McGraw Hill Education

国外优秀数学教材系列

微积分与解析几何

（影印版·原书第2版）

Calculus With Analytic Geometry

[美] 乔治 F. 西蒙斯（George F. Simmons） 著

机械工业出版社
CHINA MACHINE PRESS

国外优秀数学教材系列

微积分与解析几何

(影印版·原书第2版)

[美] 乔治 F. 西蒙斯 (George F. Simmons)　著

机 械 工 业 出 版 社

图书在版编目（CIP）数据

微积分与解析几何：第2版/（美）西蒙斯（Simmons，G. F.）著．—影印本．—北京：机械工业出版社，2014.9

国外优秀数学教材系列

ISBN 978-7-111-47442-5

Ⅰ．①微… Ⅱ．①西… Ⅲ．①微积分—高等学校—教材②解析几何—高等学校—教材 Ⅳ．①O172②O182

中国版本图书馆 CIP 数据核字（2014）第167000号

机械工业出版社（北京市百万庄大街22号 邮政编码100037）

策划编辑：汤 嘉 责任编辑：汤 嘉

封面设计：张 静 责任印制：乔 宇

北京铭成印刷有限公司印刷

2015年1月第1版第1次印刷

202mm×253mm ·57印张 ·2插页·1300千字

0001—2500册

标准书号：ISBN 978-7-111-47442-5

定价：128.00元

凡购本书，如有缺页、倒页、脱页，由本社发行部调换

电话服务	网络服务
社服务中心：（010）88361066	教材网：http：//www.cmpedu.com
销售一部：（010）68326294	机工官网：http：//www.cmpbook.com
销售二部：（010）88379649	机工官博：http：//weibo.com/cmp1952
读者购书热线：（010）88379203	**封面无防伪标均为盗版**

ABOUT THE AUTHOR

George F. Simmons has the usual academic degrees (CalTech, Chicago, Yale) and taught at several colleges and universities before joining the faculty of Colorado College in 1962. He is also the author of *Introduction to Topology and Modern Analysis* (McGraw-Hill, 1963), *Differential Equations with Applications and Historical Notes* (McGraw-Hill, 1972, 2nd edition 1991), *Precalculus Mathematics in a Nutshell* (Janson Publications, 1981), and *Calculus Gems: Brief Lives and Memorable Mathematics* (McGraw-Hill, 1992).

When not working or talking or eating or drinking or cooking, Professor Simmons is likely to be traveling (Western and Southern Europe, Turkey, Israel, Egypt, Russia, China, Southeast Asia), trout fishing (Rocky Mountain states), playing pocket billiards, or reading (literature, history, biography and autobiography, science, and enough thrillers to achieve enjoyment without guilt). One of his personal heroes is the older friend who once said to him, "I should probably spend about an hour a week revising my opinions."

PREFACE TO THE INSTRUCTOR

It is a curious fact that people who write thousand-page textbooks still seem to find it necessary to write prefaces to explain their purposes. Enough is enough, one would think. However, every textbook—and this one is no exception—is both an expression of dissatisfaction with existing books and a statement by the author of what he thinks such a book ought to contain, and a preface offers one last chance to be heard and understood. Furthermore, anyone who adds to the glut of introductory calculus books should be called upon to justify his action (or perhaps apologize for it) to his colleagues in the mathematics community.

THE CALCULUS TURMOIL

I borrow this phrase from my old friend Paul Halmos as a handy label for the noise and confusion that have agitated the calculus community for the past dozen years or so. Regardless of one's attitude toward these debates and manifestoes, it seems reasonably clear that two opinions lie at the center of it all: first, too many students fail calculus; and second, our calculus textbooks are so bad that it's natural for these students to fail.

About the books, I completely—or almost completely—disagree. By and large, our calculus textbooks are written by excellent teachers who love their subject and write clear expository English. Naturally, each author has a personal agenda, and this is what separates their books from one another and provides diversity and choice for a healthy marketplace. Some writers prefer to emphasize the theoretical parts of calculus. Others are technology buffs. Yet others (like myself) want a modest amount of biography and history, and believe that interesting and substantial applications from other parts of mathematics and other sciences are highly desirable.

