

“十二五”国家重点图书

网络科学与工程丛书

N S E

7

# Introduction to Complex Networks

## Models, Structures and Dynamics

(Second Edition)

英文版

## 复杂网络引论

——模型、结构与动力学

(第二版)

■ 陈关荣 汪小帆 李翔 编著

高等教育出版社

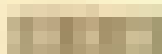
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Models, Structures  
and Dynamics

Alfred A. Bertalanffy



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## 作者简介

**陈关荣**，1981年获广州中山大学计算数学硕士学位，1987年获美国Texas A&M 大学应用数学博士学位，现任香港城市大学讲座教授及混沌与复杂网络学术研究中心主任。于1996年当选为IEEE Fellow，被ISI评定为工程学及物理学高引用率研究人员，2008年和2012年获国家自然科学二等奖，2011年获俄罗斯圣彼得堡国立大学授予荣誉博士学位和俄罗斯欧拉基金会颁发欧拉金质奖章，2014年获法国诺曼底大学授予荣誉博士学位，并当选为欧洲科学院院士。



**汪小帆**，1996年获东南大学工学博士学位，现为上海交通大学电子信息与电气工程学院教授、致远学院常务副院长。2008年受聘为教育部长江学者特聘教授。近年一直从事复杂网络系统分析与控制研究。获2002年国家杰出青年科学基金、2005年IEEE电路与系统汇刊最佳论文奖、2008年上海市自然科学一等奖和2010年上海市自然科学牡丹奖。任2014—2017年 IFAC Large-Scale Complex Systems技术委员会主席。



**李翔**，2002年获南开大学控制理论与控制工程博士学位，现为复旦大学信息科学与工程学院教授、电子工程系主任。近年一直从事复杂网络与系统控制的理论与应用研究。先后获得2005年IEEE电路与系统汇刊最佳论文奖、2008年上海市自然科学一等奖、2010年上海市青年科技英才奖、2014年国家杰出青年科学基金等。



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# 序

随着以互联网为代表的网络信息技术的迅速发展，人类社会已经迈入了复杂网络时代。人类的生活与生产活动越来越多地依赖于各种复杂网络系统安全可靠和有效的运行。作为一个跨学科的新兴领域，“网络科学与工程”已经逐步形成并获得了迅猛发展。现在，许多发达国家的科学界和工程界都将这个新兴领域提上了国家科技发展规划的议事日程。在中国，复杂系统包括复杂网络作为基础研究也已列入《国家中长期科学和技术发展规划纲要(2006—2020年)》。

网络科学与工程重点研究自然科学技术和社会政治经济中各种复杂系统微观性态与宏观现象之间的密切联系，特别是其网络结构的形成机理与演化方式、结构模式与动态行为、运动规律与调控策略，以及多关联复杂系统在不同尺度下行为之间的相关性等。网络科学与工程融合了数学、统计物理、计算机科学及各类工程技术科学，探索采用复杂系统自组织演化发展的思想去建立全新的理论和方法，其中的网络拓扑学拓展了人们对复杂系统的认识，而网络动力学则更深入地刻画了复杂系统的本质。网络科学既是数学中经典图论和随机图论的自然延伸，也是系统科学和复杂性科学的创新发展。

为了适应这一高速发展的跨学科领域的迫切需求，中国工业与应用数学学会复杂系统与复杂网络专业委员会偕同高等教育出版社出版了这套“网络科学与工程丛书”。这套丛书将为中国广大的科研教学人员提供一个交流最新

研究成果、介绍重要学科进展和指导年轻学者的平台，以共同推动国内网络科学与工程研究的进一步发展。丛书在内容上将涵盖网络科学的各个方面，特别是网络数学与图论的基础理论，网络拓扑与建模，网络信息检索、搜索算法与数据挖掘，网络动力学（如人类行为、网络传播、同步、控制与博弈），实际网络应用（如社会网络、生物网络、战争与高科技网络、无线传感器网络、通信网络与互联网），以及时间序列网络分析（如脑科学、心电图、音乐和语言）等。

