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Quotient Space Based Problem Solving

A Theoretical Foundation of Granular Computing

基于商空间的问题求解

粒度计算的理论基础

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内容简介

本书针对人类问题求解的特点建立一个基于商空间的数学模型，这个模型也是分层多粒度计算的理论基础。该理论能有效地解析目前已有的多粒度分析方法，如小波分析、分形几何和模糊集理论等；不仅适用于以问题求解为代表的人类深思熟虑的行为，同时也适用于人类的感知，如视觉信息处理等。

本书共分 7 章和 2 个附录。第 1 章讲述问题的描述方法，关键是不同粒度世界的描述。第 2 章讲述分层与多粒度计算，重点是其数学模型，多粒度计算与计算复杂性、模糊分析的关系，以及它的应用。第 3 章多粒度计算中信息合成的数学模型，并由此导出合成的原则和方法。第 4 章多粒度世界中的推理，包括推理模型，不确定性与粒度的关系，推理网络、定性推理与模糊推理等。第 5 章自动空间规划，包括装配序列的自动产生，运动规划中的几何与拓扑方法，降维法及其应用。第 6 章介绍统计启发式搜索方法，分析它的理论、计算复杂性、算法的实现，这种算法的特点及其与多粒度计算的关系。第 7 章商空间问题求解理论的推广，包括将理论推广到非等价关系，该理论与小波分析与分形几何的关系，以及在系统分析中的应用。最后，在附录中介绍若干与本书内容关系密切的数学内容，主要是统计推断与点集拓扑的某些概念和结论，供不熟悉这部分数学内容的读者阅读时参考。

本书是从事计算机、人工智能、模式识别以及粒计算等领域的科学工作者的有益参考书。

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Preface

The term problem solving is used in many disciplines, sometimes with different perspectives. As one of the important topics in artificial intelligence (AI) research, it is a computerized process of human problem-solving behaviors. So the aim of problem solving is to develop techniques that program computers to find solutions to problems that can properly be described.

In the early stage of AI, symbolists play a dominant role. They believe that all human cognitive behaviors, including problem solving, can be modeled by symbolic representation and reasoning and do not advocate the use of strict mathematical models. The most general approach to tackle problem-solving processes is "generation and test". Applying an action to an initial state, a new state is generated. Whether the state is the goal state is tested; if it is not, repeat the procedure, otherwise stop and the goal is reached. This principle imitates human trial-and-error behaviors in problem solving sufficiently. The principle has widely been used to build AI systems such as planning, scheduling, diagnosis, etc. and to solve a certain kind of real problems. Therefore, the heuristic and scratch method is misunderstood as a unique one in AI for many people. We believe that more and more modern sciences such as mathematics, economics, operational research, game theory and cybernetics would infiltrate into AI when it becomes mature gradually. Over the years, we devoted ourselves to introducing mathematics to AI. Since 1979 we have introduced statistical inference methods to heuristic search, topological dimension reduction approach to motion planning, and relational matrix to temporal planning. Due to the introduction of these mathematical tools, the efficiency and performance of AI algorithms have been improved significantly. There are two main trends in AI research recently. One is attaching importance to the usage of modern scientific methods, especially mathematics; the other is paying attention to real-world problem solving. Fortunately, our efforts above are consistent with these new trends.

Based on these works, we explored further the theoretical framework of problem solving. Inspired by the following basic characteristics in human problem solving, that is, the ability to conceptualize the world at different granularities, translate from one abstraction level to the others easily and deal with them hierarchically, we establish an algebraically quotient space model to represent the multi-granular structures of the world so that it's easy for computers to deal with them hierarchically. Certainly, this model can simulate the above characteristics of human problem-solving behaviors in a certain extent. We expect more human characteristics to merge into the model further. The system is used to describe the hierarchical and multi-granular structure of objects being observed and to solve the problems that are faced in inference, planning, search, etc. fields. Regarding the relation between computers and human problem solvers, our standpoint is that the computer problem solver should learn some things from human beings but due to the difference between their physical structures they are distinguishing.

