中国科学院教材建设专家委员会立项项目
土木工程科学研究暨研究生教学丛书

TORSIONAL ANALYSIS OF STEEL STRUCTURAL MEMBERS THEORY AND DESIGN

（钢结构构件的扭转分析——理论与设计）

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内 容 简 介

本书介绍了薄壁开口截面扭转屈曲或弯扭屈曲、畸变屈曲、扭转塑性破坏，即均匀扭转塑性破坏和翘曲扭转塑性破坏的理论分析、试验研究和设计问题，包括均匀受扭、约束受扭、沙堆比拟方法和翘曲扭转诸多试验研究和设计问题。

本书给出了两端简支焊接工形截面，轴心受压柱的扭转屈曲荷载和轴心受压冷弯开口加劲卷边槽钢构件的弹性畸变屈曲荷载的计算实例，并按照澳大利亚冷弯薄壁规范 AS/NZS 4600-2005 2nd Ed. 确定弹性畸变屈曲荷载的计算实例。

本书可作为高等院校土建类和力学类专业的研究生教材，也可供相关领域的工程技术人员和科研人员学习和参考。

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PREFACE

With the enhanced property of steel materials and design requirements of twisting members in steel systems, thin-walled cold-formed steel structures are widely used in practical engineering projects. Therefore, teaching and engineering technical staffs who are engaged in the theoretical research and design of steel structures urgently need to understand the torsional, flexural-torsional and distortional buckling in terms of bending and torsion of thin-walled members. This text book is written towards post-graduates who major in steel structure.

Commonly, readers deal with the torsional and flexural-torsional buckling of thin-walled members while ignoring the distortional buckling. So staffs of steel structures need to have full master of the buckling modes of thin-walled members. In order to address this need, combined with years of practical teaching experience and engineering design projects, Professor Chen Ji stresses much importance on the elastic and elastic-plastic theoretical analysis of thin-walled torsional members, which can make the attention of readers towards the distortional buckling. The changes inside the structure and manifestation form of deformation of members under various external force are stressed in this book to predict the failure modes of structures.

The contents in this book are as follows: Torsion of thin-walled cold-formed open section, elastic torsion and bending-torsion of axial compression members, distortional buckling of cold-formed lipped channel section, distortional buckling under in-plane uniform bending of monosymmetric and bisymmetric I-shape beams and the latest scientific research results. Also contained in this book are references related of each chapter. The contents in this book have a common focal point that post-graduates should pay attention to the study of fundamental theory and practical use. The examples in this book have detailed explanation and provide readers with elaborate deriving processes, making readers fully understand the problems of thin-walled members.

Xu, S. J., Song, W. J.

2014.5

FOREWORD

This book is written for graduate students in colleges and universities as a text book and also can be used by structural engineers and researchers, since the various design provisions of specifications are discussed in this book such as AS/NZS 4600-2005^{2nd} Ed. Standards Australian/ New Zealand Stand on Cold-Formed Steel Structures, Sydney Australia 2005; CSA Standards S16-2009 Design of Steel Structures, Canadian Standards Association, Mississauga, Ontario, 2009; Din 18800-2: Stahbauten, Stabilitatsfalle, knicken von Staben und Stabwerken, 1990; GB 50017—2014 Code for the Design of Steel Structures, China Plan Press, Beijing, 2014 (in Chinese) and GB 50018—2013 Specification of the Design of Cold-From Thin Walled Steel Structures, China Plan Press, Beijing, 2013 (in Chinese).

