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Bose-Einstein Condensation in Dilute Gases

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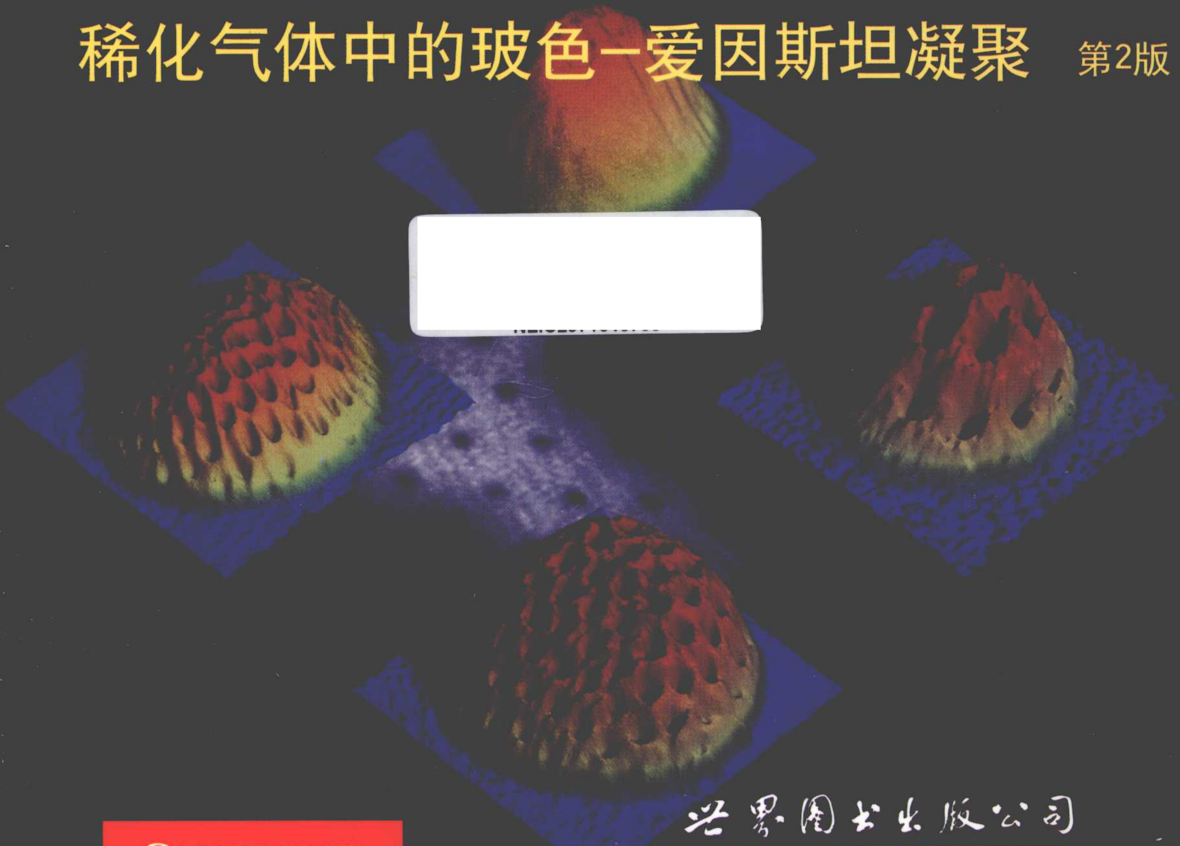

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Preface

The experimental discovery of Bose–Einstein condensation in trapped atomic clouds opened up the exploration of quantum phenomena in a qualitatively new regime. Our aim in the present work is to provide an introduction to this rapidly developing field.

The study of Bose–Einstein condensation in dilute gases draws on many different subfields of physics. Atomic physics provides the basic methods for creating and manipulating these systems, and the physical data required to characterize them. Because interactions between atoms play a key role in the behaviour of ultracold atomic clouds, concepts and methods from condensed matter physics are used extensively. Investigations of spatial and temporal correlations of particles provide links to quantum optics, where related studies have been made for photons. Trapped atomic clouds have some similarities to atomic nuclei, and insights from nuclear physics have been helpful in understanding their properties.

In presenting this diverse range of topics we have attempted to explain physical phenomena in terms of basic principles. In order to make the presentation self-contained, while keeping the length of the book within reasonable bounds, we have been forced to select some subjects and omit others. For similar reasons and because there now exist review articles with extensive bibliographies, the lists of references following each chapter are far from exhaustive.

