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航天器姿态动力学中的混沌

Chaos in Attitude Dynamics of Spacecraft

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内 容 简 介

航天器混沌姿态运动的识别和控制问题在航天科学中具有重要实际意义。本书致力于总结该领域的近期发展，提供研究航天器姿态运动的新方法和观点，也为该领域进一步的深入分析研究提供有明确工程背景的新的数学模型。读者可从本书获得混沌和混沌控制理论及其在航天器姿态运动中应用的知识，包括基本概念，主要方法以及最新进展。

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Preface

The development of spacecraft has drawn considerable attentions in the field of dynamics since the 1950s. The spacecraft can be regarded as a particle or as a body, depending on whether one focuses on the spacecraft's orbital motion or on its rotational motion about the center of mass. Spacecraft attitude dynamics deals with the rotational motion of spacecraft. In the discussion of attitude dynamics, the rotation of spacecraft is usually assumed not to alter the orbit, while the orbit sometimes influences the rotational motion. Almost all spacecraft have some attitude requirements, either explicit pointing requirements for antennas or cameras, requirements for solar panel orientation, or simply a requirement for a given spin-axis direction. All the requirements are implemented by the design of attitude controls. The strategies chosen in the control process may limit the useful lifetime of the spacecraft, since an all-thruster control system depletes its propellant supply. Attitude dynamics forms a theoretical basis of the design and control of spacecraft. The present monograph is concerned with spacecraft attitude motion, although essential elements of orbital dynamics will be introduced and the effects of orbital motion will be included in a few cases.

With the development of nonlinear dynamics, chaos in spacecraft attitude dynamics has stirred renewed interests since the 1990s. In fact, for astronautical investigations, the predictability of spacecraft rotations is critical, and thus chaotic motions must be avoided. On the other hand, there are scientific experiments that require the whole celestial sphere to be scanned, and in those cases the chaotic rotation may be desirable. Therefore chaos theory offers a new method and viewpoint for designing spacecraft. In addition, spacecraft attitude dynamics also provides new mathematical models for engineering application of chaos analysis. Although there are some excellent monographs and textbooks on spacecraft attitude dynamics, there are few treatises on chaotic attitude motion. The present monograph focuses on chaos in spacecraft attitude dynamics.

The monograph begins with the necessary fundamentals. Chapter 1 provides a primer on spacecraft dynamics, and Chapter 2 presents a survey of chaos theory. Different chaotic attitude motions are treated in Chapters 3 and 4. Chapter 3 considers only the planar motion of spacecraft, while Chapter 4 covers the spatial motion. The monograph ends with Chapter 5, dealing with controlling chaotic attitude motion.

The main goal of the monograph is to provide readers with the knowledge of theory and application of chaos and its control in spacecraft attitude dynamics, including the basic concepts, main approaches and the latest research progress. The material is appropriate for university teachers, scientists, engineers, and graduate students in the fields of mechanics, applied mathematics, and aerospace science.

Except for some background presented in Chapters 1 and 2, as well as Sections 4.1 and 5.1, all other materials contained in the monograph are adopted from research papers of the authors and their co-workers. The research work was financially supported by the National Natural Science Foundation of China (Project Nos. 19782003 and 10082003), the National Outstanding Young Scientists Foundation of China (Project No. 10725209), Shanghai Municipal Development Foundation of Science and Technology (Project Nos. 98JC14032 and 98SHB1417), Shanghai Municipal Education Commission Scientific Research Project (No. 2000A12), and Shanghai Leading Academic Discipline Project (No. Y0103). The first author thanks his former PhD students Professor Peng Jianhua, Professor Chen Liqun, Dr. Cheng Gong, and his postdoctoral fellow Professor Yu Hongjie for their collaborations on related research. The second author thanks Professor Liu Yanzhu, who, serving as his PhD supervisor, introduced him to this field. He also thanks his hosts, Professor Jean W. Zu (University of Toronto) and Professor C. W. Lim (City University of Hong Kong) for their assistance during his visit to their institutes so that he could complete his portions of the book.

