普通高等教育“十二五”规划教材

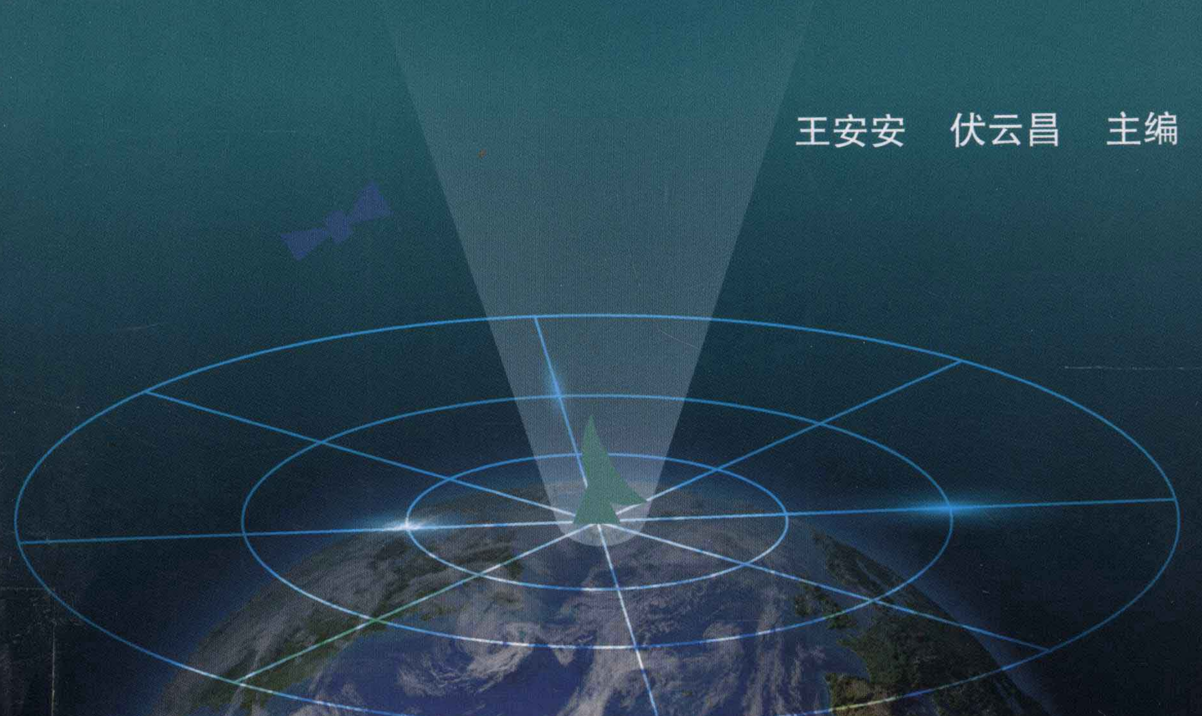
A Concise Course in University Physics

Second Edition Volume 1

大学物理简明教程（英文版）

第二版

上册



王安安 伏云昌 主编

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王安安 伏云昌 主编
王安安 樊则宾 编写

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北京

内 容 简 介

本书是按照《理工科类大学物理课程教学基本要求(2010 年版)》修订的,涵盖了所有 A 类的内容,选择了部分 B 类的内容,如非惯性系、质心、气体的范德瓦尔斯方程、玻尔兹曼分布、几何光学、固体能带论和激光简介等.为方便上、下两学期的教学安排,本次改版将原三册改编为上、下两册.全书共 19 章:上册为力学和电磁学;下册为热学、振动与波动、光学和近代物理.本书配有双语课件光盘.

本书可供理工科非物理专业 112～128 学时双语教学使用,也可供在某一部分内容进行双语教学试点选用,还可供对英文物理感兴趣的广大读者自学或作参考书用.

