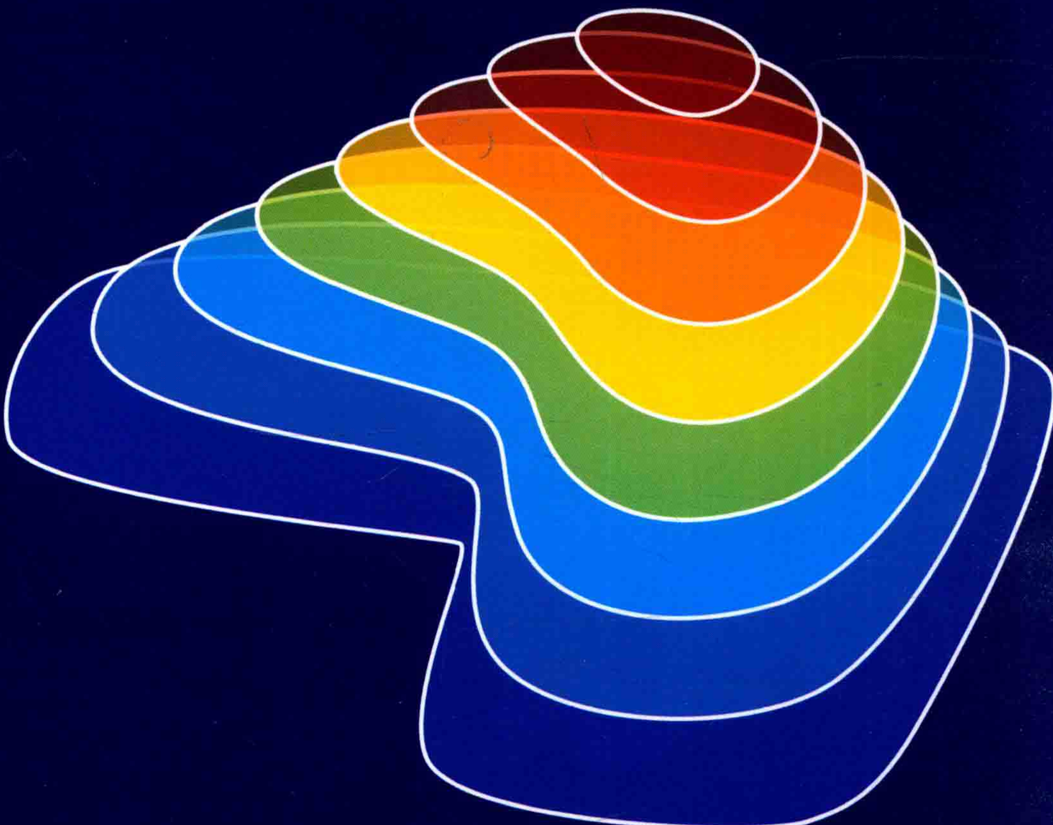
Numerical Computing with MATLAB

Revised in 2013

MATLAB数值计算（2013修订版 · 英文版）



Cleve B. Moler

MATLAB®
examples

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This draft version is intended for use in short courses at MathWorks and in academic courses during the 2004 Spring quarter at several specific universities. Use in courses elsewhere is permitted, provided the instructor notifies the authors via email to moler@mathworks.com.

Preface

Numerical Computing with MATLAB is a textbook for an introductory course in numerical methods, MATLAB, and technical computing. The emphasis is on informed use of mathematical software. We want you learn enough about the mathematical functions in MATLAB that you will be able to use them correctly, appreciate their limitations, and modify them when necessary to suit your own needs. The topics include

- introduction to MATLAB,
- linear equations,
- interpolation,
- zero and roots,
- least squares,
- quadrature,
- ordinary differential equations,
- random numbers,
- Fourier analysis,
- eigenvalues and singular values,
- partial differential equations.

George Forsythe initiated a software-based numerical methods course at Stanford University in the late 1960s. The textbooks by Forsythe, Malcolm, and Moler [20] and Kahaner, Moler, and Nash [34] that evolved from the Stanford course were based upon libraries of Fortran subroutines.

This textbook is based upon MATLAB. NCM, a collection of over 70 M-files, forms an essential part of the book. Many of the over 200 exercises involve modifying and extending the programs in NCM. The book also makes extensive use of computer graphics, including interactive graphical expositions of numerical algorithms.

The prerequisites for the course, and the book, include

- calculus,
- some familiarity with ordinary differential equations,
- some familiarity with matrices,
- some computer programming experience.