Already 20 years has passed since the English version of the book published in 1992. Meanwhile, we found that the three important applied mathematical methods, i.e., fuzzy mathematics, fractal geometry and wavelet analysis, have a close connection with quotient space based analysis. Briefly, the representational method of fuzziness by membership functions in fuzzy mathematics is equivalent to that based on hierarchical coordinates in the quotient space model; fractal geometry rooted in the quotient approximation of spatial images; and wavelet analysis is the outcome of quotient analysis of attribute functions. The quotient space theory of problem solving has made new progress and been applied to several fields such as remote sensing images analysis, cluster analysis, etc. In addition, fuzzy set and rough set theories have been applied to real problems for managing uncertainty successively. The computational model of uncertainty has attracted wide interest. Therefore, we expanded the quotient space theory to non-equivalent partition and fuzzy equivalence relation. We explored the relation between quotient space theory and fuzzy set (rough set) theory. The quotient space theory is also extended to handling uncertain problems. Based on these works, we further proposed a new granular computing theory based on the quotient space based problem solving. The new theory can cover and solve problems in more domains of AI such as learning problems so as to become a more general and universal theoretical framework. The above new progress has been included in the second version of the book.

The quotient space based problem solving that we have discussed mainly deals with human deliberative behaviors. Recently, in perception, e.g., visual information processing, the multi-level analysis method is also adopted. So the quotient space model can be applied to these fields as well. But they will not be involved in the book.

There are seven chapters and two addenda in the book. In Chapter 1, we present a quotient space model to describe the world with different grain-sizes. This is the theoretical foundation throughout the book and is the key to problem solving and granular computing. The principle of "hierarchy" as an important concept has been used in many fields such as control, communication theory. In Chapter 2, we discuss the principle starting with the features of the human problem-solving process and pay attention to its mathematical modeling and relation to computational complexity. In Chapter 3, we discuss synthetic methods that involve the inverse of top-down hierarchical analysis, that is, how to combine the information from different viewpoints and different sources. Since synthetic method is one of main measures for human problem solving we present a mathematical model and induce the corresponding synthetic rules and methods from the model. Although there have been several inference models in AI, the model presented in Chapter 4 is a new network-based one. The new model can carry out inference at different abstraction levels and integrates deterministic, non-deterministic and qualitative inferences into one framework. And the synthetic and propagation rules of network inference are also introduced. In Chapter 5, the application of quotient space theory to spatial planning is presented. It includes robot assembly sequences and motion planning. For example, in motion planning instead of widely adopted geometry-based planning we pay attention to a topology-based one that we propose, including its principles and applications. The statistically heuristic search algorithms are presented in Chapter 6, including theory, computational complexity, the features and realization of the algorithms, and their relation to hierarchical problem-solving principles and multi-granular computing. In Chapter 7, the original equivalence relation based theory is expanded to including tolerant relations and relations defined by closure operations. Also, a more general quotient space approximation principle is presented. Finally, the basic concepts and theorems of mathematics related to the book are introduced in addenda, including point set topology and statistical inference.

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Contents

CHAPTER 1

Problem Representations

Chapter Outline

1.1 Problem Solving

The term problem solving was used in many disciplines, sometimes with different perspectives (Newell and Simon, 1972; Bhaskar and Simon, 1977). As one of the main topics in artificial intelligence (AI), it is a computerized process of human problem-solving behaviors. It has been investigated by many researchers. Some important results have been provided (Kowalski, 1979; Shapiro, 1979; Nilson, 1980). From an AI point of view, the aim of the problem solving is to develop theory and technique which enable the computers to find, in an efficient way, solutions to the problem provided that the problem has been described to computers in a suitable form (Zhang and Zhang, 1992; 2004).

Problem-solving methods and techniques have been applied in several different areas. To motivate our subsequent discussions, we next describe some of these applications.

1.1.1 Expert Consulting Systems

Expert consulting systems have been used in many different areas to provide human users with expert advice. These systems can diagnose diseases, analyze complex experimental data and arrange production schedule, etc.

In many expert consulting systems, expert knowledge is represented by a set of rules. The conclusion can be deduced from initial data by successively using these rules.

1.1.2 Theorem Proving

The aim of theorem proving is to draw a potential mathematical theorem from a set of given axioms and previously proven theorems by computers. It employs the same rule-based deduction principle as in most expert systems.

1.1.3 Automatic Programming

Automatic programming, automatic scheduling, decision making, robotic action planning and the like can be regarded as the following general task. Given a goal and a set of constraints, find a sequence of operators (or actions) to achieve the goal satisfying all given constraints.

All the problems above can be regarded as intelligent problem-solving tasks. In order to enable computers to have the ability of finding the solution of these problems automatically, AI researchers made every effort to find a suitable formal description of problem-solving process. It is one of the central topics in the study of problem solving.

