钱学森力学手稿

8

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西安交通大学出版社
XI'AN JIAOTONG UNIVERSITY PRESS

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出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长，学习他的科学思想和卓越风范，展示其丰功伟绩和人格魅力，西安交通大学举办了"纪念钱学森诞辰100周年"系列活动：作为制片方之一，参与西部电影集团摄制传记故事片《钱学森》；与中央电视台合作，出品纪录片《实验班的故事——沿着钱学森走过的路》；扩建钱学森生平业绩展馆，向校内外开放；举办钱学森科学与教育思想研讨会；出版发行《钱学森力学手稿》、《钱学森年谱（初编）》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿，这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风，是钱老勇攀科学高峰和严谨治学的集中体现。这里，我们将部分原稿整理汇集成册，出版《钱学森力学手稿》，作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷，包含两部分内容。第一部分是草稿，包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时，为了求得圆柱壳体的临界压力，在有关的五百多页草稿中，对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算，最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表，每个数字有效位数都长达八位，在手摇机械式计算机作为主要计算工具的年代，这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿，以航空航天工程为核心，涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下，由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助，在此深表感谢。

Contents

Section 1

Two Dimensional Subsonic Flow of Compressible Fluids

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Summary

The basic concept of the present paper is to use ~~the~~ a tangent line to the adiabatic pressure-volume curve as an approximation to the curve itself. First, the general characteristics of such ~~an~~ fluid are shown. Then in Section I, a theory is developed which in ~~main features~~ principle is similar to that of Demtchenko and Busemann but is more general and can be applied to flow with velocity approaching that of sound.]] The theory is then applied to calculate the flow over elliptic cylinders. In Section II the work of H. Bateman is applied to this approximate adiabatic fluid and ~~two equations are given which express the relations between the velocity and the pressure distributions over a body in compressible flow to the velocity and the pressure distributions over the same body in incompressible flow. These equations are then used to predict the high speed characteristics of airfoils.~~ the results obtained are essentially the same as those obtained in Section I.

]] The theory is put into a form, by which, knowing the incompressible flow over a body, the compressible flow over a similar body ~~near to the one in incompressible flow~~ can be calculated.

TWO-DIMENSIONAL SUBSONIC FLOW

OF COMPRESSIBLE FLUIDS

by

Hsue-shen Tsien
California Institute of Technology

Introduction

Assuming that the pressure is a single-valued function of density only, the equations of two-dimensional irrotational motion of compressible fluids can be reduced to a single non-linear equation of the velocity potential. In the supersonic case, that is, in the case when ~~the~~ flow velocity is everywhere greater than that of local sound velocity, the problem is solved by Meyer & Prandtl and Busemann using the method of characteristics. The essential difficulty of this problem lies in the subsonic case, that is, in the case when ~~the~~ flow velocity is everywhere smaller than but near ~~to~~ the local sound velocity, because then the method of characteristics cannot be used. Glauert & Prandtl (Ref. 1) treated the case when the disturbance to ~~the~~ parallel rectilinear flow due to presence of a solid body is small. They were then able to linearize the differential equation for the velocity potential and obtained an equation very similar to ~~these~~ that for ~~the~~ incompressible fluids. But there are usually stagnation points either in the surface of the body or in the field of flow, where the disturbance is no longer small. Hence, it is doubtful whether the linear theory can be applied to the flow near a stagnation point. On the same ground, the theory breaks down in the case of bodies whose dimension across the stream is not small compared with the dimension parallel to the stream.

To treat these cases ~~in which the body is blunt-nosed~~, Janzen and Rayleigh developed the method of successive approximations. This method ~~was explained physically and~~ was put into a more convenient form by L. Poggi ~~(Ref. 2)~~ and P.A. Walther ~~(Ref. 3)~~. Recently C. Kaplan (Ref. 2) treated the case of flow over Joukowsky airfoils and elliptic cylinders, using Poggi's method. However, the method is rather tedious and the convergence very slow if the local velocity of sound is approached.

Begin Molenbroek ~~(Ref. 5)~~ and Tschapligin ~~(Ref. 6)~~ suggested the use of the magnitude of velocity w and inclination β of velocity to a chosen axis as independent variables, and were thus able to reduce the equation of velocity potential to a linear equation. This equation was solved by Tschapligin. ~~(Ref. 6) and recently put into a more convenient form by F. Clauser and M. Clauser (Ref. 7)~~. The solution is essentially a series each term of which is a product of a hypergeometric function of w and a trigonometric function of β. The main difficulty in practical application of this solution is to obtain a proper set of boundary conditions in the plane of independent variables w, β and to put the solution in a closed form.