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第二版前言

根据《理工科类大学物理课程教学基本要求(2010年版)》对教学过程的基本要求第5点"双语教学——在保证教学效果的前提下,有条件的学校可开展物理课程的双语教学,以提高学生查阅外文资料和科技外语交流的能力",为满足坚持双语教学的师生对英文物理教材的需要,我们对 *A Concise Course in University Physics*(《大学物理简明教程(英文版)》,1998~2000年出版)进行了部分修订. 按2010年版教学基本要求,第二版涵盖了所有A类的内容,保留原有的并新选了部分B类内容,如增加了非惯性系和惯性力、质心和质心运动定理、热力学第零定律、范德瓦尔斯方程及几何光学等内容. 为方便目前工科物理两学期教学内容的安排,将第一版的三册改编为上、下两册,供非物理专业112~128学时双语物理教学之用. 第二版保留了第一版的主要特色,力求系统完整、理论叙述准确、文字简明易懂,以达到教师好用、学生易学的目的. 因篇幅所限,第二版省去了各章总结和三个阅读材料.

全书19章编写分工如下:

上册:力学(第1~5章)由王安安编写;电磁学(第6~9章)主要由樊则宾编写.

下册:热学(第10,11章)由王安安编写;振动与波动部分第12、14章及第15章中的几何光学由伏云昌编写,第15章中的波动光学由陈劲波编写,第13章由王安安和陈劲波编写;近代物理(第16~19章)由伏云昌编写. 王安安和伏云昌负责全书的统稿、修改和定稿工作. 为方便本书读者进行双语多媒体课堂教学,伏云昌编制了配套教学课件.

北京大学陆果教授对第二版全书进行了认真的审定,并提出了宝贵的意见和建议,谨此致以诚挚谢意! 编者也对昆明理工大学校领导多年来对我们工作的支持表示衷心的感谢!

双语教学贵在坚持,我们编写及使用英文物理教材的初衷未改,通过双语教学提高学生的综合素质始终是我们的目标,编者愿与致力于这项事业的同行共同努力,持之以恒,为物理教学改革尽绵薄之力!

因时间仓促、水平有限,书中难免有不当之处,恳请同行与读者提出宝贵意见.

编　者

2011年12月于昆明

第一版前言摘要

我们正处在一个高新技术飞速发展、科技信息量激增、知识更新加快、国际交流日益广泛的时代. 我国的进一步改革开放,社会主义市场经济的建立都要求高校毕业生有更强的适应能力,在人才市场上,有效强的外语应用能力、交流型、综合型的毕业生供不应求. 在这种形势下,我们的高等教育正向着重视素质教育的方向转变,而素质与能力是密切相关的,素质的培养要以一定的知识和能力为基础,其中包括独立获取知识的能力. 毋庸置疑,直接用外语为工具获取知识、进行交流的能力是人才素质的一个重要方面.

然而,由于历史原因、文化背景、经济基础、外语教育模式和各类师资外语水平等诸多因素的影响,我们在应用外语进行教学方面的基础性工作十分薄弱. 在普通高校本科生教育中,教材和教学过程基本上只使用中文这一单一语种,实际上已经制约了学生应用外语(主要是英语)获取知识能力的发展. 为了改变这种现状,跟上时代的步伐,试用英文教材,使用英语进行教学的改革便应运而生了.

本教材的编写是编者主持的"试用英文物理教材"教改试点工作的继续,也是编者大学物理教学经验的总结. 从国外引进的教材,虽有诸多优点,但在系统上与我国的大学物理教学基本要求不完全对应,为了满足师生对英文物理教材的需要,编写一套根据我国工科物理教学基本要求,顺应工科物理教学改革的形势,反映编者在多年物理教学实践中总结出来的教学方法与经验,与我们的学生在一、二年级的英文水平相适应的英文"简明物理学教程"的计划就提到日程上来了,这就是我组织编写这套教材的初衷. 这套英文物理教材是 1996 年经国家教委批准列入正式出版计划的. 本教材可供普通高等工科院校本科生物理课 130～140 学时使用.

……

全书 21 章,具体编写分工如下:

第一册:力学(第 1～5 章),分子运动论和热力学基础(第 6、7 章)由王安安编写.

第二册:电学及稳恒电流的磁场(第 8～10 章)由吴光敏编写,磁介质和电磁感应(第 11、12 章)由樊则宾编写,麦克斯韦方程组(第 13 章)由王安安编写.