In the early stage of AI, symbolists play a dominant role. They believe that all human cognitive behaviors, including problem solving, can be modeled by symbols and symbolic reasoning. The most general approach to tackle problem solving is generation and test. Applying an action to an initial state, a new state is generated. Whether the state is the goal state is tested; if it is not, repeat the procedure, otherwise stop and the goal is reached. This principle imitates human trial-and-error behaviors in problem solving sufficiently. The principle has widely been used to build AI systems. The problem-solving process is generally represented by a graphical (tree) search or an AND/OR graphical (tree) search.

1.1.4 Graphical Representation

A graphically causal model (Pearl, 2000) is an abstract model that describes the causal mechanisms of a system. So some problem-solving processes can be regarded as inference over the graphically causal model. For example, automatic reasoning, theorem proving and the like can be considered as searching a goal node in the model. And robotic action planning, automatic programming, etc., can be formalized as searching a path in the model; and the path being found is the solution of the problem and called a solution path.

Let us take the robot's indoor path-planning problem as an example. Assuming that the initial position of the robot is in room *X* and the goal position is in room *Y*, the aim is to find a path from room *X* to room *Y*. Fig. 1.1 shows the graphical representation of the problem-solving process. The nodes shown in Fig. 1.1 represent subsets of potential solutions. For example, the node denoted by *A* represents all potential paths from room *X* to room *Y* by going through room *A*; while the node *C* all potential paths by going through rooms *A* and *C*; and so on. The arcs linking two nodes are planning rules for finding a path from one room to another. The path that links *X* and *Y* is the solution path.

1.1.5 AND/OR Graphical Representation

Some problem-solving processes may be represented more conveniently by the so-called AND/OR graph. In this representation, a complex original problem is divided into a conjunction of several subproblems. These subproblems are simpler than the original one and can generally be solved in isolation. The subproblems can be further decomposed into still more simple sub-subproblems until they can be easily solved.

In fact, the problem-solving processes above are regarded as an AND/OR graph search. The graph is similar to the general graph except that there are two kinds of links. One, called OR link, is the same as that in the general graphs. The other, called AND link, is special to the AND/OR graphical representation.

All nodes in an AND/OR graph represent subproblems to be solved or subgoals to be reached. The situation is the same as in the general graph. But in AND links, although the individual subproblems are represented by separate nodes, they all must be solved before

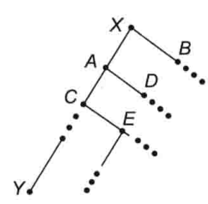

Figure 1.1: The Graphical Representation of a Problem

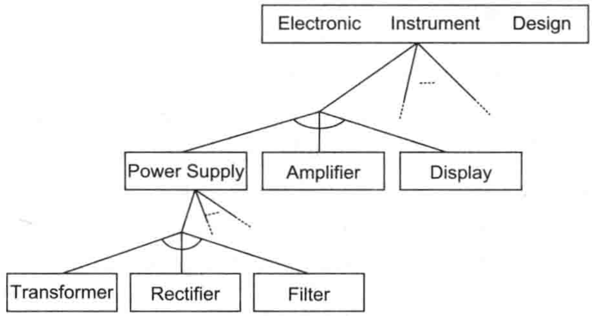

Figure 1.2: AND/OR Graphical Representation of a Problem

their parent problem is considered solved. The curved arcs between links are drawn to show this fact (see Fig. 1.2).

A solution to the problem represented by a general graph is a terminal node of the graph. However, the complete solution in an AND/OR graphical representation is represented by an AND/OR subgraph, called a solution graph (see Chapter 6 for more details).

As an example shown in Fig. 1.2, the initial problem is to design an electronic instrument. The task can be divided into several subtasks called component designs, such as power supply, amplifier and display component design. Furthermore, each subtask can be divided into several sub-subtasks called part designs. For example, power supply design consists of transformer, rectifier and filter designs, etc.

Although a wide range of problems can be described by the above representations, there is still a big gap between the formal description and human behavior in problem solving so that generally the computer solver cannot find the solution in an efficient way as a human does.

One of the basic characteristics in human problem solving is the ability to conceptualize the world at different granularities and translate from one abstraction level to the others easily, i.e. deal with them hierarchically (Hobbs, 1985). It is the hierarchy that underlies the human power in problem solving. Suppose that a manager sitting in his office drafted a production plan for his factory. In his early mental planning stage, only a coarse-grained model of the factory is needed. The factory in his mind may be encoded as a 'block diagram' consisting of several workshops while ignoring all details within the workshops. When a plan has briefly been sketched out, he must enter a more fine-grained model to consider the details within the workshops, i.e., he needs a fine coding of the factory. In some planning stage, if global information is needed, he will immediately switch to the coarse-grained representation again. This ability is one of the human intelligence.