This book originated in a set of lecture notes written for a graduate-level one-semester course on Bose–Einstein condensation at the University of Copenhagen. The first edition was completed in 2001. For this second edition we have updated the manuscript and added three new chapters on optical lattices, lower dimensions and molecules. We employ SI units throughout the text. As for mathematical notation we generally use $\sim$ to indicate 'is of order', while $\simeq$ means 'is asymptotically equal to' as in $(1-x)^{-1} \simeq 1+x$.

The symbol $\approx$ means 'is approximately equal to'. Definitions are indicated by $\equiv$, and $\propto$ means 'is proportional to'. However, the reader should be aware that strict consistency in these matters is not possible.

We have received much inspiration from contacts with our colleagues in both experiment and theory. In particular we thank Gordon Baym, Georg Bruun, Alexander Fetter, Henning Heiselberg, Andreas Isacsson, George Kavoulakis, Pietro Massignan, Ben Mottelson, Jörg Helge Müller, Alexandru Nicolin, Nicolai Nygaard, Olav Syljuåsen, Gentaro Watanabe and Mikhail Zvonarev for many stimulating and helpful discussions over the past few years. Wolfgang Ketterle kindly provided us with the cover illustration and Fig. 13.2, and we thank Eric Cornell for allowing us to use Fig. 9.3. We are grateful to Mikhail Zvonarev for providing us with the data for Figs. 15.2–4. The illustrations in the text have been prepared by Janus Schmidt and Alexandru Nicolin, whom we thank for a pleasant collaboration. It is a pleasure to acknowledge the support of Simon Capelin, Susan Francis, Lindsay Barnes, and Jonathan Ratcliffe at the Cambridge University Press, and the careful copy-editing of the manuscript by Brian Watts and Jon Billam.

Contents

1
Introduction

The experimental realization in 1995 of Bose–Einstein condensation in dilute atomic gases marked the beginning of a very rapid development in the study of quantum gases. The initial experiments were performed on vapours of rubidium [1], sodium [2], and lithium [3].[1] So far, the atoms ^{1}H, ^{7}Li, ^{23}Na, ^{39}K, ^{41}K, ^{52}Cr, ^{85}Rb, ^{87}Rb, ^{133}Cs, ^{170}Yb, ^{174}Yb and ^{4}He* (the helium atom in an excited state) have been demonstrated to undergo Bose–Einstein condensation. In related developments, atomic Fermi gases have been cooled to well below the degeneracy temperature, and a superfluid state with correlated pairs of fermions has been observed. Also molecules consisting of pairs of fermionic atoms such as ^{6}Li or ^{40}K have been observed to undergo Bose–Einstein condensation. Atoms have been put into optical lattices, thereby allowing the study of many-body systems that are realizations of models used in condensed matter physics. Although the gases are very dilute, the atoms can be made to interact strongly, thus providing new challenges for the description of strongly correlated many-body systems. In a period of less than ten years the study of dilute quantum gases has changed from an esoteric topic to an integral part of contemporary physics, with strong ties to molecular, atomic, subatomic and condensed matter physics.

The dilute quantum gases differ from ordinary gases, liquids and solids in a number of ways, as we shall now illustrate by giving values of physical quantities. The particle density at the centre of a Bose–Einstein condensed atomic cloud is typically 10^{13}–10^{15} cm^{-3}. By contrast, the density of molecules in air at room temperature and atmospheric pressure is about 10^{19} cm^{-3}. In liquids and solids the density of atoms is of order 10^{22} cm^{-3}, while the density of nucleons in atomic nuclei is about 10^{38} cm^{-3}.

To observe quantum phenomena in such low-density systems, the tem-

[1] Numbers in square brackets are references, to be found at the end of each chapter.

perature must be of order 10^{-5} K or less. This may be contrasted with the temperatures at which quantum phenomena occur in solids and liquids. In solids, quantum effects become strong for electrons in metals below the Fermi temperature, which is typically 10^4–10^5 K, and for phonons below the Debye temperature, which is typically of order 10^2 K. For the helium liquids, the temperatures required for observing quantum phenomena are of order 1 K. Due to the much higher particle density in atomic nuclei, the corresponding degeneracy temperature is about 10^{11} K.