If you've never used MATLAB before, the first chapter will help you get started. If you're already familiar with MATLAB, you can glance over most of the first chapter quickly. Everyone should read the section in the first chapter about floating-point arithmetic.

There is probably too much material here for a one-quarter or one-semester course. Plan to cover the first several chapters and then choose the portions of the last four chapters that interest you.

Make sure that the NCM collection is installed on your network or your personal computer as you read the book. The software is available from a Web site devoted to the book [47]:

```
http://www.mathworks.cn/moler
```

There are three types of NCM files:

- `gui` files: interactive graphical demonstrations;
- `tx` files: textbook implementations of built-in MATLAB functions;
- others: miscellaneous files, primarily associated with exercises.

When you have NCM available,

```
ncmgui
```

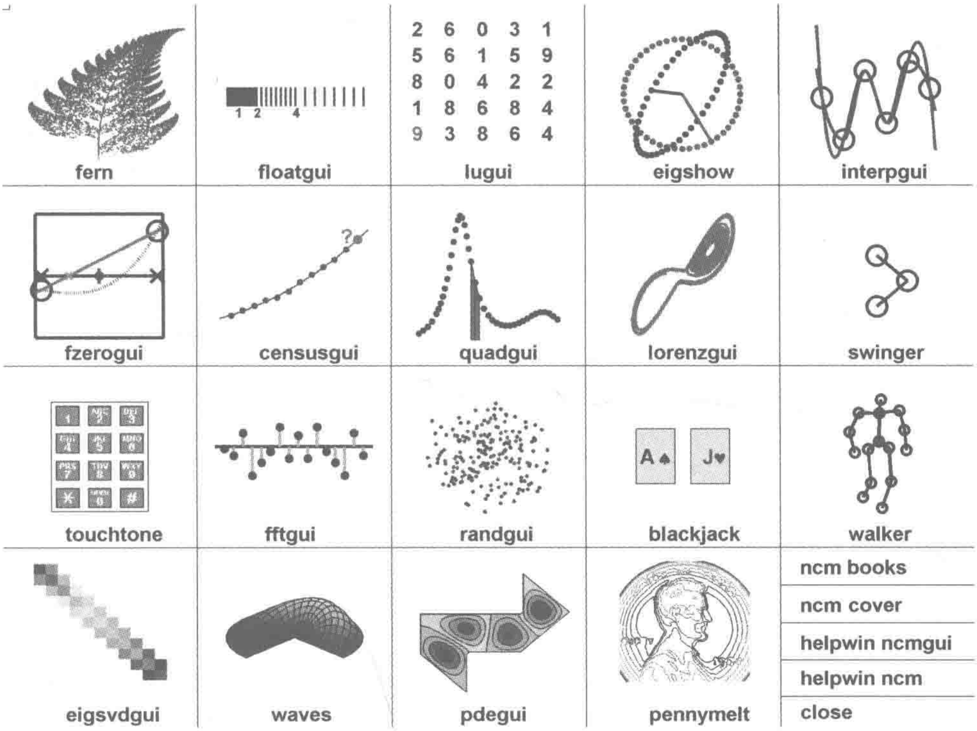
produces the figure shown on the next page. Each thumbnail plot is actually a push button that launches the corresponding `gui`.

This book would not have been possible without the people at The MathWorks and at SIAM. Both groups are professional, creative, and delightful to work with. They have been especially supportive of this book project. Out of the many friends and colleagues who have made specific contributions, I want to mention five in particular. Kathryn Ann Moler has used early drafts of the book several times in courses at Stanford and has been my best critic. Tim Davis and Charlie Van Loan wrote especially helpful reviews. Lisl Urban did an immaculate editing job. My wife Patsy has lived with my work habits and my laptop and loves me anyway. Thanks, everyone.

A revised reprint in 2008 included a change in the section on Google PageRank that improves the handling of web pages with no out links, a short new section in the Random Numbers chapter, removal of material on `inline` and `feval`, and correction of a few dozen minor typographical errors.

A significant update in September 2013 incorporates over 60 changes. Many of them have been recommended by Professor Zhiyong Zhang of Nanjing University of Posts and Telecommunications in China (NJUPT), who prepared a Chinese translation for BUAA press. The census example in section 5.3 includes the 2010 census. Output from `format long` shows 16 significant digits. Symbolic Toolbox usage reflects the MuPad connection. Many thanks to Prof. Zhang.