Tschapligin ~~(Ref. 6)~~ has shown that a great simplification of the equation in the hodograph plane results if the ratio of the specific heats of the gas is equal to -1. Since all real gases have their ratio of specific heats between 1 and 2, the value -1 seems without practical significance. It was Demtchenko (Ref. 3) and Busemann (Ref. 4) who clarified the meaning of this specific value of -1. They found that this really means to take the tangent of the pressure-volume curve as an approximation to the curve itself. However, they limit themselves to use the tangent at the state of ~~rest of~~ the gas, corresponding to the stagnation point of flow. Thus their theory can only be applied to a flow with velocities up to about one half ~~that of sound velocity~~. Recently, during a personal discussion, Th. von Kármán suggested to the author that the theory can be generalized ~~to~~ use the tangent at the state of gas corresponding to ~~the~~ undisturbed parallel flow. Thus the range of usefulness of the theory can be greatly extended. This is carried out in the first ~~part~~ of the present paper.

~~In the second part of this paper~~ this theory based upon Demtchenko and Busemann's work is applied to the case of flow over elliptic cylinders and the results compared with those of S.G. Hooker (Ref. ~~10~~) and C. Kaplan (Ref. ~~4~~). Furthermore, results calculated by Glauert-Prandtl linear theory are also included for comparison.

Recently, H. Bateman (Ref. ~~11~~) demonstrated a remarkable reciprocity of two fields of flow of two fluids related by a certain point transformation. It will be shown in the ~~third part~~ of this paper that the flow of incompressible fluid and the flow of compressible fluid approximated by the use of tangent to adiabatic pressure-volume curve can be interpreted as such a point transformation. It is thus possible to obtain a solution for compressible flow whenever a solution of incompressible flow is known. ~~The difficulty, however, lies in~~

This transformation from a flow of incompressible fluid to a flow of compressible fluid is found, however, essentially to be same as that developed ~~in Section I~~ from Demtchenko's and Busemann's work. ~~The only difference~~ is

~~body at any Mach's number whenever the low speed characteristics of the flow over the same body are known. The characteristics of the~~

incompressible flow can either be obtained by the well-known method of conformal mapping or by experiments. Due to the fact that practical aerodynamic engineers usually have the low speed characteristics at hand and that high speed data have to be obtained by use of a costly high speed wind tunnel the above mentioned relations are believed to be of considerable use to them.

~~In the fourth part of the paper, the~~ This theory ~~developed in Part III~~ is ~~applied~~ then used to correlate airfoil data obtained by J. Stack (Ref. ~~12~~ 7) in the N.A.C.A. 24" high speed wind tunnel. The agreement with theory is found to be satisfactory. Then this theory is applied to predict the compressibility effect on the lift and moment of N.A.C.A. 4412 airfoil using experimentally determined pressure distribution over the same airfoil at low speed. The result is again compared with the more simple Glauert-Prandtl theory.

Approximation to the Adiabatic Relation

~~Approximation to Adiabatic~~

If p is the pressure, v is the specific volume and γ is the ratio of specific heats of a gas, the adiabatic relation $p v^{-\gamma} =$ constant is a curve in the p-v plane as shown in Fig. 1a. ~~Now~~ The conditions near the point (p_1, v_1) which corresponds to ~~the~~ a state of ~~[illegible]~~ undisturbed flow can be approximated by the tangent to the curve at that point. The equation of the tangent at this point can be written as:

$$p_1 - p = C(v_1 - v) = C(\rho_1^{-1} - \rho^{-1}) \qquad (1)$$

a state of undisturbed ~~fluid~~ flow —

where C is the slope of the tangent and ρ is the density of the fluid. The slope C must be equal to the slope of the curve at the point (p_1, v_1), therefore,

$$C = \left(\frac{dp}{dv}\right)_1 = \left(\frac{dp}{d\rho}\frac{d\rho}{dv}\right)_1 = -\left(\frac{dp}{d\rho}\right)_1 \rho_1^2 = -a_1^2\rho_1^2$$

where a_1 is the sound velocity corresponding to the conditions p_1, v_1. Thus eq. (1) can be written as:

$$p_1 - p = a_1^2\rho_1^2\left(\frac{1}{\rho} - \frac{1}{\rho_1}\right) \tag{2}$$

This is an approximation pressure-density to the true adiabatic relation, and is shown in Fig. 1 together with the true adiabatic relation.