But let there be no misunderstanding: textbooks are servants of teachers, and not their masters. Any group of ten calculus teachers gathered together in a room will have ten very different views of what should be in their courses and how it should be taught. They will differ on the proper amount of theory; on how much numerical calculation is desirable; on whether or not to make regular use of graphing calculators or computer software; on whether some of the more elaborate applications to science are too difficult; on whether biography and history are interesting or boring for their students; and so on. But the bottom line is that only the teachers themselves are in a position to decide what goes on in their own classrooms—and certainly not textbook writers who are completely ignorant of local conditions.

Those of us who write these books try to provide everything we can think of that a teacher might want or need, in full awareness that some parts of what we offer have no place in the course plans of many teachers. Every teacher omits some sections (and even some chapters) and amplifies others, in accordance with individual judgment and personal taste. It is my hope that this book will be useful and agrèeable for many diverse tastes and interests. I want it to be a convenient tool for teachers that offers help when help is wanted, and gets out of the way when it is not wanted.

As for the fact that too many of our students fail—if indeed it is a fact—what are the reasons for this? To understand these reasons, let us consider for a moment what is needed for success in calculus. There are clearly three main requirements: a decent background in high school algebra and geometry, some of which is remembered and understood; the ability to read closely and carefully; and tenacity of purpose.

In the matter of preparation in algebra and geometry, our students are in deep trouble. This is suggested by the fact that a few years ago the United States ranked last among the thirteen industrialized nations for the mathematics achievement of its high school graduates. As for reading skills and tenacity of purpose, some of our young people have these qualities, but the great majority do not. Unfortunately, tenacity of purpose is especially important for genuine success in calculus, because this is a subject in which almost every stage depends on having a reasonable command of all that went before, and which therefore requires steady application day after day, week after week, for many months.

We know from our own experience as teachers that calculus is very difficult for most students, and we fully understand the reasons why this is so. But improving our high school mathematics education, and arresting the decline of serious reading and instilling tenacity of purpose among the majority of our young people, are only remote possibilities. Obviously help from outside is not coming, so we must look within ourselves for better ways of doing our jobs.

Most of these ways are familiar to us. Regular class meetings over periods of many months, with frequent quizzes, are intended to encourage steady application to the task of learning. We praise (whenever possible), plead, cajole, and warn. We constantly review the elementary mathematics our students either never learned or have forgotten. We do today's homework problems for them in class, continually thinking out loud and welcoming questions, in the hope that some of the useful ways of thought will rub off to smooth the path for their efforts on tomorrow's homework. However, there is one big thing we can do but rarely do.

Most calculus courses concentrate on the technical details, on developing in students the ability to differentiate and integrate lots of functions. We turn out many students who can perform these somewhat routine tasks. However, if we regularly pause to ask these successful differentiators and integrators just what derivatives and integrals actually are, and what they are for, we rarely get a satisfactory answer—by which I mean an answer that reveals genuine understanding on the part of the student. Many can give the standard limit definitions, but we should expect more than parroted formal definitions. I believe we ought to do a better job of conveying a solid sense of what calculus is really about, what its purpose is, why we need the elaborate machinery of methods for computing derivatives and integrals, and why the Fundamental Theorem of Calculus is truly "fundamental." In a word, we need to communicate what calculus is *for*. More generally, we ought to do more toward encouraging students to learn *why* things are true, rather than merely memorizing ways of solving a few problems to pass examinations. It is clear to us, but not to them, that the only way to learn calculus is to understand it—it is much too massive and complex for mere memorizing to be more than a temporary stopgap—and we have an obligation to help students get this message.

If we can give more attention to these matters, we have a good chance of making calculus less frightening and more relevant for many more students than we have in the past. One of the main purposes of this book is to help us move our teaching in this direction, to convey more light to our students—and less mystery.

CHANGES FROM THE FIRST EDITION

1. **Early Trig.** In the First Edition, I thought it preferable to place trigonometry just before methods of integration. I still agree with myself, but most users think otherwise. I have therefore inserted an account of sines and cosines in Chapter 1, with the calculus of these functions at appropriate places in the following chapters. Since a solid command of trigonometry is so essential for methods of integration, a full review is still given just before the chapter on these methods (Chapter 10).

