主编 夏冬生 张爱锋
主审 赵颖华

材料力学

Mechanics of Materials

大连海事大学出版社

材 料 力 学

Mechanics of Materials

主编　夏冬生　张爱锋
主审　赵颖华

大连海事大学出版社

图书在版编目(CIP)数据

材料力学 = Mechanics of materials：英文 / 夏冬生，张爱锋主编．—大连：大连海事大学出版社，2015.3
ISBN 978-7-5632-3149-2

Ⅰ.①材… Ⅱ.①夏… ②张… Ⅲ.①材料力学—英文 Ⅳ.①TB301

中国版本图书馆 CIP 数据核字(2015)第 059647 号

大连海事大学出版社出版

地址：大连市凌海路1号 邮编：116026 电话：0411-84728394 传真：0411-84727996

http://www.dmupress.com E-mail：cbs@dmupress.com

大连美跃彩色印刷有限公司印装 大连海事大学出版社发行

2015 年 3 月第 1 版 2015 年 3 月第 1 次印刷

幅面尺寸：185 mm × 260 mm 印张：11.5

字数：212 千 印数：1 ~ 700 册

出版人：徐华东

责任编辑：席香吉 责任校对：华云鹏

封面设计：王　艳 版式设计：解瑶瑶

ISBN 978-7-5632-3149-2 定价：26.00 元

Preface

Mechanics of materials is a branch science of solid mechanics, which mainly deals with the internal effects and deformation caused by external forces. It provides basic principles and solutions related to strength, stiffness and stability for design in engineering.

The motivation of writing the book for the authors is to provide an appropriate bilingual-teaching material for teachers and good readability for students. The book provides a comprehensive coverage of important topics in the strength and stiffness of members in axial tension (compression), torsion and bending, strength of members subjected to combined loads, stress transformation, the strength theories and column buckling. Numerous example problems are given in each chapter to demonstrate the approach to solving problems. Through using the book, students not only should master the basic concepts, principles and solving-problem approaches, but also develop a high level of skill in solving engineering problems and designing structures or machine components based on well-understood a few of principles and solutions.

I am very grateful to Zhang Aifeng (DLMU) (Sections §7.4 and §7.5, Chapters 8 and 9) for her writing work. Special thanks are given to Professor Zhao Yinghua (Road and Bridge Engineering Institute, DLMU), who reviewed the book and offered helpful suggestions for improvement. I also appreciate the comments from both my colleagues and students who have used the earlier edition of this book.

Xia Dongsheng

2015.3

Contents

Chapter 1

Introduction

Mechanics of materials is a branch of mechanics that studies the behavior of solid bodies under loads. The way in which they react to applied forces, the resulting deflections and the stresses and strains set up within the bodies are all considered in an attempt to provide sufficient knowledge to enable any component to be designed such that it will not fail within its service life.

1.1 Tasks of Mechanics of Materials

Components of structures are generally made of solid materials that have a certain capacity to resist failure. The change in shapes and dimensions of components of structures caused by loading is named as deformation. In general, the deformations of members can be classified into the elastic and plastic deformations based on whether the deformation can be recovered or not. The recovered deformation after removing loads is named as the elastic deformation and the unrecovered deformation after removing loads is named as the plastic deformation. Statics and dynamics primarily research the external effects on rigid bodies for which deformation is negligible. In contrast, mechanics of materials mainly analyze the internal effects and deformation of structure components, where every structure component is considered as a deformable body.

As for design of structure components and mechanical elements, the following requirements should be satisfied.

(1) Components should have the strength enough to carry the applied loads without breaking or a distinct plastic deformation under the expected service conditions.

(2) Components also should be rigid enough not to deform excessively under the expected

service conditions. For example, gears are often used in mechanical drive systems to transmit power. For proper operation of the gears, they should be properly aligned so the teeth of the driving gear will mesh smoothly with those of the mating gear. If the shafts carrying gears deform excessively at the gears in service conditions, the gear teeth become misaligned and non-uniform wear occurs between gears.

(3) Components should be stable enough to avoid buckling under the expected service conditions. For example, a long and slender member, such as a piston rod in a drive device or a screw arbor of jack, will suddenly bend, losing its equilibrium straight shape, when the compressive axial load surpasses the critical load. The buckling of a member always results in a total collapse of a structure.