The generalized Bernoulli theorem for compressible fluids is

$$\frac{1}{2}w_2^2 - \frac{1}{2}w_3^2 = \int_2^3 \frac{dp}{\rho} \tag{3}$$

where w is the velocity of the gas and the subscripts 2 and 3 denote two different states of the fluid. By substituting eq. (2) into eq. (3), the following relation is obtained:

$$\frac{1}{2}w_2^2 - \frac{1}{2}w_3^2 = \frac{1}{2}a_1^2\rho_1^2\left\{\frac{1}{\rho_2^2} - \frac{1}{\rho_3^2}\right\} \tag{4}$$

Now if $w_3 = 0$, $w_2 = w$, $\rho_3 = \rho_0$, and $\rho_2 = \rho$, with the subscript 0 denoting the state of the fluid corresponding to the stagnation point of flow, eq. (4) gives:

$$\frac{a_1^2\rho_1^2}{\rho_0^2} + w^2 = \frac{\rho_1^2 a_1^2}{\rho^2} \tag{5}$$

If the square of sound velocity a^2 is defined (as is usually done) as the derivative of p with respect to ρ, eq. (2) gives:

$$a^2\rho^2 = \frac{dp}{d\rho}\rho^2 = a_1^2\rho_1^2 = \text{constant} \qquad (6)$$

Therefore, eq. (5) can be written as:

$$\left(\frac{\rho}{\rho_0}\right)^2 = 1 - \frac{w^2}{a^2} \qquad (7)$$

Similarly,

$$\left(\frac{\rho_0}{\rho}\right)^2 = 1 + \frac{w^2}{a_0^2} \qquad (8)$$

It is interesting to ~~notice~~ note that from eq. (8) the density decreases as the velocity increases, as maybe expected. Thus eq. (6) ~~shows~~ indicates that the local velocity of sound increases as the velocity increases. This is just opposite to ~~the~~ a real gas, because in the case of an adiabatic flow of a real gas it is well known that the temperature of the gas decreases as the velocity ~~of gas is increased, and~~ [of the] increases thus the local sound velocity, [being proportional to the square root of the temperature,] also decreases. However, in the present approximate theory, the ratio $\frac{w}{a}$ or Mach's number still increases as the velocity increases, as can be seen by eq. (7). But this ratio only reaches the value unity when $\rho = 0$, or from eq. (8) when $w = \infty$. It is thus seen that the entire regime of flow is subsonic and thus the differential equation of the velocity potential [is always of an elliptic type, that is, always of the same type as the differential equation of the velocity potential of incompressible fluids.] ~~of an incompressible~~. This is the reason why the complex representation of the velocity potential and the stream function is possible for all cases, as will be shown in the following paragraphs. However, one should realize that the portion of the tangent that could be used as an approximation to the true adiabatic relation is that portion which lies in the first quadrant. Thus the upper limit ~~of~~ velocity, for practical application of the theory ~~is~~ occurs at $p = 0$. By using eqs. (17) and (18), this upper limit is found to be

$$\left(\frac{w}{w_1}\right)_{max} = \frac{1}{\left(\frac{w_1}{a_1}\right)} \sqrt{\left(\frac{p_1}{a_1^2 \rho_1} + 1\right)^2 - \left\{1 - \left(\frac{w_1}{a_1}\right)^2\right\}} \tag{9}$$

Since the point (p_1, ρ_1), being the tangent point to the true adiabatic curve, lies on the ~~true adiabatic~~ curve, the relation $a_1^2 = \gamma \frac{p_1}{\rho_1}$, which is true for the adiabatic relation $p\rho^{-\gamma}$ = constant, can be used, and eq. (9) becomes:

$$\left(\frac{w}{w_1}\right)_{max} = \frac{1}{\left(\frac{w_1}{a_1}\right)} \sqrt{\left(\frac{1}{\gamma} + 1\right)^2 - \left\{1 - \left(\frac{w_1}{a_1}\right)^2\right\}} \tag{10}$$

This relation is plotted in Fig. 2 with $\gamma = 1.405$. Since for most practical cases it is not likely that the ratio $\left(\frac{w}{w_1}\right)$ will rise to values much higher than 2, p will remain positive, and this theory will be sufficient to give an approximate solution.

Section I

~~General Theory~~ Hodograph Method

If the flow is irrotational, there exists a velocity potential ϕ such that

$$\frac{\partial \phi}{\partial x} = u, \qquad \frac{\partial \phi}{\partial y} = v \tag{11}$$

where u, v are the components of w in the x and y direction, respectively. The equation of continuity,

$$\frac{\partial}{\partial x}\left(\frac{\rho}{\rho_0} u\right) + \frac{\partial}{\partial y}\left(\frac{\rho}{\rho_0} v\right) = 0$$

will be satisfied, if the stream function ψ is introduced such that

$$\frac{\rho}{\rho_0} u = \frac{\partial \psi}{\partial y}, \qquad -\frac{\rho}{\rho_0} v = \frac{\partial \psi}{\partial x} \tag{12}$$

Now if the angle of inclination of the velocity w to the x axis is β, eqs. (11) and (12) give:

$$d\phi = w\cos\beta\, dx + w\sin\beta\, dy$$
$$d\psi = -w\frac{\rho}{\rho_0}\sin\beta\, dx + w\frac{\rho}{\rho_0}\cos\beta\, dy \qquad (13)$$