2. **Homework Problems.** I have added many new problems, mostly of the routine drill type, raising the total to well over 7,000. This is an increase of more than 15 percent and provides about four times as many as most instructors will want to use for their class assignments.

3. **Chapter Summaries.** It seems to help students in their efforts to review and pull things together if they have the ideas and methods of each chapter boiled down to a few pregnant phrases. I have tried to provide this assistance in the summaries at the ends of the chapters.

4. **Appendices.** The first edition had several massive appendices totaling hundreds of pages and containing enrichment material that I thought was so interesting that others would be interested, too. Many were, but I failed to realize that students barely keeping their heads above water in the regular work of the course would take a dim view of any unnecessary burdens. The first two of these long appendices were a collection of material that I thought of as "miscellaneous fun stuff," and a biographical history of calculus. These have been removed, augmented, and published separately in a little paperback book called *Calculus Gems: Brief Lives and Memorable Mathematics* (McGraw-Hill, 1992). However, I have retained some of this material in greatly abbreviated form and placed it in unobtrusive locations throughout the present book.

5. **Theory.** The third of the long appendices in the first edition was on the theory of calculus. I have retained this appendix with a few additions because many colleges and universities offer honors sections that use this material to provide greater theoretical depth than is appropriate for regular sections. Most instructors seem to agree with me in my desire to avoid cluttering our regular courses with any more theory than is absolutely necessary. This approach says: Do not try to prove what no one doubts. However, a number of people have asked me to expand my very condensed discussion of limits and continuous functions and also to give an informal descriptive treatment of the Mean Value Theorem, pointing out its practical uses as they arise. This new material can be found at the end of Chapter 2.

6. **Infinite Series.** My idea for handling this subject in the first edition was not a good one. Most students moving from the first chapter of informal overview into the second of detailed systematic treatment were impatient because they thought they were wasting their time by studying the same concepts all over again. I have therefore completely reorganized these two chapters into a traditional treatment, with series of constants developed first, and then power series.

7. **Vector Analysis.** In the first edition I closed my discussion of vector analysis with Green's Theorem. However, there seems to be general agreement these days that multivariable calculus should go a bit further, and include Gauss's Theorem (the divergence theorem) and Stokes' Theorem. I have rewritten Chapter 21 accordingly.

8. **The Workman Logo.** I thought it would be useful for students if there were some way to signal passages in the text that always cause trouble, because most students are not accustomed to the very slow and careful reading these passages require. The logo I chose for this purpose is copied from a European road sign:

It suggests that hard work is necessary to get through the adjoining passage. I have tried to use it sparingly.

9. **Simplify, Simplify!** When writing this book the first time, I thought I was aiming at the middle of my target, but many users thought I aimed too high. During the preparation of this revision, I kept a poster with these words on it directly in my line of sight as I sat at my work, and of course I looked at this message thousands of times. I hope it worked.

GRAPHING CALCULATORS

These marvelous tools are great fun to use and can make many contributions to the teaching and learning of calculus. But like all tools they should be used wisely, and this means very different things to different people. A scythe can harvest grain or cut off a foot, depending on the skill and judgment of the user.

Some of those in the calculus reform movement believe that the role of numbers and numerical computations should be greatly increased to reach a parity with symbolic (algebraic) and geometric ways of thinking. But I believe we should stop far short of this. In my opinion, there are five subject areas of calculus in which calculators are clearly of great value:

graphing;
calculation of limits;
Newton's method;
numerical integration;
computations using Taylor's formula.

In the last four of these areas, our calculators do heavy computational labor for us, and we are all grateful. But there are dangers, and one of these is an increasing tendency to replace mathematical thinking and learning by button-pushing.

The most surprising examples of this that I've seen involve teachers whose students use graphing calculators—*instead of* factoring or the quadratic formula—to solve quadratic equations as simple as $x^2 - 2x - 3 = 0$. The procedure is to "plot" the function $y = x^2 - 2x - 3$ on the calculator by pushing suitable buttons and then look at the graph the calculator produces to see where it crosses the x-axis. These students are enthusiastic about their calculators and enjoy experimenting with them, and I applaud the teachers who take advantage of this natural interest. But unfortunately, in many cases these students *do not know* how to sketch simple graphs, or how to factor or use the quadratic formula, and are not learning these basic methods of elementary algebra. More generally, sketching the graphs of functions by *thinking* is a fundamental part of learning mathematics. Let us use calculators in our classes to supplement this thinking—but not to replace it. Let us remember that the action that matters takes place in the mind of the student.

