H. J. Carmichael

Theoretical and Mathematical Physics

Statistical Methods in Quantum Optics 1

Master Equations and Fokker-Planck Equations

量子光学中的统计方法 第1卷

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Dedication

To my mother
and to the memory of my father

"For even as He loves the arrow that flies, so He loves also the bow that is stable."

Preface to the Second Corrected Printing

The material in the first volume of this book is standard and I have changed little in revising it for the second edition. My main task has been to correct the numerous errors that, having escaped detection in the original manuscript, were brought to my attention after the book came to print. My thanks to all those who have helped in this enterprise by informing me of an insidious misspelling, a mistake in an equation, or a mislabeled figure. A few other superficial changes have been made; I have reworked many of the figures and "prettied up" the typesetting in one or two spots.

There is just one change of real substance. I have chosen to replace the designation "quantum regression theorem," which has been standard in quantum optics circles for some three decades, with the more accurate "quantum regression formula." The replacement is perhaps not perfect, since the regression procedure introduced by Lax is expressed by different formulas on different occasions. In some cases the procedure runs very much parallel to Onsager's classical regression hypothesis – i.e., when a linearized treatment of fluctuations is carried out. In others it does not. The point to be made, however, is that the formula used, whatever its specific form, is never the expression of a "theorem"; it is the expression of a Markovian open system dynamics, reached from a microscopic model in quantum optics by way of an approximation, the same Markov dynamics that one finds defined, more formally, in the semigroup approach to open quantum systems. Physicists, all too often, remain unworried about semantic accuracy in a matter like this; the common designation is historical and no doubt harmless enough. On the other hand, for some reason, which was always difficult for me to understand, the quantum regression formula has attracted an undeserved level of suspicion throughout its 30 years of use; it seems not to be appreciated that the formula for multi time averages enjoys precisely the same grounding, Markov approximation and all, as the master equation itself – just as the master equation emerges in the Schrödinger picture, so, in the Heisenberg picture, emerges Lax's quantum regression. Given, then, what I perceive to be a background of misunderstanding, I think it wise to be as accurate and clear as possible. I have therefore avoided the word "theorem" in this second edition of Vol. 1, and also in Vol. 2. I have also added some commentary relating to this point in the section of Vol. 1 devoted to the quantum regression formula.

Auckland
May 2002

Howard Carmichael

Preface to the First Edition

As a graduate student working in quantum optics I encountered the question that might be taken as the theme of this book. The question definitely arose at that time though it was not yet very clearly defined; there was simply some deep irritation caused by the work I was doing, something quite fundamental I did not understand. Of course, so many things are not understood when one is a graduate student. However, my nagging question was not a technical issue, not merely a mathematical concept that was difficult to grasp. It was a sense that certain elementary notions that are accepted as starting points for work in quantum optics somehow had no fundamental foundation, no identifiable root. My inclination was to mine physics vertically, and here was a subject whose tunnels were dug horizontally. There were branches, certainly, going up and going down. Nonetheless, something major in the downwards direction was missing—at least in my understanding; no doubt others understood the connections downwards very well.

In retrospect I can identify the irritation. Quantum optics deals primarily with dynamics, quantum dynamics, and in doing so makes extensive use of words like "quantum fluctuations" and "quantum noise." The words seem harmless enough. Surely the ideas behind them are quite clear; after all, quantum mechanics is a statistical theory, and in its dynamical aspects it is therefore a theory of fluctuations. But there was my problem. Nothing in Schrödinger's equation fluctuates. What, then, *is* a quantum fluctuation?

In reply one might explore one of the horizontal tunnels. Statistical ideas became established in thermal physics during the early period of the quantum revolution. Although the central notions in this context are things like equilibrium ensembles, partition functions and the like, every graduate student is aware of the fluctuation aspect through the example of Brownian motion. Fluctuations are described using probability distributions, correlation functions, Fokker–Planck and Langevin equations, and mathematical devices such as these. In many instances the quantum analogs of these things are obvious. So, are quantum fluctuations simply thermal fluctuations that occur in the quantum realm? Well, once again, nothing fluctuates in Schrödinger's equation; yet the standard interpretation for the state solving this equation is statistical, and speaks of fluctuations, even when the most elementary system is described. Quantum fluctuations are therefore more fundamental than thermal fluctuations. They are a fundamental part of quantum theory—though apparently absent from its fundamental equation—and unlike thermal fluctuations, not comfortably accounted for by simply reflecting on the disorganized dynamics of a complex system.