第三册:机械振动(第 14 章)由伏云昌编写,机械波(第 15 章)由王安安和陈劲波编写. 电磁振荡与电磁波(第 16 章)由吴光敏编写,波动光学部分(第 17 章)中干涉与衍射由陈劲波编写,光的偏振由樊则宾编写. 近代物理部分(第 18～21 章)由伏云昌编写.

第一册绝大部分插图由刘富华用计算机绘制,其余插图由李俊昌教授绘制,封面也由李俊昌设计,谨此致以诚挚谢意.

编写大学英文物理教材是一种大胆的尝试,由于编者水平有限,错误疏漏之处在所难免,希望同行和读者批评指正. 我们相信这本教材的出版将对物理教学的现代化和物理教学与国际接轨作出有益的贡献.

编　者

1997 年 5 月于昆明

Contents

Part One Mechanics

Part One Mechanics

Chapter 1

Kinematics

Kinematics is the study of the geometry of motion: it deals with the mathematical description of motion in terms of position, velocity, and acceleration. Kinematics serves as a prelude to dynamics which studies force as the cause of changes in motion.

1.1 Frame of Reference and Particle

1.1.1 Frame of reference and coordinate system

The world we live in is made of matter, from the largest bodies, such as the Earth, one of the eight major planets in solar system of which the Sun is in the center, the galaxies in which the sun is in one of the spiral arms, and the entire universe, to the smallest particles, such as molecules, atoms and subatomic particles: electrons and nucleus composed of protons and neutrons. Each proton and each neutron is made of two kinds of quarks called up quark and down quark. Although the objects above differ in size by a factor of more than 10^{40}, they have a universality, being in endless motion, and from this point of view, we say that the motion is absolute.

In the remainder of this chapter we shall discuss the position, speed, and acceleration of various objects. To do this scientifically, the first two questions we must answer are: "What position with respect to?" and "What velocity with respect to?". If we choose different objects as the reference to describe the motion of a given body, the indications will be different. For example, if you stand on the ground in a train station and let a ball drop freely from your hand, the motion of the ball seems to be along straight line by you, but along a parabola of trajectory projected horizontally by the observer seating in a moving coach passing through the station. From this point of view, we say that motion is a relative concept and it must always be referred to a particular body that serves as a reference chosen by the observer. Since different observers may use different frames of reference, it is important to know how observations made by different observers are related. For example, when we discuss motions on the surface of the earth, this is the most cases in our course, and then it is convenient to take the earth's surface as our frame of reference. For the motion of the earth or other planets, a particular set of stars, for instance, sun is a good choice, whereas for the motion of the electrons in an atom, the nucleus of the atom is preferred. You are free to choose the frame of reference, but in all cases it is necessary to specify what reference frame is being used and you must always be aware of your choice and be careful to make all your measurements with respect to it.

In physics a frame of reference is usually pictured in terms of a coordinate system, consisting of three mutually perpendicular axes, called the x, y and z axis, relative to which position in space, velocity, acceleration and orbit can be specified. These three axes intersect at the origin O of the coordinate system. In Fig. 1-1, let us consider two observers, one of them on the sun and the other on the earth. Both observers are studying the same motion of an artificial satellite of the earth. To the observer on the earth using frame $x'y'z'$, the satellite appears to describe an almost circular path around the earth. To the solar observer using frame xyz, the satellite's orbit appears as a wavy line.