“网络科学与工程丛书”旨在出版一系列高水准的研究专著和教材，使其成为引领复杂网络基础与应用研究的信息和学术资源。我们殷切希望通过这套丛书的出版，进一步活跃网络科学与工程的研究气氛，推动该学科领域知识的普及，并为其深入发展做出贡献。

金芳蓉 (Fan Chung) 院士  
美国加州大学圣地亚哥分校  
二〇一一年元月



# Preface

The extensive study of complex networks is pervading sciences and engineering today, from physical, technological, biological, to social sciences. Their impacts on engineering and technology, in particular, are prominent and their influence is deemed to be far-reaching. Familiar complex networks include the Internet, the World Wide Web, wireless communication networks, biological neural networks, power grids, social relation and scientific cooperation networks, and so on. Research on fundamental properties and dynamical behaviors of various complex networks has recently become overwhelming.

The field of complex networks is indeed developing so fast and so wide that most newcomers typically find it quite difficult to know where to start their learning and research on the subject. Although there are some well-written textbooks and research monographs that can be adopted for studies by newcomers, these references are generally too advanced or too broad for those readers to comprehend, especially in a relatively short period of time; they are not easily used as textbooks for a short course on the subject either. Driven by such teaching and learning demands, this book has been designed to serve as a concise textbook for newcomers to the field. It is written as a one-semester introductory text for upper-division undergraduate or first-year graduate students in natural science, mathematics and engineering, or as an edited volume for self-study, or as a handy reference for research.

The book title retains the adjective “complex” to reflect the historical perspective and to emphasize the nature of the subject, which is in line with the common phrases of complex systems and complex dynamics alike, and therefore should not be seen as redundant.

The style of writing in this book is intended to be informal, emphasizing basic ideas and methodologies with elementary and sometimes heuristic mathematical arguments, easily readable by anyone having minimal knowledge of calculus, linear algebra and ordinary differential equations. In this regard, and to be self-contained, a preliminary chapter on graph theory, probability and statistics as well as dynamical systems is included after the first overview chapter. The book is divided into two parts: Part I *Fundamental Theory* is a detailed text consisting of three chapters, presenting background information and basic materials needed to learn the subject, with a variety of exercises for illustrating fundamental concepts and familiarizing related modeling and analysis techniques. Part II *Applications – Selected Topics* contains several selected application-oriented topics, which are all independent of each other, in the sense that one can choose any chapter to teach or to learn individually without referring to the contents of the other chapters in this part. Of course, the current arrangement of chapters would also be a logical ordering if one decided to read through this whole portion of the book. The last chapter of this part provides only outlines of several emerging topics which are believed important and promising, with sufficient numbers of key references provided for interested readers’ future studies.

This book is a slightly modified and extended version of the same authors’ earlier version of the book entitled *Introduction to Complex Networks: Models, Structures and Dynamics*, published by the Higher Education Press, Beijing in 2012. The basic materials of this book have been used in teaching a post-graduate course on *Complex Networks: Modeling, Dynamics and Control* at the City University of Hong Kong since 2007, received valuable feedback from students which has made the present version better suited to a text for both teaching and learning.

Owing to the introductory nature of the book, it does not cover the most advanced developments in the field, especially those in the last five years or so. It is the authors' hope that after learning this elementary text, readers are ready to read recent literature so as to pursue state-of-the-art research in the field of network science and engineering. For Chinese readers, there is a complementary volume written in Chinese by the same authors, entitled *Network Science: An Introduction* (Higher Education Press, Beijing, 2012), which has very little overlapping with the present book and hence should be fairly referential and informative.

Guanrong Chen  
*City University of Hong Kong*

Xiaofan Wang  
*Shanghai Jiao Tong University*

Xiang Li  
*Fudan University*

Summer 2014

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Guanrong Chen  
*City University of Hong Kong*

Xiaofan Wang  
*Shanghai Jiao Tong University*

Xiang Li  
*Fudan University*

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# Part One

## Fundamental Theory