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Contents

Chapter 1 Primer on Spacecraft Dynamics

Abstract This chapter provides a fundamental theory of spacecraft dynamics. After a brief survey of gravitational field, the two-body problem is summarized as a simplified model of orbit motion of a spacecraft around the Earth. The main environmental torques acting on spacecraft, the gravitational torque and magnetic torque are introduced. The dynamical equations of attitude motion of a spacecraft are established, where the Euler's equations and Poisson's equations are applied for a rigid spacecraft in gravitational field. The stability problems of the relative equilibrium of a rigid spacecraft in circular orbit under gravitational torque are analyzed by using the first approximation method and the Lyapunov's direct method. The attitude motions of a gyrostat are analyzed as a model of spacecraft with axisymmetric rotors. The permanent rotations and its stability of a spinning spacecraft are discussed under torque-free assumption.

Keywords orbit dynamics, two-body problem, gravitational torque, magnetic torque, Euler's equations, Poisson's equations, torque-free rigid bodies, gyrostats

This chapter presents fundamental theory of spacecraft dynamics that will be needed in chapters 3 and 4. The chapter begins with elementary orbit dynamics, which provides necessary background for attitude dynamics in addition to its own merits. After a brief survey of gravitational field of a particle and a rigid body, a satellite around the Earth is modeled as the two-body problem, and the first integrals are derived from the dynamical equations as the energy integral, the momentum integral, the Laplace integral and the time integral. The Keplerian orbit is discussed with the emphasis on the elliptic motion. Then the chapter turns to attitude dynamics by introducing main environmental torques acting on spacecraft, the gravitational torque and the magnetic torque. Euler's equations and Poisson's equations are applied to the attitude motion of spacecraft in the gravitational field. As two significant special solutions to Euler's equations and Poisson's equations, planar libration and spatial relative equilibrium are analyzed. The dynamical equations of a gyrostat are also developed. The chapter ends with attitude motion of torque-free rigid bodies and gyrostats. The influence of energy dissipation to spinning spacecraft is investigated. The complete treatments of spacecraft dynamics can be found in [1–10].

1.1 Orbital Motion of Spacecraft

1.1.1 Gravitational Field of a Particle

According to Newton's law of universal gravitation, a particle m is attracted by another particle m_e by a force

$$\boldsymbol{F} = -G\frac{mm_e}{r^2}\left(\frac{\boldsymbol{r}}{r}\right) \tag{1.1.1}$$

where $\boldsymbol{r}$ denotes the position vector of point m with respect to point m_e, m and m_e stand for the masses of corresponding points, and $G = 6.67\times10^{-11}\ \mathrm{m^3/kg\cdot s^2}$ is the universal gravitational constant (Fig. 1.1).

Define the potential function U of the gravitational field produced by the point m_e as

$$U = \frac{Gm_e}{r} = \frac{\mu}{r} \tag{1.1.2}$$

where $\mu = Gm_e$ is a constant depending only on point m_e. The gravitational force $\boldsymbol{F}$ acting on point m can be written as

$$\boldsymbol{F} = m\nabla U \tag{1.1.3}$$

where $\nabla = (\partial/\partial x)\boldsymbol{i} + (\partial/\partial y)\boldsymbol{j} + (\partial/\partial z)\boldsymbol{k}$, and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ represent basis-vectors of a reference coordinate frame $(O\text{-}xyz)$ with point m_e as the origin O. Define $V = -mU$ as the potential energy of point m in the gravitational field of particle m_e.

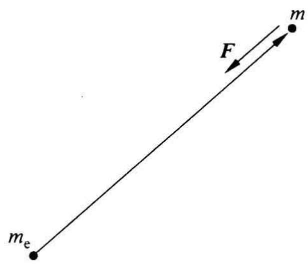

Figure 1.1 Gravitational force of a particle

1.1.2 Gravitational Field of a Rigid Body

To discuss the gravitational field of a rigid body, the central principal axes of a body are established as the reference coordinate frame $(O_e\text{-}xyz)$, where O_e is the mass center of a body. The position vectors of a particle m with respect to the point O_e and an arbitrary point P of the body are denoted as $\boldsymbol{r}$ and $\boldsymbol{r}'$, respectively. It follows that $\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{\rho}$, where $\boldsymbol{\rho}$ is the position vector of point P with respect to point O_e (Fig. 1.2). Let α_i $(i = 1,2,3)$ be the direct cosines of the vector $\boldsymbol{r}$ relative to axes of $(O_e\text{ - }xyz)$, and x, y, z be the coordinates of point P in $(O_e\text{ - }xyz)$. Then the vector $\boldsymbol{r}'$ can be written as