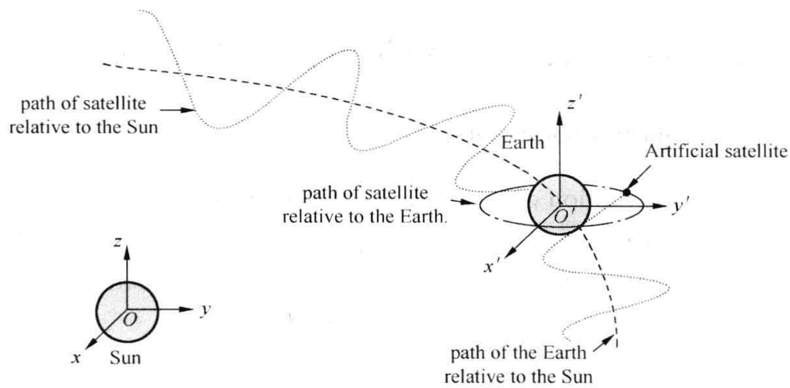

Fig. 1-1 The orbits of a satellite relative to the Earth and to the Sun

1.1.2 Particles

The moving objects that we might examine are among countless possibilities. We shall restrict our concentration on a simple case—translational motion of a particle first, which is defined as the change of position of the particle as a function of time. In the case of an ideal particle—a body with mass, but with no size and no shape, therefore position as a function of time gives a complete description. We can represent an object as a particle (that is as a mass point) if every small part of the object moves in exactly the same way. The concept of particle is an ideal model, the motion of objects are usually more complicated. In some circumstances we are not interested in the size, orientation, and internal structure of a body, and then we can treat the body as a particle, concentrate on its translational motion and ignore all the other motions. For example, we can describe the motion of a ship sailing down a river or a car traveling on a street as a particle motion—for most purposes it is sufficient to know the position of the center of the ship or the car as a function of time.

You must be aware that an object can be treated as a particle in one situation but not in another. The earth behaves pretty much like a particle if we are interested only in its orbital motion around the sun. If we study the rotation of the earth revolving on its own axis, however, the earth is not a particle at all.

It is a very useful method in physics to simplify an object as an ideal model which helps us to solve the major problem in a subject. You will use more ideal models in the other parts of this course.

1.1.3 Time interval and time

It is necessary to distinguish two concepts, time interval and time. When we say time in physics, we mean a give instant. For example, some scheduled flight takes off at 8:00 am from Beijing, lands at 11:00 am on Kunming, 8 o'clock is an instant and so is 11 o'clock. The 3 hours that the whole flying lasts is a time interval. The position of a moving particle is corresponding to a given instant labeled with t while the distance it passed is corresponding to a given time interval labeled with Δt.

1.2 Displacement, Velocity, and Acceleration

1.2.1 Position vector and Position function

When we describe the motion of a particle, the first question is: "Where is it?". In three dimensional world, we need a vector to answer this question. We locate a particle by a vector $\boldsymbol{r}$, extending from the origin of the coordinate system to the particle's position as in Fig. 1-2. Thus,

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \tag{1-1}$$

in which, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors and x, y and z are the components of the vector $\boldsymbol{r}$. The components can be positive, negative or zero.

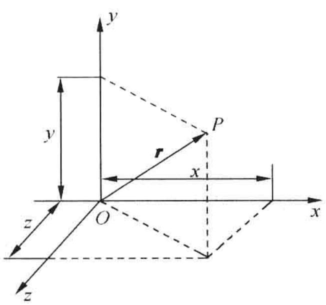

Fig. 1-2 Position vector

We shall define position, displacement, velocity and acceleration for the general case of three dimensions. To simplify the figures, we shall illustrate them in two dimensions in the rest of this chapter.

Mechanical motion is defined as the process of changes in position with time. In principle, the position vector can be correlated with the time by means of a vector function

$$\boldsymbol{r} = \boldsymbol{r}(t) \tag{1-2a}$$

Its three components are written by the following scalar functions

$$x = x(t), \quad y = y(t), \quad z = z(t) \tag{1-2b}$$

Eq. (1-2a) or Eq. (1-2b) is defined as the position function that determines the location of a particle at any given time. Combining Eq. (1-1) and Eq. (1-2b), we have

$$\boldsymbol{r}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} + z(t)\boldsymbol{k} \tag{1-3}$$

which is equivalent with Eq. (1-2a) and Eq. (1-2b).

The path equation can be obtained by eliminating t from Eq. (1-2b)

$$f(x, y, z) = 0$$

If the path of a particle is a straight line, the motion is called as a rectilinear motion; if the path is a curve, the motion is called as a curvilinear motion.