These wonderful graphing calculators are superb instruments when used in the right way. It is sobering to reflect that Leibniz himself would perhaps have given a year of his life to possess one—Leibniz who not only (along with Newton) created calculus, but also invented the first calculating machine that could multiply and divide as well as add and subtract.

The many problems in this book that require the use of a calculator are signaled by the standard symbol .

THE PURPOSE OF THIS BOOK

This book is intended to be a mainstream calculus text that is suitable for every kind of course at every level. It is designed particularly for the standard course of three semesters for students of science, engineering, or mathematics. Students are expected to have a background of high school algebra and geometry, and hopefully, some trigonometry as well.

The text itself—that is, the 21 chapters without considering Appendix A—is traditional in subject matter and organization. I have placed great emphasis on *motivation* and *intuitive understanding,* and the refinements of theory are downplayed. Most students are impatient with the theory of the subject, and justifiably so, because the essence of calculus does not lie in theorems and how to prove them, but rather in tools and how to use them. My overriding purpose has been to present calculus as a problem-solving art of immense power that is indispensable in all the quantitative sciences. Naturally, I wish to convince students that the standard tools of calculus are reasonable and legitimate, but not at the expense of turning the subject into a stuffy logical discipline dominated by extra-careful definitions, formal statements of theorems, and meticulous proofs. It is my hope that every mathematical explanation in these chapters will seem to the thoughtful student to be as natural and inevitable as the fact that water flows downhill (rather than uphill) along a canyon floor. The main theme of our work is what calculus is good for—what it enables us to do and understand—and not what its logical nature is as seen from the specialized (and limited) point of view of the modern pure mathematician.

There are several additional features of the book that it might be useful for me to comment on.

Precalculus Material Because of the great amount of calculus that must be covered, it is desirable to get off to a fast start and introduce the derivative quickly, and to spend as little time as possible reviewing precalculus material. However, college freshmen constitute a very diverse group, with widely different levels of mathematical preparation. For this reason I have included a first chapter on precalculus material, which I urge teachers to skim over as lightly as they think advisable for their particular students. This chapter is written in enough detail so that individual students who need to spend more time on the preliminaries should be able to absorb most of it on their own with a little extra effort.*

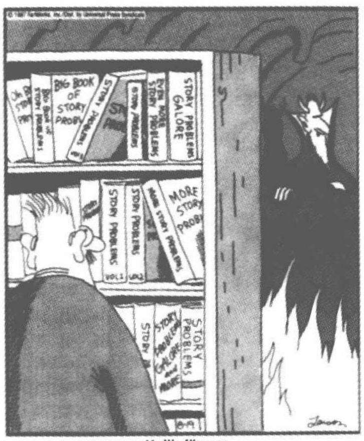
Problems For students, the most important parts of their calculus book may well be the problem sets, because this is where they spend most of their time and energy. There are more than 7,000 problems in this book, including many old standbys familiar to all calculus teachers and dating back to the time of Euler and even earlier. I have tried to repay my debt to the past by inventing new problems whenever possible. The problem sets are carefully constructed, beginning with routine drill exercises and building up to more complex problems requiring higher levels of thought and skill. The most challenging problems are marked with an asterisk (*). In general, each set contains approximately twice as many problems as most teachers will want to assign for homework, so that a large number will be left over for students to use as review material.

Most of the chapters conclude with long lists of additional problems. Many of these are intended only to provide further scope and variety to the problems sets at the ends of the sections. However, teachers and students alike should treat these additional problems with special care, because a few are quite subtle and difficult and should be attacked only by students with ample reserves of drive and tenacity.

I should also mention that there are several sections scattered throughout the book with no corresponding problems at all. Sometimes these sections occur in small groups and are merely convenient subdivisions of what I consider a single topic and intend as a single assignment, as with Sections 6.1, 6.2, 6.3, and 6.4, 6.5. In other cases (e.g., Sections 15.5 and 19.4), the absence of problems is a tacit suggestion that the subject matter of these sections should be touched upon only lightly and briefly.