The path that led in 1995 to the first realization of Bose–Einstein condensation in dilute gases exploited the powerful methods developed since the mid 1970s for cooling alkali metal atoms by using lasers. Since laser cooling alone did not produce sufficiently high densities and low temperatures for condensation, it was followed by an evaporative cooling stage, in which the more energetic atoms were removed from the trap, thereby cooling the remaining atoms.

Cold gas clouds have many advantages for investigations of quantum phenomena. In a weakly interacting Bose–Einstein condensate, essentially all atoms occupy the same quantum state, and the condensate may be described in terms of a mean-field theory similar to the Hartree–Fock theory for atoms. This is in marked contrast to liquid ^{4}He, for which a mean-field approach is inapplicable due to the strong correlations induced by the interaction between the atoms. Although the gases are dilute, interactions play an important role as a consequence of the low temperatures, and they give rise to collective phenomena related to those observed in solids, quantum liquids, and nuclei. Experimentally the systems are attractive ones to work with, since they may be manipulated by the use of lasers and magnetic fields. In addition, interactions between atoms may be varied either by using different atomic species or, for species that have a Feshbach resonance, by changing the strength of an applied magnetic or electric field. A further advantage is that, because of the low density, 'microscopic' length scales are so large that the structure of the condensate wave function may be investigated directly by optical means. Finally, these systems are ideal for studies of interference phenomena and atom optics.

The theoretical prediction of Bose–Einstein condensation dates back more than 80 years. Following the work of Bose on the statistics of photons [4], Einstein considered a gas of non-interacting, massive bosons, and concluded that, below a certain temperature, a non-zero fraction of the total number of particles would occupy the lowest-energy single-particle state [5]. In 1938 Fritz London suggested the connection between the superfluidity of liquid ^{4}He and Bose–Einstein condensation [6]. Superfluid liquid ^{4}He is the prototype Bose–Einstein condensate, and it has played a unique role in the development of physical concepts. However, the interaction between helium atoms is strong, and this reduces the number of atoms in the zero-momentum state even at absolute zero. Consequently it is difficult to measure directly the occupancy of the zero-momentum state. It has been investigated experimentally by neutron scattering measurements of the structure factor at large momentum transfers [7], and the results are consistent with a relative occupation of the zero-momentum state of about 0.1 at saturated vapour pressure and about 0.05 near the melting pressure [8].

The fact that interactions in liquid helium reduce dramatically the occupancy of the lowest single-particle state led to the search for weakly interacting Bose gases with a higher condensate fraction. The difficulty with most substances is that at low temperatures they do not remain gaseous, but form solids or, in the case of the helium isotopes, liquids, and the effects of interaction thus become large. In other examples atoms first combine to form molecules, which subsequently solidify. As long ago as in 1959 Hecht [9] argued that spin-polarized hydrogen would be a good candidate for a weakly interacting Bose gas. The attractive interaction between two hydrogen atoms with their electronic spins aligned was then estimated to be so weak that there would be no bound state. Thus a gas of hydrogen atoms in a magnetic field would be stable against formation of molecules and, moreover, would not form a liquid, but remain a gas to arbitrarily low temperatures.

Hecht's paper was before its time and received little attention, but his conclusions were confirmed by Stwalley and Nosanow [10] in 1976, when improved information about interactions between spin-aligned hydrogen atoms was available. These authors also argued that because of interatomic interactions the system would be a superfluid as well as being Bose–Einstein condensed. This latter paper stimulated the quest to realize Bose–Einstein condensation in atomic hydrogen. Initial experimental attempts used a high magnetic field gradient to force hydrogen atoms against a cryogenically cooled surface. In the lowest-energy spin state of the hydrogen atom, the electron spin is aligned opposite the direction of the magnetic field (H$\downarrow$), since then the magnetic moment is in the same direction as the field. Spin-polarized hydrogen was first stabilized by Silvera and Walraven [11]. Interactions of hydrogen with the surface limited the densities achieved in the early experiments, and this prompted the Massachusetts Institute of Technology (MIT) group led by Greytak and Kleppner to develop methods for trapping atoms purely magnetically. In a current-free region, it is impossible to create a local maximum in the magnitude of the magnetic field. To trap atoms by the Zeeman effect it is therefore necessary to work with a state of hydrogen in which the electronic spin is polarized parallel to the magnetic field (H↑). Among the techniques developed by this group is that of evaporative cooling of trapped gases, which has been used as the final stage in all experiments to date to produce a gaseous Bose–Einstein condensate. Since laser cooling is not feasible for hydrogen, the gas was precooled cryogenically. After more than two decades of heroic experimental work, Bose–Einstein condensation of atomic hydrogen was achieved in 1998 [12].