1.2.2 Displacement

Displacement is the change in position during a given time interval. In Fig. 1-3, at time t, the particle is at point A, given by position vector $\boldsymbol{r} = \overrightarrow{OA}$. At a later time t_1, the particle will be at B with $\boldsymbol{r}_1 = \overrightarrow{OB}$. Although the particle has moved along the arc $\widehat{AB} = \Delta s$, the displacement is the vector given by

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

or

$$\Delta \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r} \tag{1-4}$$

note that the displacement indicates the change in position not in the path length over the same time interval. Displacement is a vector, its magnitude $|\Delta \boldsymbol{r}|$ is the length of the chord AB; the path is a scalar Δs, the length of the arc AB. In most cases, $|\Delta \boldsymbol{r}| \neq \Delta s$ (Fig. 1-3), only in the limiting case of $\Delta t \to 0$, $|\Delta \boldsymbol{r}|$ can be regarded equal to Δs. For example, a man walks from point A along the rim of a circle of radius R for half a round, his displacement is $2R$, but path is πR. A particle moves back and forth in x axis for one period, its displacement is zero, but path equals to $2A$ (A is the amplitude). You should also be aware of the difference between $|\Delta \boldsymbol{r}|$ and Δr.

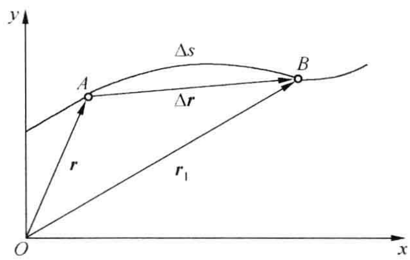

Fig. 1-3 Displacement $\Delta \boldsymbol{r}$ during time interval Δt

1.2.3 Velocity

The second question to describe the motion of a particle is: "How fast is the change of positions?". If $\Delta \boldsymbol{r}$ is the displacement that occurs during the time interval Δt, the average velocity for this interval is defined as

$$\bar{\boldsymbol{v}} = \frac{\Delta \boldsymbol{r}}{\Delta t}$$

The direction of average velocity points in the same direction of displacement (Fig. 1-3); the magnitude of it equals $|\Delta \boldsymbol{r}|/\Delta t$. Obviously, average velocity is related to the specified time interval Δt, and it takes into account only the net displacement in the time interval Δt, ignores the details of the motion, and gives no credits for back and forth motion or the length of the path.

To describe the motion of a particle at a given time t or at a given point, we must make Δt very small. The instantaneous velocity at time t is obtained by evaluating $\Delta \boldsymbol{r}/\Delta t$ in the limit that Δt approaches zero

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{r}}{\Delta t} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} \tag{1-5}$$

Thus, the instantaneous velocity is defined as the time derivation of the position vector.

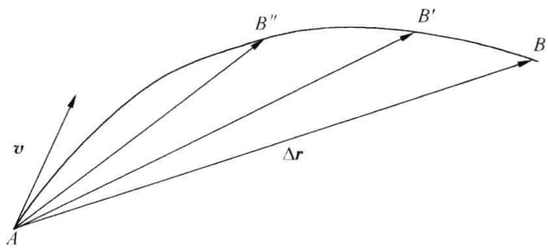

Fig. 1-4 Velocity is tangent to the path at A

Direction of instantaneous velocity:

To determine the direction of instantaneous velocity $\boldsymbol{v}$ at point A, let us see Fig. 1-4. When Δt approaches 0, point B approaches point A, as indicated by B', B'', … with the vector $\overrightarrow{AB}$ changing continuously in both magnitude and direction, in the limit when B

is very close to A, $\overrightarrow{AB}=\Delta\boldsymbol{r}$ coincides in direction with the tangent at A, therefore, the instantaneous velocity is a vector tangent to the path, and points to the advance direction.