There are a great many so-called "story problems" spread through the entire book. All teachers know that students shudder at these problems, because they usually require nonroutine thinking. However, the usefulness of mathematics in the various sciences demands that we try to teach our students how to penetrate into the meaning of a story problem, how to judge what is relevant to it, and how to translate it from words into sketches and equations. Without these skills—which are equally valuable for students who will become doctors, lawyers, financial analysts, or thinkers of any kind—there is no mathematics education worthy of the name.†

THE FAR SIDE By GARY LARSON



Hell's library

*A more complete exposition of high school mathematics that is still respectably concise can be found in my little book, *Precalculus Mathematics in a Nutshell* (Janson Publications, Dedham, MA, 1981), 119 pages.

†I cannot let the opportunity pass without quoting a classic story problem that appeared in *The New Yorker* magazine many years ago. "You know those terrible arithmetic problems about how many peaches some people buy, and so forth? Well, here's one we *like,* made up by a third-grader who was asked to think up a problem similar to the ones in his book: "My father is forty-four years old. My dog is eight. If my dog was a human being, he would be fifty-six years old. How old would my father plus my dog be if they were both human beings?"

Differential Equations and Vector Analysis Each of these subjects is an important branch of mathematics in its own right. They should be taught in separate courses, after calculus, with ample time to explore their distinctive methods and applications. One of the main responsibilities of a calculus course is to prepare the way for these more advanced subjects and take a few preliminary steps in their direction, but just how far one should go is a debatable question. Some writers on calculus try to include mini-courses on these subjects in large chapters at the ends of their books. I disagree with this practice and believe that few teachers make much use of these chapters. Instead, in the case of differential equations I prefer to introduce the subject as early as possible (Section 5.4) and return to it in a low-key way whenever the opportunity arises (Sections 5.5, 7.7, 8.5, 9.6, 17.7, 19.9); and in vector analysis I have responded to reviewers by including a discussion of Gauss's Theorem and Stokes' Theorem in Chapter 21.

Appendix A One of the major ways in which this book is unique and different from all its competitors can be understood by examining Appendix A, which I will now comment on very briefly. Before doing so, I emphasize that this material is entirely separate from the main text and can be carefully studied, dipped into occasionally, or completely ignored, as each individual student or instructor desires.

In the main text, the level of mathematical rigor rises and falls in accordance with the nature of the subject under discussion. It is rather low in the geometrical chapters, where for the most part I rely on common sense together with intuition aided by illustrations; and it is rather high in the chapters on infinite series, where the substance of the subject cannot really be understood without careful thought. I have constantly kept in mind the fact that most students have very little interest in purely mathematical reasoning for its own sake, and I have tried to prevent this type of material from intruding any more than is absolutely necessary. Some students, however, have a natural taste for theory, and some instructors feel as a matter of principle that all students should be exposed to a certain amount of theory for the good of their souls. This appendix contains virtually all of the theoretical material that by any stretch of the imagination might be considered appropriate for the study of calculus. From the purely mathematical point of view, it is possible for instructors to teach courses at many different levels of sophistication by using—or not using—material selected from this appendix.

Supplements The following supplements have been developed to accompany this Second Edition of *Calculus with Analytic Geometry.*

A *Student Solutions Manual* is available for students and contains detailed solutions to the odd-numbered problems. An *Instructor's Solutions Manual* is available for instructors and contains detailed solutions to the even-numbered problems. Also available to instructors adopting the text are a Print Test Bank and an algorithmic Computerized Test Bank.

There are a variety of texts available from McGraw-Hill that support the use of specific graphing calculators and mathematical software programs for calculus. Please contact your local McGraw-Hill representative for more information on these titles.

ACKNOWLEDGMENTS

Every project of this magnitude obviously depends on the cooperative efforts of many people.

For this second edition, the editor Jack Shira provided friendly encouragement and smoothed my way throughout. I am profoundly grateful to my friend Maggie Lanzillo, the associate editor, who was a source of skilled support, assistance, and guidance on innumerable occasions—extending even to restaurant suggestions for dining in Italy. Thanks, Maggie. I owe you more than I can express. And as another piece of extraordinary good luck, this second edition was designed by Joan O'Connor, who designed the first edition, and whose inspired artistic taste seems to work miracles on a daily basis.