For a computer, things are quite different. Despite all data about a factory, such as machines, workers, tools, buildings, etc., having been stored in its memory, it is still unknown how to generate different representations from these data, how to choose a properly representational form based on different computational requirements, how to transform a coarse-grained model into a fine one or vice versa. Neither general graphical nor AND/OR graphical representation can tackle such problems as they lack a mechanism for representing the world at different granularities. Therefore, we have to provide a precise formalization of the notion of problem representations at different granularities in order for computers to imitate the above human abilities.

1.2 World Representations at Different Granularities

1.2.1 The Model of Different Grain-Size Worlds

From the above discussion, it seems important to develop a new theory and technique which will in some way enable computers to represent the world at different granularities.

Suppose that a problem-solving space, or a problem space for short, is described by a triplet (X,f,T).

X denotes the problem domain, or universe. In the preceding example, when drafting a production plan, the factory as a whole is the domain in question.

$f(.)$ indicates the attributes of domain X or is denoted by a function $f:X\rightarrow Y$, where Y may be a real set, a set of n-dimensional space R^n, or a general space, $f(x)$ is either single-valued or multi-valued. For each element $x\in X$, $f(x)$ corresponds to a certain attribute of the x, and is called an attribute function. In the example above, value of output, work-force and profit are the attributes that depict the factory.

T is the structure of domain X, i.e. the relations among elements in X. For example, the relations among workshops, workers, machines and managers, etc. Structure T is the most complex and various part of the triplet description. One main category includes the Euclidean distance in Euclidean space, the inner product in inner product space, the metric in metric space, the semi-order in semi-order space, topology in topological space, directed graphs and undirected graphs, etc. The other is the structure that arose from some operations such as linear space, group, ring, field and lattice in algebra and logic inference. Certainly, the above two categories may be combined to form a new structure, for example, normed space, normed ring, etc.

Given a problem space (X,f,T), solving a problem implies the analysis and investigation of X, f and T. But the problems are how to choose a suitable granularity of X, what relationships exist among different grain size worlds, etc.

Suppose that X indicates a domain with the finest grain-size. By simplifying X we have a more coarse-grained domain denoted by $[X]$. So the original problem space (X,f,T) is

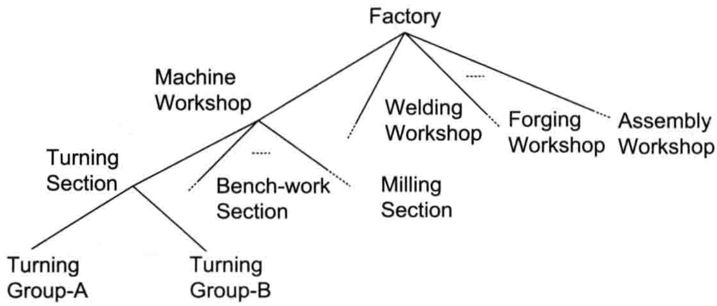

Figure 1.3: The Hierarchical Structure of a Factory

transformed into a new one $([X], [f], [T])$ with a new abstraction level. In the above 'factory' example, if groups are elements of domain X, then, in a simplified domain $[X]$, its elements may be sections, each consists of several groups, i.e., the elements with common or similar functions in X as shown in Fig. 1.3.

The hierarchical structure shown in Fig. 1.3 is quite similar to the concept of quotient set in mathematics (Eisenberg, 1974).

Assume that X is a domain, R is an equivalence relation on X, and $[X]$ is a quotient set under R. Regarding $[X]$ as a new domain, we have a new world which is coarser than X. We say that X is classified with respect to R.

Before the discussion of the domain partition, we first introduce some concepts and propositions of set theory.

Definition 1.1

Assume that X and Y are two sets, $R \subset X \times Y$ is a product set of X and Y on $X \times Y$. For $\forall (x, y) \in X \times Y$, have $(x, y) \in R$. We say that x and y have relation R denoted as xRy, or R is a relation on $X \times Y$. When $X = Y$, R is called a relation on X.

Definition 1.2

Assume that X is a set, R is a relation on X and satisfies

(1) Reflexivity: xRx,
(2) Symmetry: if xRy, then yRx,
(3) Transitivity: If xRy and yRz, then xRz,

R is called an equivalence relation on X denoted by xRy or $x \sim y$.

Definition 1.3

For $x \in X$, $[x] = \{y | x \sim y\}$ is called an equivalence class of x.