The basic requirements of the strength, rigidity and stability must be satisfied in order to ensure that components serve safely and satisfactorily in the expected working condition. During design of structures or members, even though materials with high quality and huge volume are always hoped to be applied for safety, it may bring the results of wasting materials and cumbersome structures. A qualified engineer can consider both of safety and economy (in some cases good appear, environmental protection and saving energy being involved) to find a proper design project. Therefore, how to choose the appropriate materials and determine the reasonable shape and size of sections is very crucial in design of structures or members.

1.2 Basic Assumptions of Solid Bodies

We know that the structure members are made from different materials and possess the complicated microstructures and micro-mechanical properties. Considering the difference in micro-structures, we find it very difficult to solve the mathematical and physical problems in theoretical analyses and inconvenient to apply principles. Therefore, the following three assumptions are proposed based on the properties of engineering materials.

(1) **Continuity**

A solid body is assumed to be continuously filled with material over its volume without any defect and small hole. In macro scope, it is acceptable because the small cracks and holes in a member can be negligible compared to the whole volume.

(2) **Homogeneity**

A solid body is assumed to be homogeneously filled with material over its volume so that the mechanical properties of any part are irrelevant to the locations. As for metals, they consist of numerous crystalline grains. Every crystalline grain possesses different mechanical properties. Moreover, the mechanical properties at grain boundaries are different from ones inside crystalline grains. However, the mechanical properties of any part of metals are the statistical average of ones of numerous crystalline grains in this part. Therefore, many engineering materials may be approximated as homogeneous materials.

(3) Isotropy

A solid body is assumed to possess the same mechanical properties in all directions. Isotropic materials include steel, aluminum, copper, plastic and glass. As for metals, although a single grain has different mechanical properties with direction, metal components having a physical size contain a great number of randomly oriented crystalline grains so that the mechanical properties are approximately the same in all directions. Therefore, it is quite realistic for many engineering materials to be approximated as isotropic materials.

1.3 Internal Forces, Stresses and Section Method

Internal force is the resultant of internal forces, which is acting mutually between two neighboring parts inside the body, caused by the external forces. As we know, when a body is not subjected to any external force, the mutual internal forces still exist between any two adjacent particles inside it. In mechanics of materials, the change of the internal force caused by the external forces is referred to as the internal force.

To illustrate the internal forces on a section in a bar subjected to the external forces, we pass through a section and divide it into two segments (Fig. 1-1). The internal forces must exist on the cutting section of any one segment, which balance the external forces acting on the segment. It is convinced that the internal forces on the cutting section for the left and right segments are equal and opposite according to action and reaction law.

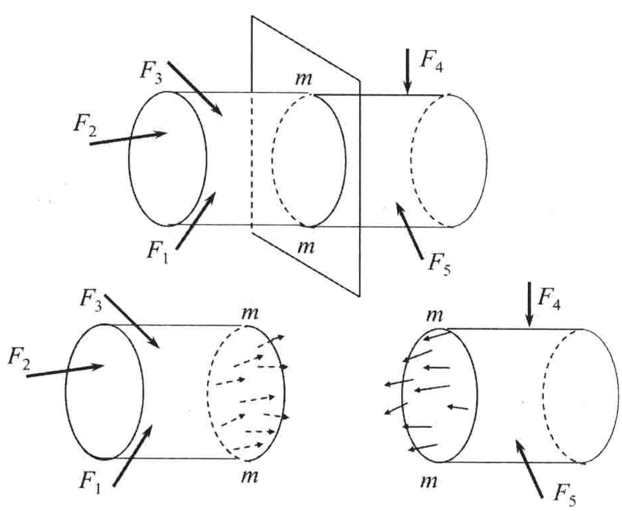

Fig. 1-1

As the internal forces reach a certain maximum value, the members will break. The strength, stiffness and stability are closely related to the internal forces and their distribution state in members.

Whether or not a bar will break under the loads depends on not only the internal forces but also the sectional area. Thus, we introduce a concept, the stress, intensity of the internal force distributed over the section, which plays a crucial role on the design of members.

To illustrate the concept of stress, considering a segment in Fig. 1-2, we select an element area ΔA (Fig. 1-2) around an arbitrary point M on the cut section $m-m$. ΔF is the resultant internal force on the element area ΔA.

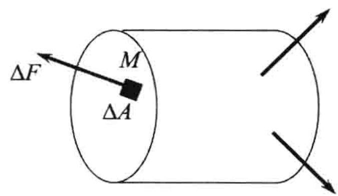

Fig. 1-2

The stress on the element area ΔA can be expressed as

$$\bar{p} = \frac{\Delta F}{\Delta A} \tag{1.1}$$

We should note that Eq. (1.1) represents the average intensity of internal force on ΔA, which is termed as the average stress.