Solving for dx and dy,

$$dx = \frac{\cos\beta}{w}d\phi - \frac{\sin\beta}{w}\frac{\rho_0}{\rho}d\psi$$
$$dy = \frac{\sin\beta}{w}d\phi + \frac{\cos\beta}{w}\frac{\rho_0}{\rho}d\psi \qquad (14)$$

So long as the correspondence between the physical ~~plane~~ and hodograph plane is one to one, or mathematically $\frac{\partial(x,y)}{\partial(u,v)} \neq 0$, ~~one can~~ expressed x and y can be as functions of w, β, and ~~as~~ ϕ and ψ as functions of w, β. Thus,

$$d\phi = \phi'_w\, dw + \phi'_\beta\, d\beta \qquad (15)$$
$$d\psi = \psi'_w\, dw + \psi'_\beta\, d\beta$$

where primes indicate the derivative with respect to the independent variables indicated as subscripts. Now substituting eq. (15) into eq. (14), ~~one has~~ the following expressions for dx + dy are obtained:

$$dx = \left(\frac{\cos\beta}{w}\phi'_w - \frac{\sin\beta}{w}\frac{\rho_0}{\rho}\psi'_w\right)dw + \left(\frac{\cos\beta}{w}\phi'_\beta - \frac{\sin\beta}{w}\frac{\rho_0}{\rho}\psi'_\beta\right)d\beta$$

$$dy = \left(\frac{\sin\beta}{w}\phi'_w + \frac{\cos\beta}{w}\frac{\rho_0}{\rho}\psi'_w\right)dw + \left(\frac{\sin\beta}{w}\phi'_\beta + \frac{\cos\beta}{w}\frac{\rho_0}{\rho}\psi'_\beta\right)d\beta \qquad (16)$$

Since the left-hand side of eqs. (16) are exact differentials, ~~one can apply~~ the reciprocity relation can be applied and ~~obtain~~ therefore:

$$\frac{\partial}{\partial\beta}\left(\frac{\cos\beta}{w}\phi'_w - \frac{\sin\beta}{w}\frac{\rho_0}{\rho}\psi'_w\right) = \frac{\partial}{\partial w}\left(\frac{\cos\beta}{w}\phi'_\beta - \frac{\sin\beta}{w}\frac{\rho_0}{\rho}\psi'_\beta\right)$$
$$\frac{\partial}{\partial\beta}\left(\frac{\sin\beta}{w}\phi'_w + \frac{\cos\beta}{w}\frac{\rho_0}{\rho}\psi'_w\right) = \frac{\partial}{\partial w}\left(\frac{\sin\beta}{w}\phi'_\beta + \frac{\cos\beta}{w}\frac{\rho_0}{\rho}\psi'_\beta\right) \qquad (17)$$

Carrying out these differentiations and simplifying with the aid of eq. (7), eq. (17) gives:

$$-\frac{\sin\beta}{w}\phi'_w - \frac{\cos\beta}{w}\frac{\rho_0}{\rho}\psi'_w = -\frac{\cos\beta}{w^2}\phi'_\beta + \frac{\sin\beta}{w^2}\frac{\rho}{\rho_0}\psi'_\beta$$
$$\frac{\cos\beta}{w}\phi'_w - \frac{\sin\beta}{w}\frac{\rho_0}{\rho}\psi'_w = -\frac{\sin\beta}{w^2}\phi'_\beta - \frac{\cos\beta}{w^2}\frac{\rho}{\rho_0}\psi'_\beta \quad (18)$$

Solving for ϕ'_w and ψ'_β,

$$\phi'_w = -\frac{\rho}{\rho_0}\frac{1}{w}\psi'_\beta$$
$$\phi'_\beta = \frac{\rho_0}{\rho} w\,\psi'_w \quad (19)$$

Eq. (19) can be further simplified by introducing a new variable ω, such that

$$d\omega = \frac{\rho}{\rho_0}\frac{dw}{w} \quad (20)$$

Then eq. (19) becomes:

$$\phi'_\omega = -\psi'_\beta$$
$$\phi'_\beta = \psi'_\omega \quad (21)$$

This can be easily recognized as the Riemann-Cauchy differential equations, and thus $\phi + i\psi$ must be an analytic function of $\omega - i\beta$. However, for convenience of calculation, another new set of independent variables $U = W\cos\beta$, $V = W\sin\beta$ are introduced where $W = a_0 e^{\omega}$

Then eq. (21) can be written as:

$$\frac{\partial\phi}{\partial U} = \frac{\partial\psi}{\partial(-V)}$$
$$\frac{\partial\phi}{\partial(-V)} = -\frac{\partial\psi}{\partial U} \quad (22)$$

By integrating eq. (20),

$$W = \frac{2a_0 w}{\sqrt{a_0^2 + w^2} + a_0} \quad (23)$$