There are eleven chapters in this book. Chapter 1 includes the history of torsion of steel structural members, such as the sand-heap analogy for the full plastic torque of solid section beam, developed by Nadai, in 1931. Chapter 2 Introduction gives the elastic-torsion shear and warping torsion stresses of I-section thin-walled steel member from Trahair, N. S. and Pi, Y. L. in 1997. Chapter 3 provides the linear elastic torsional behavior of I-section thin-walled steel member, in which the expression of thin-walled open cross-section member under torsion, graduated student on study, Xu, S. J. gives a good suggestion. Chapter 4 gives the elastic torsional buckling of axial compression under the twisting deformation as the angle α between the inclining fiber and the vertical line in which the effect of residual stress distribution and end boundary conditions on elastic torsional buckling load are considered. Chapter 5 explains the elastic section distortional buckling stress of the cold-formed lipped channel involved the twist rotation and the lateral bending displacement of the compression flange. For calculation, an equilibrium method and a numerical method are used. Chapter 6 shows the simply supported bi-symmetrically welded I-beam under in-plane uniform bending in order to explain its distortional buckling. In 1995, Papangelis, J. P., Hancock, G. J. and Trahair, N S. and in 2006, Samanta, A. and Kumar, A. indicate that, for the short mono-symmetric I-section beam, the distortional mode is predominant and for long mono-I-section beam, the flexural buckling mode is predominate. Chapter 7 shows the simply supported bi-symmetrically welded I-beam under in-plane uniform bending in order to explain its distortional buckling. In 1980, Hancock, G. J., Bradford, M. A. and Trahair, N. S. used an energy method obtain the analytical solution of the elastic lateral-distortional buckling moment of bi-symmetrical I-section beam in uniform bending. Chapter 8 I-section beams and beam-columns under bending and torsion use different methods to calculate the out-of-plane stability, sometime take place flexural and torsional deformations before they reach their ultimate state. Chapter 9 in 1953, Timoshenko, S. P. predicts the elastic torsional behavior of a thin-walled steel I-section beam, in which the combination of twist rotations and stresses is stated. Chapter 10 First yield condition of steel structural I-section beam under torsion is obtained, in which, the combined warping normal stress with shear stress due to the uniform torsion and with shear stress due to the warping torsion are stated. Chapter 11 Inelastic torsionabehavior is usually modeled as being elastic-plastic strain hardening in which the plastic collapse torsion and uniform-torsion plastic torque under combined bending and torsion are conducted by Dinno, K. S. and Merchant, W. in 1965 and testing results are given.

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SOME SYMBOLS USED IN TEXTBOOK

A	Area
B_ω	Bi-moment
E	Elastic modulus
G	Shear modulus of elasticity
H	Height
I	Moment of inertia
I_t	Torsion section constant $I_t = \sum_{i=1}^{n} I_{it}$
I_ω	I-section warping constant
M_u	Uniform-torque
M_ω	Warping torque
P_ω	Torsional buckling load
$\bar{R}$	Coefficient of Wagner effect
S	Shear center; static moment
S_ω	Warping static moment
W	Second modulus
b_f	Width of flange
b_w	Width of web
r_0	Polar radius of sectional gyration
x_0	Coordinate of shear center
ρ_0	Polar distance
σ_ω	Warping normal stress
τ_ω	Warping shear stress
φ	Angle of twist rotation
ω_s	Sectorial coordinate

CONTENTS

CHAPTER 1 HISTORY OF TORSION OF STEEL STRUCTURAL MEMBERS

Although the torsional analysis of beams has attracted the interest of many researchers since Saint-Venant[1.1] in 1855 solved the problem of elastic uniform torsion, it appears difficult to predict the ultimate strengths of steel I-beams in non-uniform torsion. Many researches have been carried out into elastic uniform torsion and expressions have been obtained for the warping displacements of the cross section. Some analytical solutions for the uniform elastic-plastic torsion of beams with various cross sections have also been obtained.

Nadai, A.[1.2] in 1931 developed the sand-heap analogy for the full plastic torque of solid section beams, Christopherson, D. G.[1.3] in 1940 obtained an elastic-plastic bending solution for an I-section, and, later, Nadai, A.[1.4] in 1954 used the rooftop membrane analogy for the elastic-plastic solution of various cross section beams. Sokolovsky, W. W.[1.5] in 1946 developed an elastic-plastic solution for an oval section beam, and Smith, J. O. and Sidebottom, O. M.[1.6] in 1965 derived an elastic-plastic bending solution for prismatic bars of rectangular sections. Billinghurst, A., Williams, J. R. L., Chen, G. and Trahais, N. S.[1.7] in 1992 used the mitre method to obtain elastic-plastic bending solutions for various cross sections.