Also, I offer my sincere thanks to the publisher's reviewers. These astute people shared their knowledge and judgment with me in many important ways.

FOR THE FIRST EDITION:

Joe Browne, *Onondaga Community College*
Carol Crawford, *United States Naval Academy*
Bruce Edwards, *University of Florida*
Susan L. Friedman, *Baruch College*
Melvin Hausner, *New York University*
Louis Hoelzle, *Bucks County Community College*
Stanley M. Lukawecki, *Clemson University*
Peter Maserick, *Pennsylvania State University*
David Zitarelli, *Temple University*

FOR THE SECOND EDITION:

Susan Jane Colley, *Oberlin College*
Preston Dinkins, *Southern University*
Vera Granlund, *University of Virginia*
Elmer Hayashi, *Wake Forest University*
Michael Hilgers, *University of Missouri, Rolla*
Herbert Kamowitz, *University of Massachusetts, Boston*
Richard Katz, *California State University, Los Angeles*
Donald LaTorre, *Clemson University*
Robert Moreland, *Texas Tech University*
Maurice Oleson, *Memorial University*
David Pengelley, *New Mexico State University*
Mark Schwartz, *Ohio Wesleyan University*
Robert Smith, *Millersville University*

As to the flaws and errors that undoubtedly remain—for there are always a pesky few that manage to hide no matter how fervently we try to find them—there is no one to blame but myself. I will consider it a great kindness if colleagues and student users will take the trouble to inform me of any blemishes they detect, for correction in future printings and editions. As Confucius said, "A man who makes a mistake and doesn't correct it is making two mistakes."

George F. Simmons

TO THE STUDENT

Appearances to the contrary, no writer deliberately sets out to produce an unreadable book; we all do what we can and hope for the best. Naturally, I hope that my language will be clear and helpful to students, and in the end only they are qualified to judge. However, it would be a great advantage to all of us—teachers and students alike—if student users of mathematics textbooks could somehow be given a few hints on the art of reading mathematics, which is a very different thing from reading novels or magazines or newspapers.

In high school mathematics courses, most students are accustomed to tackling their homework problems first, out of impatience to have the whole burdensome task over and done with as soon as possible. These students read the explanations in the text only as a last resort, if at all. This is a grotesque reversal of reasonable procedure, and makes about as much sense as trying to put on one's shoes before one's socks. I suggest that students should read the text first, and when this has been thoroughly assimilated, *then and only then* turn to the homework problems. After all, the purpose of these problems is to nail down the ideas and methods described and illustrated in the text.

How should a student read the text in a book like this? Slowly and carefully, and in full awareness that a great many details have been deliberately omitted. If this book contained every detail of every discussion, it would be five times as long, which God forbid! There is a saying of Voltaire: "The secret of being a bore is to tell everything." Every writer of a book of this kind tries to walk a narrow path between saying too much and saying too little.

The words "clearly," "it is easy to see," and similar expressions are not intended to be taken literally, and should never be interpreted by any student as a putdown on his or her abilities. These are code-phrases that have been used in mathematical writing for hundreds of years. Their purpose is to give a signal to the careful reader that in this particular place, the exposition is somewhat condensed, and perhaps a few details of calculations have been omitted. Any phrase like this amounts to a friendly hint to the student that it might be a good idea to read even more carefully and thoughtfully in order to fill in omissions in the exposition, or perhaps get out a piece of scratch paper to verify omitted details of calculations. Or better yet, make full use of the margins of this book to emphasize points, raise questions, perform little computations, and correct misprints.

George F. Simmons

本书除具有标准微积分教材的内容外，书中例子偏重实际，侧重于微积分的应用，同时补充了三角函数、极坐标等理论知识，使学生从高中到大学平稳过渡。文中穿插数学史与数学文化的相关内容，同时附录中提供了大量的补充内容以及严格的理论证明，适合不同层次的学生按需要学习。附加问题生动有趣，多是相关内容的经典结论。

本书可作为高等院校理工科专业教材，也可作为相关科研、技术人员的参考书。

George F. Simmons

Calculus With Analytic Geometry

[ISBN 978-0070576421]

北京市版权局著作权合同登记图字：01-2013-2949 号

目　　录

CONTENTS