$$\boldsymbol{r}' = (r\alpha_1 - x)\boldsymbol{i} + (r\alpha_2 - y)\boldsymbol{j} + (r\alpha_3 - z)\boldsymbol{k} \tag{1.1.4}$$

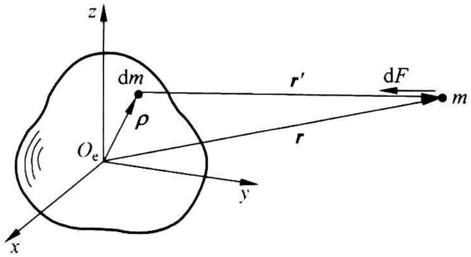

Figure 1.2 Gravitational force of a rigid body

The potential function U of a body is defined as

$$U = G\iiint_S \frac{\mathrm{d}m}{r'} \tag{1.1.5}$$

where the domain of integration S is the whole body. When $\rho \ll r$, substituting Eq. (1.1.4) into Eq. (1.1.5) and considering only the second terms of ρ/r, one obtains

$$U = \frac{Gm_e}{r}\left[1 + \frac{1}{2m_e r^2}(A + B + C - 3I)\right] \tag{1.1.6}$$

where m_e is the mass, and A, B, C are the principal moments of inertia of a body in $(O_e\text{ - }xyz)$, respectively,

$$A = \iiint_S (y^2 + z^2)\mathrm{d}m,\ B = \iiint_S (z^2 + x^2)\mathrm{d}m,\ C = \iiint_S (x^2 + y^2)\mathrm{d}m \tag{1.1.7}$$

and I is defined as

$$I = A\alpha_1^2 + B\alpha_2^2 + C\alpha_3^2 \tag{1.1.8}$$

In the case when a body is axisymmetric with respect to z-axis, let $A = B$ and introduce $\mu = Gm_e$ as the gravitational parameter. Equation (1.1.6) can be simplified as

$$U = \frac{\mu}{r}\left[1 - \frac{C - A}{2m_e r^2}(3\alpha_3^2 - 1)\right] \tag{1.1.9}$$

For a sphere-symmetric body, $A = C$. Thus

$$U = \frac{\mu}{r} \tag{1.1.10}$$

which is the same as Eq. (1.1.2). It means that the gravitational field of a spherical body is equivalent to that of a particle, in which the whole body mass is located in its mass center. Equation (1.1.9) or (1.1.10) can be used to express the gravitational field of the Earth, which has the gravitational parameter $\mu = 398\,601.19\,\text{km}^3/\text{s}^2$.

1.1.3 Dynamical Equations of Two-body System

Assume that the Earth may be simplified as a rigid sphere. The orbital motion of a satellite around the Earth can be treated as the two-body problem (m_e, m) with particle m as a satellite attracted by particle m_e as the Earth. Let O_e denote the mass center of this system, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ denote the position vectors of m and m_e with respect to O_e. Then three points m, m_e and O_e are collinear with the following relationship (Fig. 1.3):

$$m\boldsymbol{r}_1 + m_e\boldsymbol{r}_2 = 0 \tag{1.1.11}$$

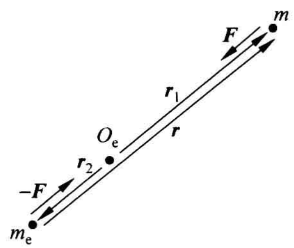

Figure 1.3 Two-body system

The dynamical equations of points m and m_e can be derived from Newton's second law as

$$m\boldsymbol{r}_1 = \boldsymbol{F}, \quad m\boldsymbol{r}_e = -\boldsymbol{F} \tag{1.1.12}$$

where $\boldsymbol{F}$ is the gravitational force acting on point m,

$$\boldsymbol{F} = -G\frac{mm_e}{r^2}\,\frac{\boldsymbol{r}}{r} \tag{1.1.13}$$

Substitution of $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ into Eq. (1.1.12) leads to

$$\boldsymbol{r} = \left(1 + \frac{m}{m_{\mathrm{e}}}\right)\boldsymbol{r}_1 = -\left(1 + \frac{m_{\mathrm{e}}}{m}\right)\boldsymbol{r}_2 \tag{1.1.14}$$

