR 模型能成功地刻画波动率的主要典型特征(长记忆性，重尾，自相似性)，且 HAR 模型具有合理的经济解释。然而，根据异质市场假说，异质市场的驱动因素有很多成分，HAR 模型实质上仅仅考虑了一种异质驱动因素——投资者投资行为，而没有考虑诸如投资者心理和市场交易机制等其他的典型异质市场驱动成分。因此，HAR 模型也就没能完全刻

画波动率的典型特征，例如：不对称效应等。

本书的第一个目的是利用高频数据的滤子技术，剔除市场微观结构效应的影响，给出纠偏后的新的已实现波动率度量。第二个目的是利用度量好的已实现波动率，在充分考虑多成分异质市场驱动因素的基础上，提出新的波动率模型——HAR-L-M 模型，它能给我们提供优越并且简单易行的波动率预测方法。

首先，我们分析了分笔日内收益率的伪自相关系数，得出股票指数和个股具有不同的市场微观结构效应，个股主要受微观结构噪音的影响，股指除了受微观结构噪音影响之外，还受非同步交易效应的影响。因此，为了纠正已实现波动率估计的偏度，我们借助于外汇市场的指数加权平均滤子(EWMA)，剔除个股的市场微观结构噪音。对于股票指数，我们提出两阶段滤子法来消除市场微观结构效应，具体来说，在第一阶段，我们利用 EWMA 滤子剔除影响股指的微观结构噪音，在第二阶段，我们使用自回归滤子(AR)剔除影响股指的非同步交易，我们将此两阶段滤子方法取名为 EWMA-AR 滤子。通过分析过滤前后已实现波动率的阶行为，我们发现上述滤子方法能有效地纠正沪市的已实现波动率偏度。

其次，我们分析了股市已实现波动率的典型特征，包括收益率分布、波动率分布、波动率的非对称效应以及长记忆性等。

最后，为了克服 HAR 模型的缺陷，基于异质市场假说，我们除了考虑 HAR 模型的投资行为异质成分，还引入了另外两个异质成分——投资者投资心理和市场交易机制。正

是由于这些的典型异质驱动因素，金融数据才显示出我们常见的典型特征，例如长记忆性、尖峰重尾性、自相似性和非对称性等。我们将此扩展的模型命名为 HAR-L-M 模型。模拟分析结果显示 HAR-L-M 模型能很好地刻画金融数据的典型特征，更符合中国股市的多重分析特征。模型估计和预测的分析结果也显示 HAR-L-M 模型具有很好的样本外预测能力，预测精度远远优于分整可积的长记忆模型以及 HAR 模型。

# PREFACE

Since Black and Scholes (1973) established the theory of option pricing, volatility has played an important role not only in the derivatives pricing but also in portfolio selection and risk management. Despite of the assumption of constant volatility in Black and Scholes (1973), it is widely recognized that volatility changes over time, and other various stylized facts about volatility have been documented (see, e.g., Ghysels, Harvey, and Renault (1996) and Poon and Granger (2003). These facts have motivated many academic researchers and practitioners to study the dynamics of volatility over the last three decades.

Since the first conditional volatility model by Engle (1982), thousands of papers concerning conditional heteroskedasticity have been published, these led to the vast ARCH-GARCH and stochastic volatility literature based on squared returns. These prevailing approaches employed (at most) daily data and considered volatility as an unobservable variable that can be estimated through models. As we all know, the traditional latent variable models have several drawbacks, for example, it is difficult to estimate its parameters, high frequency data is not utilized, standardized returns are not Gaussian, forecasting is imprecise and multivariate extensions are difficult.

Most recently, Anderson, Bollerslev, Diebold, and Laby (2000) and other economists developed a new nonparametric estimator of volatility which fully exploits intraday information to develop observable proxies for the ex-post volatility: realized volatility.

In its standard form realized volatility is nothing more than the square root of the sum of squared high-frequency returns over a given non-vanishing time interval, i. e. the second uncentered sample moment of the high-frequency returns. Under very general conditions the sum of intraday squared returns converges (as the number of intraday return increases) to the relevant notion of volatility of the interval. Thus, realized volatility provides us, in principle, with a consistent nonparametric measure of volatility.

A model-free and error-free estimation of volatility would allow us to treat volatility as an observable variable, rather than a latent one, as in the GARCH(1,1) model for example. This would open the possibility to directly analyze, model, and forecast volatility itself. Therefore, more sophisticated dynamic models can be directly estimated and optimized without having to rely on the complicated estimation procedures needed when volatility is assumed to be unobserved. For forecasting purposes, moreover, a better estimate of the target function allows to better extract the real underlying signal and then improve the forecasting performance.