I now appreciate more clearly where my question was headed: Yes it does head downwards, and it goes very deep. What is less clear is that there is a path in that direction understood by anyone very well. The direction is towards the foundations of quantum mechanics, and here one must face those notorious issues of interpretation that stimulate much confusion and contention but few definitive answers.

I must hasten to add that this book is not about the foundations of quantum mechanics—at least not in the formal sense; the subject is mentioned directly in only one chapter, near the end of Volume II. It is helpful to know, though, that this subject is the inevitable attractor to which four decades of development in quantum optics have been drawn. The book's real theme is quantum fluctuations, tackled for the most part at a pragmatic level. It is about the methods developed in quantum optics for analyzing quantum fluctuations in terms of a visualizable evolution over time. The qualifier "visualizable" is carried through as an informal connection to foundations. In view of it, I emphasize the Schrödinger and interaction pictures over the Heisenberg picture since in these pictures appropriate representations of the time-varying states (Glauber–Sudarshan or Wigner representations for example) can provide tangible access to something that fluctuates. Such mental props cannot be taken too literally, however, and the book is as much about their limitations as about their successes. I have written the book in a period when the demands for theoretical analyses of new experiments have required that the limitations be acknowledged and paid serious attention. The book meanders a bit in response to the proddings. Hopefully, though, there is always forwards momentum, towards methods of wider applicability and a more satisfying understanding of the foundations.

Quantum optics has a unique slant on quantum fluctuations, different from that of statistical physics with its emphasis on thermal equilibrium, and also differing from relativistic field theory where fluctuations refer either to virtual transitions—dressing stable objects—or little particle "explosions" (collisions) with a well-defined beginning and end. Quantum optics is concerned with matter interacting with electromagnetic waves at optical frequencies. At such frequencies, in terrestrial laboratories, it meets with quantum fluctuations that are real, and ongoing, and not inevitably buried in thermal noise; at least the latter has been the case since the invention of the laser; and it is the laser, overwhelmingly, that gives quantum optics its special perspective. The laser is basically a convenient source of coherence. Thought of simply, this is the coherence of a classical wave, but it is readily written into material systems where it must ultimately be seen as quantum coherence. The mix of coherence (waves) with the particle counting used to detect optical fields marks quantum optics for encounters with the difficult issues that arose around the ideas of Einstein and Bohr at the beginning of the century. Old issues are met with new clarity, but even more interesting are the entirely new dimensions. Seen as a quantum field, laser light is in a degenerate state, having a very large photon occupation number per mode. This property makes it easy to excite material systems far from thermal equilibrium, where simple perturbation theory is unable to account for the dynamics. In the classical limit one expects to encounter the gamut of nonlinear phenomena: instability, bifurcation, multistability, chaos. One might ask where quantum fluctuations fit in the scheme of such things; no doubt as a minor perturbation in the approach to the classical limit. But in recent years the drive in optics towards precision and application has opened up the area of cavity QED. Here the electromagnetic field is confined within such a small volume that just one photon can supply the energy density needed to excite a system far from equilibrium. Under conditions like this, quantum fluctuations overwhelm the classical nonlinear dynamics. How, then, does the latter emerge from the fluctuations as the cavity QED limit is relaxed?