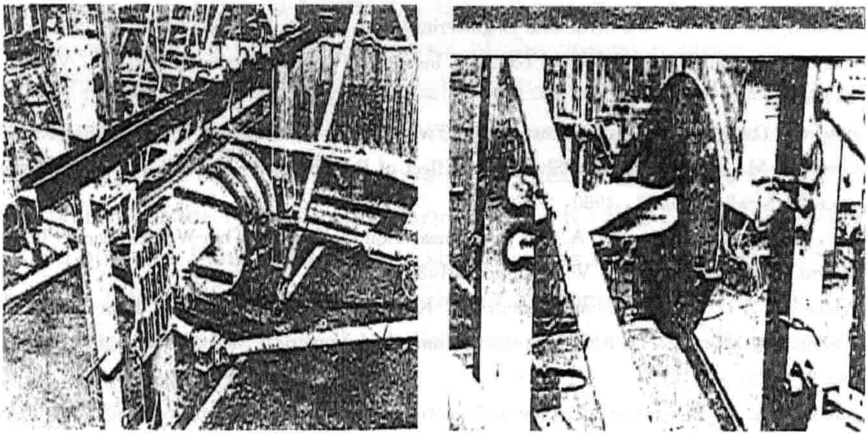
The linear elastic non-uniform torsion of thin-walled open section beams is studied by Timoshenko, N. S. and Gere, J. M.[1.8] in 1961, Vlasov, V. J.[1.9] in 1961 and Wagner, H.[1.10] in 1936 by considering small angles of cross-section rotation φ (so that $\sin\varphi \approx \varphi$, $\cos\varphi \approx 1$). Because of the complexity of the elastic-plastic analysis of the non-uniform torsion of beams, no analytical solutions have been obtained so far.

Approximate solutions for the small-rotation elastic-plastic nonuniform torsion of I-section cantilevers are suggested by Boulton, N. S.[1.11] in 1962, Dinno, K. S. and Merchant, W.[1.12] in 1965, Augusti, G.[1.13] in 1966 and Boulton, N. S. in 1962 presented a lower-bound theory, while Dinno, K. S. and Merchant, W. in 1965 developed a so-called Merchant upper bound for the full plastic torque-carrying capacities of I-section cantilevers. Augusti, G. in 1966 presented an additional theory that gave results between the upper and lower bounds and proved that the Merchant upper bounds a true upper bound.

Since thin-walled open sections have low torsional stiffness, the torsional deformations can be of such magnitudes that it is not adequate to treat the angles of rotation as small. When finite twist rotation angles are considered, the elastic uniform torsion problem becomes nonlinear. Ashwell, D. G.[1.14] in 1951 and Gregory, M.[1.15] in 1960 study both theoretically and experimentally the elastic nonlinear behavior of twisted cantilevers of different cross sections under uniform torsion conditions. Tso, W. K. and Ghobarah, A. A.[1.16] in 1971 presented a study of the nonlinear non-uniform elastic torsion of thin-walled open sections.

Recently, numerical methods have been used by some researchers to investigate the elastic plastic uniform and non-uniform torsional behavior of beams. Yamada, Y. , Katagir, S. and Takatruka, K.[1.17] in 1972, Johnson, A. F.[1.18] in 1973, and Itani, R. Y.[1.19] in 1979 studied elastic-plastic uniform torsion. Baba, S. and Kajita, T.[1.20] in 1982 used a two-node, four-degree-of-freedom beam element for the uniform torsion analysis and a four-node, 12-degree-of-freedom rectangular section element for the warping analysis of the section. Their linear and nonlinear predictions of the deformations are quite similar and they concluded that the solutions for the plastic torque of beams should lie in-between the sand heap analogy and Merchant upper bound. Bathe, K. J. and Chaudhary, A.[1.21] in 1982 used warping displacement functions for beams of rectangular cross section in the formulation of a two-node Hermitian-based beam and in the formulation of a variable number of nodes isoparametric beam for the linear and nonlinear analysis of torsion. Bathe, K. J. and Wiener, R. M.[1.22] in 1983 employed a Hermitian beam element and a nine-node shell element for the elastic-plastic non-uniform torsion of I-beams. Gellin, S. , Lee, G. C. and Chern, J. M.[1.23] in 1983 presented a strip finite-element model for the analysis of the nonlinear material behavior of thin-walled members in non-uniform torsion. May, J. M. and AI-Shaarbaf, A. J.[1.24] in 1989 used a standard three-dimensional 20-node isoparametric quadratic brick element in the elastic-plastic analysis of uniform and non-uniform torsion of members subjected to pure and warping torsion. Their results supported Baba, S. and Kajita, T's. conclusions. Chen, G. and Trahair, N. S.[1.25] in 1991 developed a finite-element model for the inelastic analysis of non-uniform torsion of I-section beams by using the mitre model to describe the shear strain distribution over the cross section.