Magnitude of instantaneous velocity :

Substituting $\boldsymbol{r}$ from Eq. (1-3) into Eq. (1-5) gives

$$\boldsymbol{v}=\frac{\mathrm{d}}{\mathrm{d}t}(x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k})=\frac{\mathrm{d}x}{\mathrm{d}t}\boldsymbol{i}+\frac{\mathrm{d}y}{\mathrm{d}t}\boldsymbol{j}+\frac{\mathrm{d}z}{\mathrm{d}t}\boldsymbol{k} \tag{1-6}$$

or

$$\boldsymbol{v}=v_x\boldsymbol{i}+v_y\boldsymbol{j}+v_z\boldsymbol{k} \tag{1-7}$$

As we see, the three components of the velocity vector are given by

$$v_x=\frac{\mathrm{d}x}{\mathrm{d}t},\quad v_y=\frac{\mathrm{d}y}{\mathrm{d}t},\quad v_z=\frac{\mathrm{d}z}{\mathrm{d}t} \tag{1-8}$$

And the magnitude of the velocity is

$$v=\sqrt{v_x^2+v_y^2+v_z^2} \tag{1-9}$$

Fore the case of the motion in a plane, angle θ formed between $\boldsymbol{v}$ and $+x$ direction in determined by $\tan\theta=v_y/v_x$ as shown in Fig. 1-5, usually used to indicate the direction of velocity.

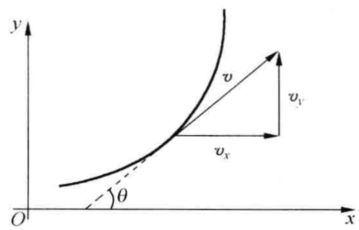

Fig. 1-5 Velocity in two dimensions

Velocity and speed:

On the other hand, the magnitude of velocity vector can be written as

$$v=|\boldsymbol{v}|=\left|\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right|=\lim_{\Delta t\to0}\frac{|\Delta\boldsymbol{r}|}{\Delta t} \tag{1-10}$$

Let Δs represent the path length over Δt, which is given by the length of the arc AB (Fig. 1-3), and the closer B is to A, the closer magnitude of $\Delta\boldsymbol{r}$ is to Δs, that is

$$\lim_{\Delta t\to0}\frac{|\Delta\boldsymbol{r}|}{\Delta s}=1$$

Therefore

$$v=\lim_{\Delta t\to0}\frac{|\Delta\boldsymbol{r}|}{\Delta t}=\lim_{\Delta t\to0}\frac{\Delta s}{\Delta t}=\frac{\mathrm{d}s}{\mathrm{d}t} \tag{1-11}$$

Where $\Delta s/\Delta t$, the path length divided by the time taken, is called the average speed, so $\mathrm{d}s/\mathrm{d}t$ is the instantaneous speed. Note that speed is a scalar, and Eq. (1-11) means that the magnitude of instantaneous velocity equals instantaneous speed, which can be briefly called as velocity and speed.

The unit of speed is m/s, that is, meter per second in SI system.

Example 1-1 The position of a particle moving in x-y plane is given by $x=R+R\cos\omega t$, $y=R\sin\omega t$, here $R=1\mathrm{m}$, $\omega=(\pi/4)\mathrm{s}^{-1}$. Calculate:

(1) the path function $f(x,y)=0$;

(2) velocity at any time;

(3) position vector at $t=0$ and $t=6\mathrm{s}$, the displacement $\Delta\boldsymbol{r}$ and path length Δs during this time interval.

Solution (1) Rearrange the position function as

$$x - R = R\cos\omega t, \quad y = R\sin\omega t$$

then we have

$$(x-R)^2 + y^2 = R^2$$

This is the path function of a circle with radius R and the position of center locates at $(R, 0)$ as Fig. 1-6 shows.

(2) From Eq. (1-8)

$$v_x = \frac{dx}{dt} = -R\omega\sin\omega t$$

$$v_y = \frac{dy}{dt} = R\omega\cos\omega t$$

$$\boldsymbol{v} = -R\omega\sin\omega t\boldsymbol{i} + R\omega\cos\omega t\boldsymbol{j}$$

$$v = \sqrt{v_x^2 + v_y^2} = R\omega = \frac{\pi}{4}\text{m/s}$$

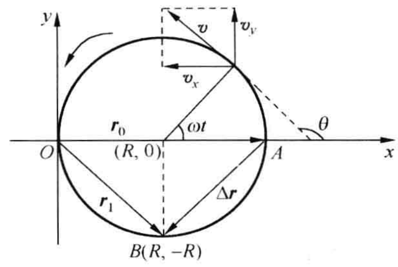

Fig. 1-6 For Example 1-1

which means that the motion is a circular motion with constant speed. The angle θ between $\boldsymbol{v}$ and $+x$ direction is given by

$$\tan\theta = v_y / v_x = -\cot\omega t$$

By inspection of the signs of v_x and v_y at the particular time, you can determine which quadrant the angle is in.