Thus the following equation can be derived from Eq. (1.1.12)

$$\ddot{\boldsymbol{r}} + \frac{\mu}{r^3}\boldsymbol{r} = 0 \tag{1.1.15}$$

where $\mu = G(m_{\mathrm{e}} + m)$, which is approximately equal to the gravitational parameter of the Earth $\mu = Gm_{\mathrm{e}}$, and the mass center O_{e} coincides with m_{e} with sufficient accuracy since $m \ll m_{\mathrm{e}}$. Introduce the velocity $\boldsymbol{v}$ of point m and then rewrite the dynamical Eq. (1.1.15) as

$$\dot{\boldsymbol{v}} + \frac{\mu}{r^3}\boldsymbol{r} = 0 \tag{1.1.16a}$$

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \tag{1.1.16b}$$

1.1.4 First Integrals

(1) Energy Integral

Dot-multiplying each term of Eq. (1.1.16a) by $\boldsymbol{v} = \dot{\boldsymbol{r}}$, and observing that $\boldsymbol{v} \cdot \dot{\boldsymbol{v}} = v\dot{v}$, $\boldsymbol{r} \cdot \dot{\boldsymbol{r}} = r\dot{r}$, one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{v^2}{2} - \frac{\mu}{r}\right) = 0 \tag{1.1.17}$$

Thus the integral of energy can be derived as

$$\frac{v^2}{2} - \frac{\mu}{r} = E \tag{1.1.18}$$

where $v^2/2$ and μ/r are, respectively, the kinetic energy and the potential energy of a satellite with unit mass, and the constant E is the conserved total specific energy.

(2) Integral of Angular Momentum

Cross-multiplying each term of Eq. (1.1.16a) with $\boldsymbol{r}$ leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r} \times \boldsymbol{v}) = 0 \tag{1.1.19}$$

from which the integral of angular momentum is derived as

$$\boldsymbol{r} \times \boldsymbol{v} = \boldsymbol{h} \tag{1.1.20}$$

where $\boldsymbol{h}$ is the conserved specific angular momentum of a satellite with respect to the Earth center O_e. Since the constant vector $\boldsymbol{h}$ is orthogonal to vectors $\boldsymbol{r}$ and $\boldsymbol{v}$, the orbital plane composed of vectors $\boldsymbol{r}$ and $\boldsymbol{v}$ has a fixed orientation in space. In order to determine the spatial position of the orbital plane, define an inertial reference frame $(O_e - X_0Y_0Z_0)$ with the mass center of the Earth O_e as the origin, where Z_0-axis is parallel to the polar axis of the Earth, the plane (X_0, Y_0) is parallel to the equatorial plane of the Earth, and X_0-axis is along the node of the ecliptic plane and the equatorial plane with direction to the first point of Aries. A celestial sphere is fixed on $(O_e - X_0Y_0Z_0)$ with center O_e and arbitrary radius. Within two intersection points of the node line of plane (X_0, Y_0) and the orbital plane with the celestial sphere, select point N corresponding to the ascension of a satellite as the ascending point. The angle Ω between O_eN and O_eX_0 is defined as the right ascension of the ascending node. The incline angle i of the orbital plane with respect to the plane (X_0, Y_0) is defined as the inclination angle of the orbital plane. Therefore, the orientation of the orbital plane can be determined by two angles Ω and i (Fig. 1.4). Denote the angle between the velocity $\boldsymbol{v}$ and