As a consequence of the dramatic advances made in laser cooling of alkali atoms, such atoms became attractive candidates for Bose–Einstein condensation, and they were used in the first successful experiments to produce a gaseous Bose–Einstein condensate. In later developments other atoms have been shown to undergo Bose–Einstein condensation: metastable ^{4}He atoms in the lowest-energy electronic spin-triplet state [13, 14], and ytterbium [15, 16] and chromium atoms [17] in their electronic ground states.

The properties of interacting Bose fluids are treated in many texts. The reader will find an illuminating discussion in the volume by Nozières and Pines [18]. A collection of articles on Bose–Einstein condensation in various systems, prior to its discovery in atomic vapours, is given in [19], while more recent theoretical developments have been reviewed in [20]. The 1998 Varenna lectures are a useful general reference for both experiment and theory on Bose–Einstein condensation in atomic gases, and contain in addition historical accounts of the development of the field [21]. For a tutorial review of some concepts basic to an understanding of Bose–Einstein condensation in dilute gases see Ref. [22]. The monograph [23] gives a comprehensive account of Bose–Einstein condensation in liquid helium and dilute atomic gases.

1.1 Bose–Einstein condensation in atomic clouds

Bosons are particles with integer spin. The wave function for a system of identical bosons is symmetric under interchange of the coordinates of any two particles. Unlike fermions, which have half-odd-integer spin and antisymmetric wave functions, bosons may occupy the same single-particle state. An estimate of the transition temperature to the Bose–Einstein condensed state may be made from dimensional arguments. For a uniform gas of free particles, the relevant quantities are the particle mass m, the number of particles per unit volume n, and the Planck constant $h = 2\pi\hbar$. The only quantity having dimensions of energy that can be formed from $\hbar$, n, and m is $\hbar^2 n^{2/3}/m$. By dividing this energy by the Boltzmann constant

k we obtain an estimate of the condensation temperature T_c,

$$T_c = C\frac{\hbar^2 n^{2/3}}{mk}. \tag{1.1}$$

Here C is a numerical factor which we shall show in the next chapter to be equal to approximately 3.3. When (1.1) is evaluated for the mass and density appropriate to liquid ^{4}He at saturated vapour pressure one obtains a transition temperature of approximately 3.13 K, which is close to the temperature below which superfluid phenomena are observed, the so-called lambda point[2] (T_λ= 2.17 K at saturated vapour pressure).

An equivalent way of relating the transition temperature to the particle density is to compare the thermal de Broglie wavelength λ_T with the mean interparticle spacing, which is of order $n^{-1/3}$. The thermal de Broglie wavelength is conventionally defined by

$$\lambda_T = \left(\frac{2\pi\hbar^2}{mkT}\right)^{1/2}. \tag{1.2}$$

At high temperatures, it is small and the gas behaves classically. Bose–Einstein condensation in an ideal gas sets in when the temperature is so low that λ_T is comparable to $n^{-1/3}$. For alkali atoms, the densities achieved range from 10^{13} cm^{-3} in early experiments to 10^{14}–10^{15} cm^{-3} in more recent ones, with transition temperatures in the range from 100 nK to a few μK. For hydrogen, the mass is lower and the transition temperatures are correspondingly higher.

In experiments, gases are non-uniform, since they are contained in a trap, which typically provides a harmonic-oscillator potential. If the number of particles is N, the density of gas in the cloud is of order N/R^3, where the size R of a thermal gas cloud is of order $(kT/m\omega_0^2)^{1/2}$, ω_0 being the angular frequency of single-particle motion in the harmonic-oscillator potential. Substituting the value of the density $n \sim N/R^3$ at $T = T_c$ into Eq. (1.1), one sees that the transition temperature is given by

$$kT_c = C_1\hbar\omega_0 N^{1/3}, \tag{1.3}$$

where C_1 is a numerical constant which we shall later show to be approximately 0.94. The frequencies for traps used in experiments are typically of order 10^2 Hz, corresponding to $\omega_0 \sim 10^3$ s^{-1}, and therefore, for particle numbers in the range from 10^4 to 10^8, the transition temperatures lie in the range quoted above. Estimates of the transition temperature based

[2] The name *lambda point* derives from the shape of the experimentally measured specific heat as a function of temperature, which near the transition resembles the Greek letter λ.