As the element area ΔA approaches zero, we obtain the stress at the point M, which reflects the intensity of internal forces at a point.

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.2}$$

The stress p at a point is a vector, which is generally neither parallel nor perpendicular to the section. In order to investigate more conveniently, we often decompose the stress vector acting at a point M into two stress components respectively along perpendicular and parallel directions to the cross section (Fig. 1-3). The normal component is defined as the normal stress, which is denoted by σ, whilst the parallel component is defined as the shear stress, which is denoted by τ. Thus

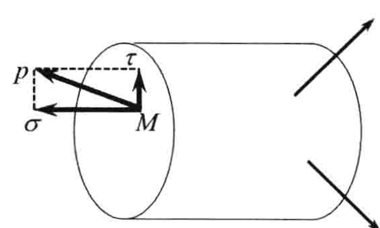

Fig. 1-3

$$p^2 = \sigma^2 + \tau^2 \tag{1.3}$$

It is noted that in SI metric units, force is measured in newtons (N) and area in square meters (m^2), from which the unit of stress is newtons per square meter (N/m^2), equivalent Pascal (Pa): 1 Pa = 1 N/m^2. Because the pascal is a too small unit in engineering practice, multiples of this unit is commonly used in expression of kPa, MPa and GPa. We have

$$1\ \text{kPa} = 10^3\ \text{Pa},\quad 1\ \text{MPa} = 10^6\ \text{Pa},\quad 1\ \text{GPa} = 10^9\ \text{Pa}$$

1.4 Strain

Both of the deformation and internal force occur under the influence of external forces and moreover, they closely correlate each other. Therefore, for studying rigidity of a member and the stress distribution, it is necessary to analyze deformation at any point in a body.

Fig. 1-4 shows that two points A and B in a body are displaced to A' and B', respectively, due to deformation. The original length of the line segment AB is Δs before deformation and the length of the line segment $A'B'$ is $\Delta s + \Delta u$ after deformation. We define the average normal strain ε_m as

$$\varepsilon_m = \frac{l_{A'B'} - l_{AB}}{l_{AB}} = \frac{\Delta u}{\Delta s} \tag{1.4}$$

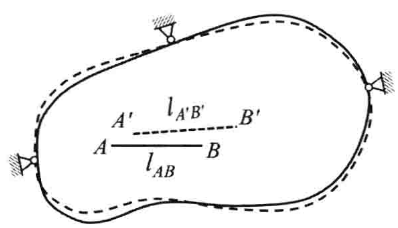

Fig. 1-4

where ε_m represents the average elongation or contraction per unit length of the line segment AB. If the deformation degree along AB is not uniform at any point of the line segment AB, ε_m changes with the length of the selected line segment AB.

When the point B is infinitely approached to the point A, we get the limit value of ε_m.

$$\varepsilon = \lim_{l_{AB}\to 0} \frac{l_{A'B'} - l_{AB}}{l_{AB}} = \lim_{\Delta s\to 0} \frac{\Delta u}{\Delta s} \tag{1.5}$$

where ε represents the normal strain along AB at the point A. If the deformation degree along AB is uniform at any point of the line segment AB, the average normal strain ε_m is equal to the normal strain ε at the point A.

As a body deforms due to action of the forces, the change in angle between two originally orthogonal line segments is defined as shear strain. Fig. 1-5 shows that line segment AB is perpendicular to line segment AC before deformation and the angle $\angle CAB$ between these two line segments is changed to $\angle C'A'B'$ after deformation. The change in angle between these two line segments is $\frac{\pi}{2} - \angle C'A'B'$. When the two points B and C are infinitely approached to the point A, the limit value of the angle change between the two orthogonal line segments AB and AC is expressed as

$$\gamma = \lim_{\substack{l_{AB}\to 0 \\ l_{AC}\to 0}} \left(\frac{\pi}{2} - \angle C'A'B'\right) \tag{1.6}$$

where γ represents the shear strain at the point A on the plane CAB, which is measured in radians.