Cleve Moler
Sept. 16, 2013



ncmgui

Contents

Chapter 1

Introduction to MATLAB

This book is an introduction to two subjects: MATLAB and numerical computing. This first chapter introduces MATLAB by presenting several programs that investigate elementary, but interesting, mathematical problems. If you already have some experience programming in another language, we hope that you can see how MATLAB works by simply studying these programs.

If you want a more comprehensive introduction, there are many resources available. You can select the **Help** tab in the toolstrip atop the MATLAB command window, then select **Documentation**, **MATLAB** and **Getting Started**. A MathWorks Web site, *MATLAB Tutorials and Learning Resources* [44], offers a number of introductory videos and a PDF manual entitled *Getting Started with MATLAB*.

An introduction to MATLAB through a collection of mathematical and computational projects is provided by Moler's free online *Experiments with MATLAB* [48].

A list of over 1500 MATLAB-based books by other authors and publishers, in several languages, is available at [45]. Three introductions to MATLAB are of particular interest here: a relatively short primer by Sigmon and Davis [56], a medium-sized, mathematically oriented text by Higham and Higham [31], and a large, comprehensive manual by Hanselman and Littlefield [29].

You should have a copy of MATLAB close at hand so you can run our sample programs as you read about them. All of the programs used in this book have been collected in a directory (or folder) named

```
NCM
```

(The directory name is the initials of the book title.) You can either start MATLAB in this directory or use

```
pathtool
```

to add the directory to the MATLAB path.

1.1 The Golden Ratio

What is the world's most interesting number? Perhaps you like π, or e, or 17. Some people might vote for ϕ, the *golden ratio*, computed here by our first MATLAB statement.

```
phi = (1 + sqrt(5))/2
```

This produces

```
phi =
    1.6180
```

Let's see more digits.

```
format long
phi

phi =
   1.618033988749895
```

This didn't recompute ϕ, it just displayed 16 significant digits instead of 5.

The golden ratio shows up in many places in mathematics; we'll see several in this book. The golden ratio gets its name from the golden rectangle, shown in Figure 1.1. The golden rectangle has the property that removing a square leaves a smaller rectangle with the same shape.

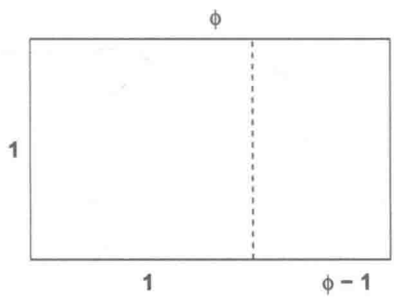


Figure 1.1. *The golden rectangle.*

Equating the aspect ratios of the rectangles gives a defining equation for ϕ:

$$\frac{1}{\phi} = \frac{\phi - 1}{1}.$$

This equation says that you can compute the reciprocal of ϕ by simply subtracting one. How many numbers have that property?

Multiplying the aspect ratio equation by ϕ produces the polynomial equation

$$\phi^2 - \phi - 1 = 0.$$

The roots of this equation are given by the quadratic formula:

$$\phi = \frac{1 \pm \sqrt{5}}{2}.$$

The positive root is the golden ratio.

If you have forgotten the quadratic formula, you can ask MATLAB to find the roots of the polynomial. MATLAB represents a polynomial by the vector of its coefficients, in descending order. So the vector

```
p = [1 -1 -1]
```

represents the polynomial

$$p(x) = x^2 - x - 1.$$

The roots are computed by the `roots` function.

```
r = roots(p)
```

produces

```
r =
  -0.618033988749895
   1.618033988749895
```

These two numbers are the only numbers whose reciprocal can be computed by subtracting one.

You can use the Symbolic Toolbox, which connects MATLAB to a computer algebra system, to solve the aspect ratio equation without converting it to a polynomial. The equation involves a symbolic variable and a double equals sign. The `solve` function finds two solutions.

```
syms x
r = solve(1/x == x-1)
```

produces

```
r =
5^(1/2)/2 + 1/2
1/2 - 5^(1/2)/2
```

The `pretty` function displays the results in a way that resembles typeset mathematics.

```
pretty(r)
```

produces

```
 +-              -+
 |    1/2         |
 |  5             |
```

```
|  ---- + 1/2  |
|   2          |
|              |
|         1/2  |
|        5     |
|  1/2 - ----  |
|         2    |
+-            -+
```