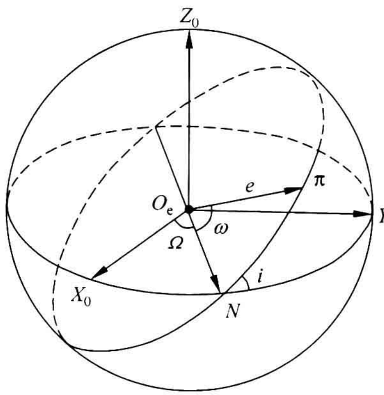

Figure 1.4 Celestial sphere and orbital plane

the local horizontal plane by θ. Then the area $\mathrm{d}S$ swept by the position vector $\boldsymbol{r}$ in time interval $\mathrm{d}t$ can be calculated as (Fig. 1.5)

$$\mathrm{d}S = \frac{1}{2} r(v\mathrm{d}t)\cos\theta = \frac{1}{2}|\boldsymbol{r} \times \boldsymbol{v}|\mathrm{d}t = \frac{1}{2} h\mathrm{d}t \tag{1.1.21}$$

Then the magnitude of vector $\boldsymbol{h}$ is equal to two times the area velocity swept by position vector $\boldsymbol{r}$.

$$h = 2\frac{\mathrm{d}S}{\mathrm{d}t} \tag{1.1.22}$$

It means that the satellite moves in the orbit with a constant area velocity.

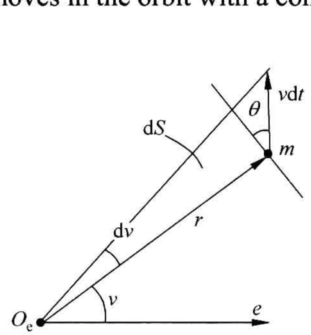

Figure 1.5 Area swept by vector $\boldsymbol{r}$

(3) Laplace's Integral

Cross-multiplying each term of Eq. (1.1.16a) by $\boldsymbol{h}$, one obtains

$$\dot{\boldsymbol{v}}\times\boldsymbol{h}+\frac{\mu}{r^3}\boldsymbol{r}\times\boldsymbol{h}=\frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{v}\times\boldsymbol{h}-\frac{\mu\boldsymbol{r}}{r}\right)=0 \tag{1.1.23}$$

from which the Laplace's integral is derived as follows

$$\boldsymbol{v}\times\boldsymbol{h}-\frac{\mu\boldsymbol{r}}{r}=\boldsymbol{e} \tag{1.1.24}$$

Since both vectors $\boldsymbol{v}\times\boldsymbol{h}$ and $\boldsymbol{r}$ lie in the orbital plane, the constant vector $\boldsymbol{e}$ should be also restricted to the same plane (Fig. 1.4). The magnitude of vector $\boldsymbol{e}$ can be determined by constants E and h as

$$e^2=\frac{1}{\mu^2}\left(\boldsymbol{v}\times\boldsymbol{h}-\frac{\mu\boldsymbol{r}}{r}\right)^2=1+\frac{2Eh^2}{\mu^2} \tag{1.1.25}$$

Consequently, the Laplace's integral provides only one scalar relationship to specify the location of vector $\boldsymbol{e}$ in the orbital plane. The angle ω between $\boldsymbol{e}$ and O_eN is selected as an independent constant, which is called the orbit angle of perigee. Dot-multiplying $\boldsymbol{r}$ by $\boldsymbol{e}$ leads to

$$\boldsymbol{r}\cdot\boldsymbol{e}=\boldsymbol{r}\cdot\left(\frac{1}{\mu}\boldsymbol{v}\times\boldsymbol{h}-\frac{\boldsymbol{r}}{r}\right)=p-r \tag{1.1.26}$$

where p is called the semi-parameter expressed as

$$p=\frac{h^2}{\mu} \tag{1.1.27}$$

Let the angle ν between the position vector $\boldsymbol{r}$ and vector $\boldsymbol{e}$ be the angular coordinate of point m in the orbital plane. ν is called the true anomaly of a satellite. Since $\boldsymbol{r}\cdot\boldsymbol{e}=re\cos\nu$, comparing it with Eq. (1.1.26) yields

$$r=\frac{p}{1+e\cos\nu} \tag{1.1.28}$$

Equation (1.1.28) determining the position of point m in the orbital plane is a conic section with eccentricity e. Therefore, vector $\boldsymbol{e}$ is called the eccentricity vector. The angle u between vector $\boldsymbol{r}$ and O_eN is used as another angular coordinate to specify the location of point m in the orbital plane.