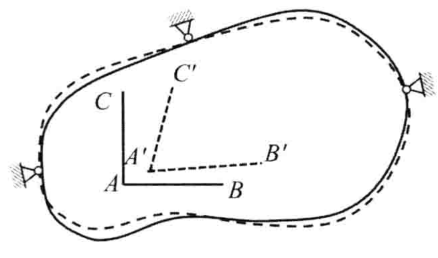

Fig. 1-5

The deformation degree at any point in a body may be measured by the normal strain ε and the shear strain γ. It is found from the definition expressions of normal strain ε and shear strain γ (Eq. (1.5) and (1.6)) that the dimensions of normal strain ε and shear strain γ are 1.

1.5 Basic Deformation Forms of Members

Axial Tension (Compression). Two equivalent and opposite resultant forces F along the axial line, are applied on both of the ends of a straight bar, resulting in the elongation or contraction along the axis, as shown in Fig. 1-6.

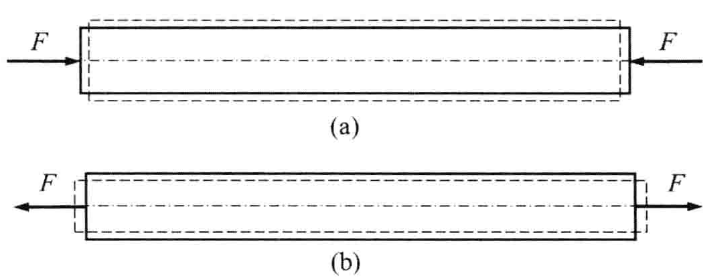

Fig. 1-6

Shear. Two equivalent and opposite forces *F* with a close distance are transversely applied on a member, causing sections between the points of application of the loads to slide past its adjacent sections along the loads, as shown in Fig. 1-7.

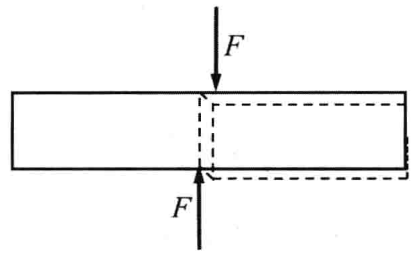

Fig. 1-7

Torsion. When a member is subjected to two equivalent couples *T* whose action planes are perpendicular to the longitudinal axis, it demonstrates relative rotation of any two cross sections about the longitudinal axis, as shown in Fig. 1-8.

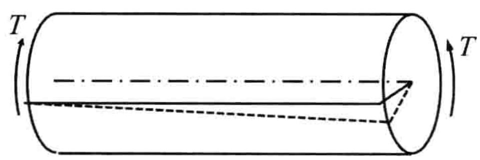

Fig. 1-8

Bending. When two equivalent couples *M* or the forces perpendicular to the axial line are applied in the longitudinal symmetric plane of a member, the straight axial line becomes curving, as shown in Fig. 1-9.

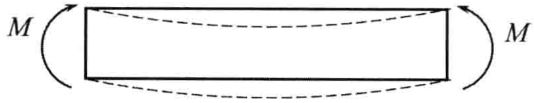

Fig. 1-9

Chapter 2

Axial Tension (Compression), Mechanical Properties of Materials and Shear

In engineering, there are numerous structural members subjected to the loads along the axial line (the line connecting the centroids of cross sections), such as a bridge truss (Fig. 2-1 (a)), a connecting bar in a gas engine (Fig. 2-1(b)) and a fasten bolt of cylinder head.

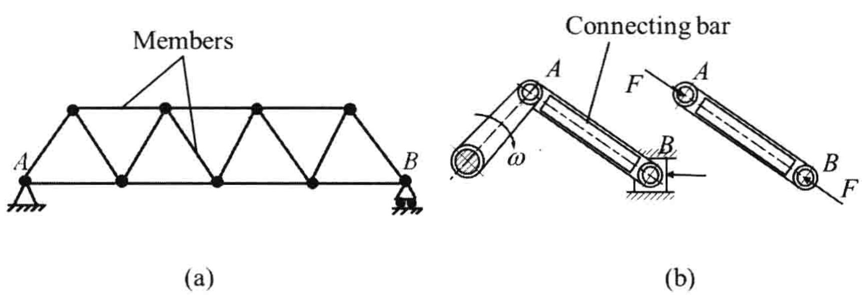

Fig. 2-1

As for the members shown in Fig. 2-1, if their configuration and loads are simplified, the free-body diagrams of the members are presented in Fig. 2-2. It is found that two equivalent resultant forces coinciding with the axial line, acts on the ends of the members, resulting in elongation or contraction along the axial line. The loads along the axial line of member are called as the axial loads.