(3) When $t = 0$, we have

$$\boldsymbol{r}_0 = 2R\boldsymbol{i}$$

represented by $\overrightarrow{OA}$, and at $t = 6$ s

$$\boldsymbol{r}_1 = \left(R + R\cos\frac{3}{2}\pi\right)\boldsymbol{i} + R\sin\frac{3}{2}\pi\boldsymbol{j} = R\boldsymbol{i} - R\boldsymbol{j}$$

represented by $\overrightarrow{OB}$, the displacement during $\Delta t = 6$s is

$$\Delta\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_0 = -R\boldsymbol{i} - R\boldsymbol{j}$$

represented by $\overrightarrow{AB}$ in Fig. 1-6.

$$|\Delta\boldsymbol{r}| = \sqrt{(-R)^2 + (-R)^2} = \sqrt{2}R = 1.41 \text{ m}$$

While the path length during the same Δt is

$$\Delta s = \text{arc } \widehat{AOB} = \frac{3}{2}\pi R = 4.71 \text{ m}$$

1.2.4 Acceleration

The path of a particle moving in two or three dimensions is a curve in general, its velocity changes in both of magnitude and direction. The magnitude of the velocity changes when the particle speeds up or slows down. The direction of the velocity changes because the velocity is tangent to the path and the path bend continuously. Fig. 1-7 indicates the velocity $\boldsymbol{v}$ at time t, and $\boldsymbol{v}_1$ at t_1, corresponding to the position A and B, respectively. The change in velocity during the time interval $\Delta t = t_1 - t$, is represented by $\Delta\boldsymbol{v}$ in the vector triangle in which $\boldsymbol{v} + \Delta\boldsymbol{v} = \boldsymbol{v}_1$, then $\Delta\boldsymbol{v} = \boldsymbol{v}_1 - \boldsymbol{v}$. To describe the average rate of change in velocity for the time interval Δt, the average acceleration is defined by

$$\bar{\boldsymbol{a}} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

Using the same method as in definition of the velocity, the instantaneous acceleration at time t, refereed simply as acceleration is given by

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{1-12}$$

which is the time derivation of velocity vector.

Direction of acceleration:

Acceleration vector has the same direction as the limit direction of the change in velocity when $\Delta t \to 0$, which is always pointing toward the concavity of the curve, and because $\Delta \boldsymbol{v}$ is always in the direction in which the curve bends, as shown in Fig. 1-7. Suppose that the direction of acceleration is at an angle of α to the velocity, $\alpha < 90°$, $\alpha > 90°$, and $\alpha = 90°$ corresponding to the cases of $|\boldsymbol{v}_1| > |\boldsymbol{v}|$, $|\boldsymbol{v}_1| < |\boldsymbol{v}|$, and $|\boldsymbol{v}_1| = |\boldsymbol{v}|$, respectively. It is important to be aware that there is an acceleration whenever the velocity changes in either magnitude or direction.