$$u=\nu+\omega \tag{1.1.29}$$

(4) Time Integral

In order to determine the relationship between the position and the time, Eq. (1.1.22) is rewritten as

$$r^2\frac{\mathrm{d}\nu}{\mathrm{d}t}=h \tag{1.1.30}$$

Thus the angular velocity of radius-vector $\boldsymbol{r}$ in the orbital plane can be expressed as

$$\frac{\mathrm{d}\nu}{\mathrm{d}t}=\frac{\sqrt{\mu p}}{r^2} \tag{1.1.31}$$

Substitution of Eq. (1.1.28) into Eq. (1.1.31) leads to the time integral as

$$t=t_0+\sqrt{\frac{p^3}{\mu}}\int_0^\nu\frac{\mathrm{d}\nu}{(1+e\cos\nu)} \tag{1.1.32}$$

where t_0 is the time at $\nu=0$, i.e. the time of passing the perigee.

Aforementioned first integrals contain 8 integration constants: E, h, Ω, I, ω, p, e, and t_0, in which only 6 constants are needed in order to determine the motion of point m. When 6 constants are chosen, the other 2 can be calculated by Eqs. (1.1.25) and (1.1.27). The 6 independent integration constants are called orbital elements.

1.1.5 Characteristics of Keplerian Orbit

Establish a reference frame (O_e - $\xi\eta\zeta$) in the orbital plane with O_e as the origin, $O_e\xi$ along the eccentricity vector $\boldsymbol{e}$, and $O_e\zeta$ normal to the plane. Since $r(\nu)=r(-\nu)$, the orbit curve is symmetrical with respect to $O_e\xi$. The intersection point of the orbit and the vector $\boldsymbol{e}$ is called the perigee, and denoted by π, which has a minimum distance to point O_e. The distance between m and O_e is equal to the semi-parameter p when the orbit intersects $O_e\eta$ (Fig. 1.6).

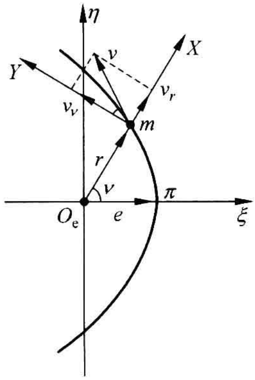

Figure 1.6 Keplerian orbit

According to the character of conic section, the orbit curve belongs to different types determined by the eccentricity e: ellipse $(e<1)$, parabola $(e=1)$, or hyperbola $(e>1)$. As the hyperbolic curve is unrestricted in space, in order to ensure the positiveness of v^2, the constant of the energy integral (1.1.18) should be $E>0$. On the contrary, when $E<0$, the range of point m is restricted by $r\leqslant \mu/|E|$ and corresponds to an ellipse. The parabola is a critical case when $E=0$. The velocity v_p of a parabolic orbit can be obtained from Eq. (1.1.18) as

$$v_p=\sqrt{\frac{2\mu}{r}} \tag{1.1.33}$$

which is called the parabolic velocity or the escape velocity. Thus the following criteria can be derived:

$$E<0,\ v<v_p:\ \text{ellipse}$$

$$E=0,\ v=v_p:\ \text{parabola}$$

$$E>0,\ v>v_p:\ \text{hyperbola}$$

Establish a cylindrical coordinates frame (O_e - XYZ) with radial axis O_eX along the vector $\boldsymbol{r}$, transverse axis O_eY towards the advanced direction of motion, and the normal axis O_eZ parallel to $O_e\zeta$. The true anomaly ν is the angle between two coordinate planes (X,Y) and (ξ,η). (O_e - XYZ), rotating around O_e with angular velocity $\mathrm{d}\nu/\mathrm{d}t$, is called the orbital reference frame. Equations (1.1.31) and (1.1.28) yield, respectively, the radial velocity v_X and transverse velocity v_Y of point m