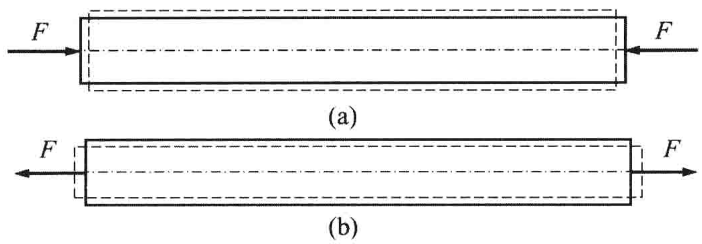

Fig. 2-2

2.1 Axial Force and Diagram of Axial Force

To determine the internal force on the cross section of a bar subjected to two axial loads F applied to both of the ends, the bar is supposed to be divided into two segments along an arbitrary cross section $m - m$ as shown in Fig. 2-3(a). The mutual action force on the cross section $m - m$ between both of the left and right segments is a distributed force system, the resultant force of which is the internal force.

Firstly, we consider the left segment and draw its free-body diagram, as shown in Fig. 2-3(b). To keep this left segment in equilibrium, the internal force F_N on the cross section $m - m$ must act along the axial line because the external loads F acting on both of the ends coincide with the axial line.

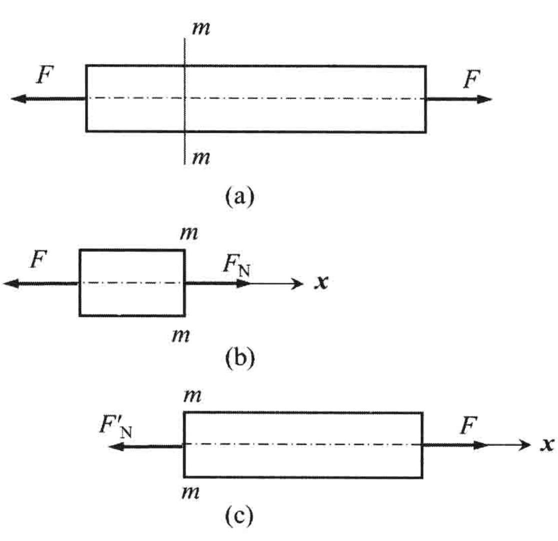

Fig. 2-3

Then set up the equilibrium equation of the left segment as follows.

$$\sum F_x = 0, \quad F - F_N = 0$$

$$F_N = F$$

If we consider the right segment, its free-body diagram is shown in Fig. 2-3(c).

Set up the equilibrium equation of the right segment as follows.

$$\sum F_x = 0, \quad -F + F'_N = 0$$

$$F'_N = F$$

An internal force F_N acting on the cross section of a bar in axial compression or tension is termed as the axial force, due to the internal force acting along the axial line.

Sign Convention for Axial Forces

Tensile axial forces (axial forces are directed away from the cross section) are considered to be *positive*.

Compressive axial forces (axial forces are directed to the cross section) are considered to be *negative*.

It is noticed that no matter we consider the left or right segment to the cutting section, the axial forces on the cutting section have the same calculated values with the sign convention of axial forces.

The above method is usually applied to determine the internal forces of a member, which is called as the section method. The section method includes three steps as follows.

(1) Assume to separate the member into two segments along a selected section.

(2) Consider an arbitrary segment (i. e. the either segment that is on the left or right side with respect to the cross section), draw its free-body diagram and substitute the action of another segment on it by the corresponding internal force on the cutting section.

(3) Set up the equilibrium equations and determine the unknown internal forces.

Fig. 2-3 indicates that the axial force F_N on the cross sections is constant along the entire length when the two axial loads are applied to both of the ends of a bar. However, when a bar is subjected to more than two axial loads or the distributed axial loads, the axial force F_N is different in different segment or change along the length. It is helpful to show the change of the axial force F_N on the cross sections along the axial line by drawing an axial force diagram. Specifically, this diagram is a plot of the axial force F_N versus its position x along the bar's length.

Example 2-1

A circular bar carries a series loads as shown in Fig. 2-4. Determine the axial force on the cross sections and plot the diagram of axial force.

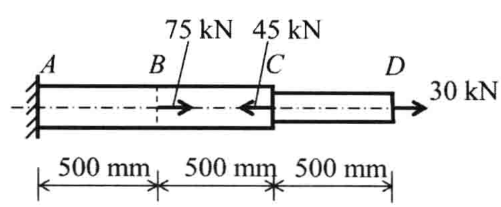

Fig. 2-4