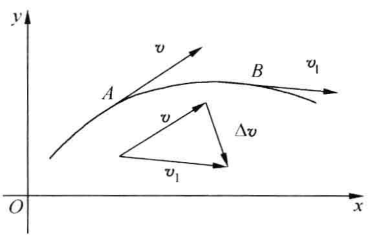

Fig. 1-7 Acceleration in curvilinear motion

Magnitude of acceleration:

From Eq. (1-5), we can also write Eq. (1-12) in the form

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2 \boldsymbol{r}}{\mathrm{d}t^2} \tag{1-13}$$

Substituting Eq. (1-3) into Eq. (1-13) gives

$$\boldsymbol{a} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}\boldsymbol{i} + \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}\boldsymbol{j} + \frac{\mathrm{d}^2 z}{\mathrm{d}t^2}\boldsymbol{k}$$

or

$$\boldsymbol{a} = a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k} \tag{1-14}$$

The three components of acceleration are given by

$$a_x = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}, \quad a_y = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}, \quad a_z = \frac{\mathrm{d}^2 z}{\mathrm{d}t^2} \tag{1-15}$$

And the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{1-16}$$

The unit of acceleration is $\mathrm{m/s^2}$ in SI system.

In the case of a motion in x-y plane, suppose φ is the angle formed by $\boldsymbol{a}$ and $+x$ direction, thus

$$\tan \varphi = \frac{a_y}{a_x} \tag{1-17}$$

Example 1-2 Suppose the position function is the same as in Example 1-1. Find the acceleration at any time.

Solution From the result of Example 1-1, we have

$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t} = -R\omega^2 \cos \omega t$$

$$a_y = \frac{dv_y}{dt} = -R\omega^2 \sin\omega t$$

$$a = \sqrt{a_x^2 + a_y^2} = R\omega^2 = \frac{v^2}{R} \text{ m/s}^2 = 0.62 \text{ m/s}^2$$

which means that the magnitude of $\boldsymbol{a}$ is a constant, the direction of it can be represented by angle α between $\boldsymbol{a}$ and $+x$ direction, and

$$\tan\alpha = \frac{a_y}{a_x} = \tan \omega t$$

For example, if $t = 3$ s, $\tan\alpha = \tan\frac{3}{4}\pi = -1$, because $a_x > 0$, $a_y < 0$, so that $\alpha = -\pi/4$, in the forth quadrant, as shown in Fig. 1-8. On the other hand, we can rewrite a_x and a_y as

$$a_x = -(x-R)\omega^2, \quad a_y = -y\omega^2$$

that is

$$\boldsymbol{a} = -\omega^2[(x-R)\boldsymbol{i} + y\boldsymbol{j}]$$

Note that, there is a vector

$$\boldsymbol{R} = (x-R)\,\boldsymbol{i} + y\,\boldsymbol{j}$$

which is pointing from the center of the circle to the position of particle in Fig. 1-8, therefore

$$\boldsymbol{a} = -\omega^2 \boldsymbol{R}$$

which means the acceleration is always pointing toward the center of the circle. So it is called as centripetal acceleration.

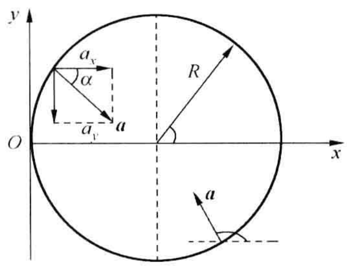

Fig. 1-8 For Example 1-2

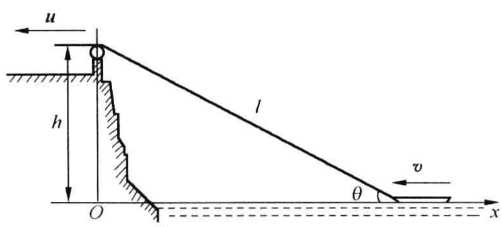

Fig. 1-9 For Example 1-3

Example 1-3 A seaman standing on a cliff, pulls a boat by a pulley, as shown in Fig. 1-9. Suppose that the height of the cliff is h, the rate of the rope pulled is u. Find:

(1) the velocity of the boat;

(2) the acceleration of the boat.

Solution Because the motion of the boat is in one dimension, set x axis pointing right, choose the origin at the foot point of the pulley, and let l representing the variable length of the rope at any time. So that, position vector of the boat is $\boldsymbol{r} = x\boldsymbol{i}$, note that $x = \sqrt{l^2 - h^2}$ in which x and l are changing with time, take time derivation of x, we have

$$\frac{dx}{dt} = \frac{l\frac{dl}{dt}}{\sqrt{l^2 - h^2}} = -\frac{lu}{x}$$