

Gabriele F. Giuliani and Giovanni Vignale

Quantum Theory of the Electron Liquid

电子液体量子理论

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QUANTUM THEORY OF THE ELECTRON LIQUID

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QUANTUM THEORY OF THE ELECTRON LIQUID

Modern electronic devices and novel materials often derive their extraordinary properties from the intriguing, complex behavior of large numbers of electrons forming what is known as an electron liquid. This book provides an in-depth introduction to the physics of the interacting electron liquid in a broad variety of systems, including metals, semiconductors, artificial nano-structures, atoms, and molecules.

One-, two- and three-dimensional systems are treated separately and in parallel. Different phases of the electron liquid, from the Landau Fermi liquid to the Wigner crystal, from the Luttinger liquid to the quantum Hall liquid, are extensively discussed. Both static and time-dependent density functional theory are presented in detail. Although the emphasis is on the development of the basic physical ideas and on a critical discussion of the most useful approximations, the formal derivation of the results is highly detailed and based on the simplest, most direct methods. A self-contained, comprehensive presentation of the necessary techniques, from second quantization to canonical transformations to both zero and finite temperature Green's functions is provided.

This comprehensive text will be of value to graduate students in physics, electrical engineering and quantum chemistry, as well as practicing researchers in those areas.

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中国台湾。不得出口。

To my parents Ada and Federico
GV

To Pamela, Daniela, Adriana and Giuseppe
GFG

Preface

Don't listen to what I say; listen to what I mean!

“R. P. Feynman”

The electron liquid paradigm is at the basis of most of our current understanding of the physical properties of electronic systems. Quite remarkably, the latter are nowadays at the intersection of the most exciting areas of science: materials science, quantum chemistry, nano-electronics, biology, and quantum computation. Accordingly, its importance can hardly be overestimated. The field is particularly attractive not only for the simplicity of its classic formulation, but also because, by its very nature, it is still possible for individual researchers, armed with thoughtfulness and dedication, and surrounded by a small group of collaborators, to make deep contributions, in the best tradition of “small science”.

When we began to write this book, more than five years ago, our goal was to bring up to date the masterly treatise of the 1960s by Pines and Nozières on quantum liquids – the very same book on which we had first studied the subject. There were good reasons for wanting to do this. During the past 40 years the field has witnessed momentous developments. Advances in semiconductor technology have allowed the realizations of ultra-pure electron liquids whose density, unlike that of the ones spontaneously occurring in nature, can be tuned by electrical means, allowing a systematic exploration of both strongly and weakly correlated regimes. Most of these systems are two- or even one-dimensional, and can be coupled together in the form of multi-layers or multi-wires, opening observational possibilities that were undreamed of in the 1960s. On the theoretical side, quantum Monte Carlo methods, implemented on powerful computers, have allowed an essentially exact determination of the ground-state energy of the electron liquid, and have provided partial answers to the still open question of the structure of its phase diagram. The Landau theory of the Fermi liquid, which in the 1960s was in its infancy, has been fully vindicated by detailed and often painstaking microscopic calculations. The emergence of density functional theory as the standard tool for the calculation of the electronic structure of matter has anointed the electron liquid as the holder of the prototypical correlations in electronic systems.

Starting from the 1980s some truly revolutionary concepts have emerged, which we wanted to be well represented in our book: for example, the notion of fractionally charged excitations in one-dimensional systems and in the quantum Hall liquid, the Luttinger liquid model for one-dimensional systems and for the edges of a quantum Hall liquid, and the beautiful composite-fermion theory of the quantum Hall effect. These concepts transcend the traditional Landau picture of the interacting electron liquid as the “continuation” of

the noninteracting one. What makes these developments particularly significant is the fact that the new scenarios have been found to emerge in the low-energy and low-temperature limit, subverting a traditional wisdom which saw in the high-energy limit the true frontier of physics.

As we advanced in the project, the natural desire to make the book truly accessible to graduate students and as self-contained as possible, and the explicit design to discuss openly and critically the approximations on which the theory is based, caused the length of the manuscript to grow beyond our original intentions. We hope that the reader will find the length of the treatise justified by a corresponding increase in clarity and readability. In the end, however, we just had to throw in the towel, and accept to live with many imperfections we were not able to get rid of. To assuage this problem we point the reader to the book web site <http://www.missouri.edu/~physvign/qtel.htm> where we will post the corrections and clarifications that will undoubtedly prove necessary a few seconds after publication. We apologize in advance to all the authors whose important work has not been properly referenced.

A few words concerning our choice of topics are now in order. As a rule, we have refrained from treating in any depth a topic when we had nothing to add to treatments already in print. Examples of such reasoned omissions are the electron-phonon interaction, superconductivity, weak localization theory, the renormalization group, classical plasma analogies, and lattice models of strong correlation. For most of these topics, we have limited ourselves to broad-brush discussions, summarizing the main results of more technical treatments. On the other hand, the reader will find in this book several in-depth discussions of topics never presented before in a pedagogical form, such as the time-dependent current density functional theory, the visco-elastic description of the collective dynamics of the electron liquid, with and without a magnetic field, and the renormalized hamiltonian approach to Fermi liquid theory.

Many people from around the world have in a variety of ways helped us to complete this work. Our special thanks go to David Ceperley, Bahman Davoudi, Paola Gori-Giorgi, Jainendra Jain, Albert Overhauser, Marco Polini, George Simion, and Carsten Ullrich. It is also a pleasure to thank Klaus Capelle, Stefano Chesi, Irene D'Amico, Roberto D'Agosta, Maurizio Ferconi, Michael Geller, Matt Grayson, Catalina Marinescu, Gerardo Ortiz, Vincenzo Piazza, Vittorio Pellegrini, Zhixin Qian, Roberto Raimondi, Stefano Roddaro, Gaetano Senatore, Carlos Wexler and, of course, the Purdue and UMC graduate students who for the last few years have had to put up with lectures based on early, unpolished drafts of this book. GV also thanks the National Science Foundation for providing continuous support during the completion of this work.

As in any endeavor of this magnitude motivations must exist that come from the depths of one's soul. In our case love for the still intriguing field of interacting electrons and inspiration for this work have sprang from our fortunate and early interaction with our mentors and electron gas theory pioneers Franco Bassani, Mario P. Tosi, Albert W. Overhauser, and our beloved Kundan S. Singwi who is no longer with us to see this.

We finally must also express our gratitude to and hope for forgiveness from our families, especially the children who have endured for much too long a time high doses of paternal absenteeism.

PS: Due to life's serendipitous nature, this book has already met with a great deal of success, having afforded one of us (GFG) the possibility of remaining in touch with a professional endeavor during particularly challenging times. In this respect GFG must also heartily thank Geoffrey B. Thompson, John H. Edmonson and Leonard L. Gunderson for having given him, through their singular abilities a chance of seeing the completion of this work.

Gabriele F. Giuliani and Giovanni Vignale
West Lafayette (IN) and Columbia (MO), May 2004

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