Unfortunately, because of market microstructures effects, the assumption that log asset prices evolve as a diffusion process becomes less realistic as the time scale reduces. At the tick time scale, the empirical data differ from the frictionless continuous-time price process assumed in the standard theory of realized volatility. Thus, simple realized volatility measures computed with very short time intervals are no longer unbiased and consistent estimators of daily volatilities.

Although some economists have developed and investigated the measuring and forecasting methods of realized volatility under effects of market microstructure, there remain some deficiencies.

First of all, because data of very high frequency would bring more errors due to microstructure effects, most authors thought the frequency of data used to estimate volatility should not be too high to erase market microstructure effects. Andersen and his coworker chose a data interval of 5 minutes when studying the volatility of DAIJ30 stocks. Later, they developed a method called "signature plot" to select proper data interval. However, the optimum data interval to balance the usual measuring error and the microstructure effects would not be unique along all the sample periods. It seems to vary in different periods and different markets. At the same time, this method wasted a lot of high frequency data.

Secondly, long memory volatility is usually obtained by employing fractional difference operators like in the FIGARCH models of returns or ARFIMA models of realized volatility. However, fractionally integrated models pose some problems. Fractional integration is a convenient mathematical trick but completely lacks a clear economic interpretation. Moreover, the application of the fractional difference operator requires a very long build up period which results in a loss of many observations. Finally, these kinds of models are able to reproduce only the unifractal type of scaling but not the empirical multi-fractal behavior found in many recent works. So, inspired by the Heterogeneous Market Hypothesis (HMH) and drawbacks of fractional integration, Corsi (2004) propose an additive cascade of different volatility components generated by the actions of different types of market participants, and termed this model HAR. In spite of the simplicity of its structure, simulation results and empirical analyses seemed to confirm that the HAR model successfully achieved the purpose of reproducing the main empirical features of volatility (long memory, fat tail, self-similarity) in a very simple and parsimoniously way, and bears a clear economic interpretation. According to HMH, heterogeneous market is driven by several components, however, HAR model only considered one heterogeneous component of market-actors investment behavior, and did not include other heterogeneous component, such as actors' investment psychology and market trading mechanism etc. So, HAR model did not reproduce other main features of volatility, such as asymmetry etc.

The first purpose of this thesis is to develop new realized volatility estimators of Chinese Shanghai Stock Market which, while fully exploiting all the available information contained in very high frequency data, are able to effectively correct for the bias induced by microstructure effects. The second purpose is to extend HAR model and develop, by building on such highly accurate realized volatility measures, new conditional volatility models able to provide superior and easy-to-implement volatility forecasts.

First, we analyses spurious autocorrelation of intraday tick-by-tick returns, and draw a conclusion that stock index and single stock have different market microstructure effects, single stock is mainly affected by microstructure noise and stock index is also affected by non-synchronized transaction besides noise. So, in order to remove the cause of the bias from the raw tick-by-tick time series, with exponential weighting moving average (EWMA) filter which was used in Foreign Exchange market, we eliminate effects of noise on single stock, and regarding stock index, we propose two stage filter method to eliminate effects of market microstructure. Specifically, at the first stage, we eliminate effects of noise on stock index using EWMA filter, at the second stage, we use autoregression (AR) filter to eliminate effects of non-synchronized transaction, therefore, we term this two stage filter EWMA-AR. The result is an effective reduction of the realized volatility bias for Chinese Shanghai Stock Market data, particularly for the most liquid stocks by analyzing the scaling law of realized volatility.

Secondly, we investigate some characteristics of the volatility for Chinese Shanghai Stock Market data, such as the distribution of return, the distribution of volatility, the asymmetric effect of volatility, and the long memory effect of volatility.

Thirdly, inspired by the HMH and drawbacks of HAR model, we consider three heterogeneous components of market, and they are actors' investment behavior which is only component in the HAR model, actors' investment psychology and market trading mechanism. Owing to these heterogeneous driven components, financial data show some stylized features, such as long memory, fat tail, self-similarity, asymmetry etc. we term this extended model HAR-L-M. Simulation results seem to confirm that the HAR-L-M model successfully achieves the purpose of reproducing the main empirical features of financial data and multifractal market. Results on the estimation and forecast of the HAR-L-M model on Chinese Shanghai Stock Market data, show remarkably good out of sample forecasting performance which seems to steadily and substantially outperform those of standard models.

# 目 录