The variable `r` is a vector with two components, the symbolic forms of the two solutions. You can pick off the first component with

```
phi = r(1)
```

which produces

```
phi =
5^(1/2)/2 + 1/2
```

This expression can be converted to a numerical value in two different ways. It can be evaluated to any number of digits using variable-precision arithmetic with the `vpa` function.

```
vpa(phi,50)
```

produces 50 digits.

```
1.6180339887498948482045868343656381177203091798058
```

It can also be converted to double-precision floating point, which is the principal way that MATLAB represents numbers, with the `double` function.

```
phi = double(phi)
```

produces

```
phi =
    1.618033988749895
```

The aspect ratio equation is simple enough to have closed-form symbolic solutions. More complicated equations have to be solved approximately. In MATLAB an *anonymous function* is a convenient way to define an object that can be used as an argument to other functions. The statement

```
f = @(x) 1./x-(x-1)
```

defines $f(x) = 1/x - (x-1)$ and produces

```
f =
    @(x) 1./x-(x-1)
```

The graph of $f(x)$ over the interval $0 \le x \le 4$ shown in Figure 1.2 is obtained with

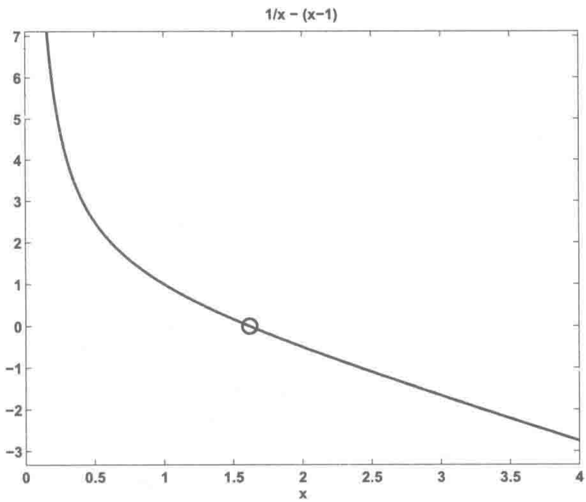


Figure 1.2. $f(\phi) = 0$.

```
ezplot(f,[0,4])
```

The name `ezplot` stands for "easy plot," although some of the English-speaking world would pronounce it "e-zed plot." Even though $f(x)$ becomes infinite as $x \to 0$, `ezplot` automatically picks a reasonable vertical scale.

The statement

```
phi = fzero(f,1)
```

looks for a zero of $f(x)$ near $x = 1$. It produces an approximation to ϕ that is accurate to almost full precision. The result can be inserted in Figure 1.2 with

```
hold on
plot(phi,0,'o')
```

The following MATLAB program produces the picture of the golden rectangle shown in Figure 1.1. The program is contained in an M-file named `goldrect.m`, so issuing the command

```
goldrect
```

runs the script and creates the picture.

```
% GOLDRECT  Plot the golden rectangle

phi = (1+sqrt(5))/2;
x = [0 phi phi 0 0];
y = [0 0 1 1 0];
u = [1 1];
```

```
v = [0 1];
plot(x,y,'b',u,v,'b--')
text(phi/2,1.05,'\phi')
text((1+phi)/2,-.05,'\phi - 1')
text(-.05,.5,'1')
text(.5,-.05,'1')
axis equal
axis off
set(gcf,'color','white')
```

The vectors `x` and `y` each contain five elements. Connecting consecutive (x_k, y_k) pairs with straight lines produces the outside rectangle. The vectors `u` and `v` each contain two elements. The line connecting (u_1, v_1) with (u_2, v_2) separates the rectangle into the square and the smaller rectangle. The `plot` command draws these lines—the $x - y$ lines in solid blue and the $u - v$ line in dashed blue. The next four statements place text at various points; the string `'\phi'` denotes the Greek letter. The two `axis` statements cause the scaling in the x and y directions to be equal and then turn off the display of the axes. The last statement sets the background color of `gcf`, which stands for *get current figure*, to white.

A *continued fraction* is an infinite expression of the form

$$a_0 + \cfrac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \cdots}}}.$$

If all the a_k's are equal to 1, the continued fraction is another representation of the golden ratio:

$$\phi = 1 + \cfrac{1}{1 + \frac{1}{1 + \frac{1}{1 + \cdots}}}.$$

The following MATLAB function generates and evaluates truncated continued fraction approximations to ϕ. The code is stored in an M-file named `goldfract.m`.

```
function goldfract(n)
%GOLDFRACT   Golden ratio continued fraction.
%  GOLDFRACT(n) displays n terms.

p = '1';
for k = 1:n
   p = ['1+1/(' p ')'];
end
p

p = 1;
q = 1;
for k = 1:n
   s = p;
   p = p + q;
```