Some experimental studies for the elastic-plastic uniform torsion of I-sections are carried out by Boulton, N. S. in 1962 and Dinno, S. S. and Gill, K. S. [1.26] in 1964 included two tests with ends free to warp. The experimental torque results are much higher than those predicted by the Merchant upper bound and very large rotation angles are sustained before failure occur. The experimental torque result conducted by Farwell, C. R. Jr, and Galambos, T. V. [1.27] in 1969 is shown in Fig. 1. 1.



(a) (b)

Fig 1. 1 Test setup of steel wide flange I-section beam under non-uniform torsion

(a) Under a load applied mid-span of beam 5 before testing; and (b) After testing until to failure

The purposes of this paper are to present an elastic-plastic model for analyzing large torsional deformations of I-section beams, and to investigate the elastic-plastic behavior of I-section beams in non-uniform torsion.

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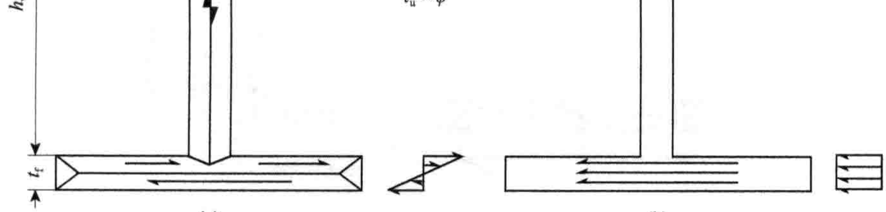
CHAPTER 2 INTRODUCTION

Torsion effects are rarely considered in the design of steel structures, usually because of the difficulty in analyzing them. Even when elastic computer analyses are used to predict torsion effects, there is a lack of advice on how to design for these except by a first-yield analysis of the critical member cross sections, which are difficult to determine and analyze. Torsion in a thin-walled steel member is resisted by a combination of the resistance to uniform torsion. Fig. 2. 1 (a) developed by shear stresses that vary almost linearly across the thickness of the section wall, and the resistance to warping torsion [Fig. 2. 1 (b)] developed by equal and opposite flange bending and shear actions from Trahair, N. S. and Pi, Y. L[2.1] in 1997. Although an elastic theory for combining these two torsion actions is well developed, manual solutions are sufficiently difficult to discourage its use in routine design.



(a) (b)

Fig. 2. 1 Elastic-torsion shear stresses of I-section thin-walled steel member

(a) Uniform torsion shear stresses τ_u; and (b) Warping torsion shear streses τ_w

Furthermore, the apparently significant combinations of the shear stresses of uniform torsion with the normal and shear stresses of warping torsion lead to further difficulties. A first difficulty is the combinations of these different stresses, which cause first yield. More difficult is the determination of the most heavily stressed point in the member, because these stresses vary in different ways around the cross section and along the member.

On the other hand, designs based on elastic analysis are likely to be extremely conservative, not only because of the significant difference between the first yield in a cross section and full plasticity, but also because of the unaccounted for yet significant reserves of strength that are not mobilized in redundant members until after inelastic redistribution takes place.