The book is divided into two volumes. This first volume deals with the statistical methods used in quantum optics up to the late 1970s. The material included here is based on a series of lectures I gave at the University of Texas at Austin during the fall semester of 1984. In this early period, methods for treating open systems in quantum optics were developed around two principal examples: the laser and resonance fluorescence. The two examples represent two defining themes for the subject, each identified with an innovation that extended the ways of thinking in some more established field. The laser required thinking in QED to be extended, beyond its focus on few-particle scattering to the treatment of many particle fields approaching the classical limit. The innovation was Glauber's coherence theory and the phase-space methods based on coherent states. The revival of the old topic of resonance fluorescence moved in the opposite direction. At first its concern was strong excitation—the nonperturbative limit which had been inaccessible to experiments before the laser was invented. Soon, however, a second theme developed. Contrasting with laser light and its approximation to a classical field, resonance fluorescence is manifestly a quantum field; its intensity fluctuations display features betraying their origin in *particle* scattering. The innovation here was in the theory of photoelectron counting—in the need to go beyond the semiclassical Mandel formula which holds only for statistical mixtures of coherent states. Thus, the study of resonance fluorescence began the preoccupation in quantum optics with the so-called nonclassical states of light.

Resonance fluorescence is treated in Chap. 2 and there are two chapters in this volume, Chaps. 7 and 8, on the theory of the laser. My aim with the example of resonance fluorescence is to illustrate the utility of the master equation and the quantum regression theorem for solving a significant problem, essentially exactly, with little more than some matrix algebra. Chapter 1 and the beginning of Chap. 2 fill in the background to the calculations. Here I provide derivations of the master equation and the quantum regression theorem. I think it important to emphasize that the quantum regression theorem is a derived result, equal in the firmness of its foundations to the master equation itself, and indeed a necessary adjunct to that equation if it is to be used to calculate anything other than the most trivial things (i.e. one-time operator averages).

Chapters 3–7 all lead up to a treatment of laser theory by the phase-space methods in Chap. 8. My purpose in Chap. 8 has been to carry through a systematic application of the phase-space methods to a nonequilibrium system of historical importance. Some readers will find the treatment overly detailed and be satisfied to simply skim the calculations. I would recommend the option, in fact, when the book is used as the basis for a course. In taking it, nothing need be lost with regard to the physics since the more useful results in laser theory are presented in Chap. 7 in a more accessible way. The earlier chapters have wide relevance in quantum optics. They deal with the properties of coherent states and the Glauber–Sudarshan P representation (Chap. 3), the Q and Wigner representations (Chap. 4), and the extension of these phase-space representations to two-state atoms (Chap. 6). Chapter 5 makes a short excursion to review those results from classical nonequilibrium statistical physics that are imported into quantum optics on the basis of the phase-space methods.

Volume I ends with Chap. 8 and the phase-space treatment of the laser. The treatment provides a rigorous basis for the standard visualization of amplitude and phase fluctuations in laser light. The visualization, however, is essentially classical, and the story of quantum fluctuations cannot be ended here. Being aware of the approximations used to derive the laser Fokker–Planck equation and having seen the example of resonance fluorescence, for which a similar simplification does not hold, it is clear that such classical visualizations cannot generally be sustained. Volume II will deal with the extension of the basic master equation approach to situations in which the naive phase-space visualization fails, where the quantum nature of the fluctuations has manifestations in the actual form of the evolution over time. Modern topics such as squeezing, the positive P representation, cavity QED, and quantum trajectory theory will be covered there.

I have sprinkled exercises throughout the book. In some cases they are included to excuse me from carrying through a calculation explicitly, or to repeat and generalize a calculation that has just been done. The exercises are integrated with the development of the subject matter and are intended, literally, as exercises, exercises for the practitioner, rather than an introduction to problems of topical interest. Their level varies. Some are quite difficult. The successful completion of the exercises will generally be aided by a detailed understanding of the calculations worked through in the book.

Numerous students and colleagues have read parts of this book as a manuscript and helped purge it of typographical errors or made other useful suggestions. I know I will not recall everyone, but I cannot overlook those whom I do remember. I am grateful for the interest and comments of Paul Alsing, Robert Ballagh, Young-Tak Chough, John Cooper, Rashed Haq, Wayne Itano, Jeff Kimble, Perry Rice, and Murray Wolinsky.

Eugene, Oregon
August 1998

Howard Carmichael

Volume 2. Modern Topics