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CHAPTER 3 LINEAR ELASTIC TORSIONAL BEHAVIOR

The linear elastic methods of torsion analysis assume that the material has a linear elastic stress-strain relationship and that the twist rotations remain small, so that the stresses and twists are proportional to the applied torques, as shown in Fig. 3. 1. The elastic methods of linear torsion analysis are well established by Timoshenko, S. P.[3.1] in 1953; Timoshenko, S. P. and Goodier, S. N.[3.2] in 1970, Vlasov, V. Z.[3.3] in 1961, Zbirohowski-koscia, K.[3.4] in 1967, Kollbrunner, C. F. and Basler, K.[3.5] in 1969, Trahair, N. S. and Bradford, M. A.[3.6] in 1991 and Trahair, N. S.[3.7] in 1993. The engineering method consists of two parts; cross-section analysis that relates the stresses to the stress resultants and linear member analysis that relates the twist relations and stress resultants to the applied torsional loading. The combination of these two parts allows the twist rotations and stresses to be predicated.

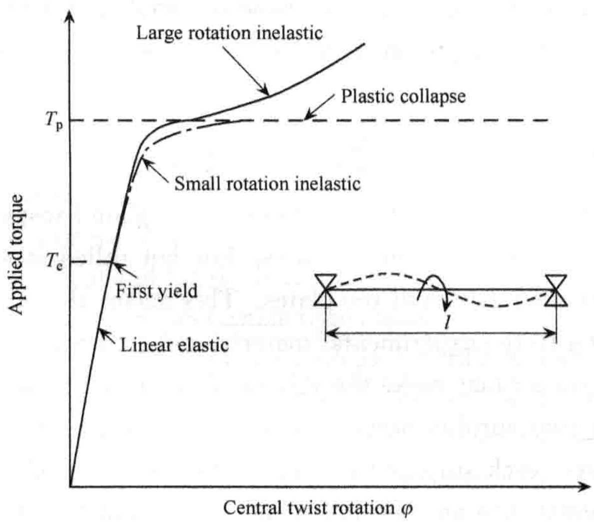

Fig. 3. 1 Torsional behavior of I-section thin-walled steel member

The linear elastic method of torsion analysis is most logically used for serviceabity design. Under service loading, most if not all of the member remains elastic and the linear elastic analysis closely predicts twist rotations. These and any related deflections can then be assumed by comparing them with what are considered to be limiting values.

Linear elastic torsion analysis is less logically used for strength design because yielding usually takes place well before the ultimate torsion capacity is reached. Nevertheless, the absence of accepted methods offailure prediction has forced designers to use the linear elastic method to predict the stresses caused by the strength design loads and to compare them with limiting values that are usually related to the yield stress. This method generally gives very conservative strength predications.

Cross-section analysis under uniform torsion consists of the uniform-torsion of the shear stresses τ_u and the uniform-torque M_u of the stress resultant as follows.

$$M_u = GI_t \frac{d\varphi}{dz} \tag{3.1}$$

Eq. (3.1) is related to $d\varphi/dz$, in which G = shear modulus of elasticity; $I_t = bt^3/3$ = torsion section constant, φ = angle of twist rotation and z = axial distance along the member.

For a thin-walled open section composed of several long and narrowplates, such as the I-shape, channel shape, T shape and angle-shape etc., the total torsional constants may be approximately taken as the sum of the torsional constant I_{it} of each plate.

$$I_t = \sum_{i=1}^{n} I_{it} = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{3.2}$$

in which b_i and t_i denote the width and thickness of plate i respectively and n denotes the number of the component plates. For hot rolled steel, there is some surplus area near the junction of the plates. This makes the torsion constant increase. According to the experimental material and empirical formula, each surplus part at the corner may make the value of I_t increase 6% for the rolled channel section with two surplus parts. But for the rolled I-section and H-section with surplus parts, each surplus part may make the value of I_t increase 7.5%. There, the torsional constant of a hot-rolled I-section may be taken as the result of Eq. (3.2) time a coefficient 1.3.

Very thin-walled open sections have low values of I_t and their uniform torsion resistance may be neglected. Closed sections have large values of I_t and uniform torsion dominates their resistance to torsion.