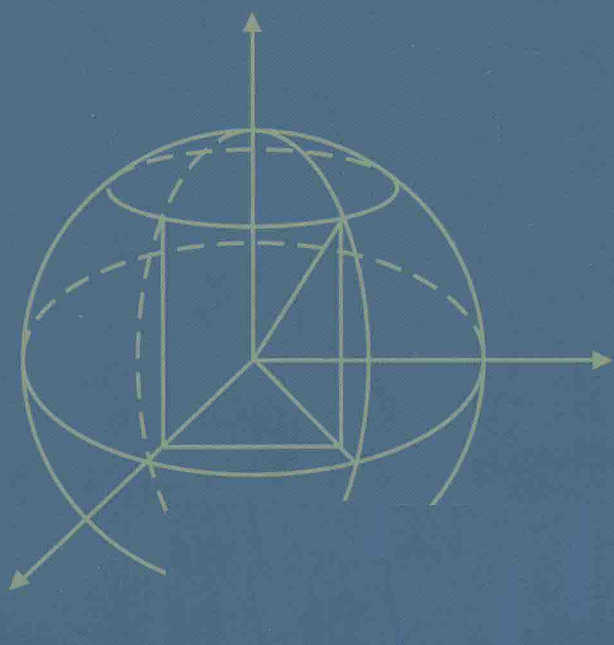
Anisotropic Mathematical Physics and Complex Special Functions

Zhang Chengzong



中国宇航出版社
China Astronautic Publishing House

Anisotropic Mathematical Physics and Complex Special Functions

Zhang Chengzong

中国宇航出版社
·北京·

图书在版编目（CIP）数据

数学物理方法与复数特殊函数 = Anisotropic mathematical physics and complex special functions：英文 / 张承宗著. -- 北京:中国宇航出版社，2014. 12

978 - 7 - 5159 - 0849 - 6

ISBN 978 - 7 - 5159 - 0850 - 2

Ⅰ. ①数… Ⅱ. ①张… Ⅲ. ①数学物理方法-英文②复数-特殊函数-英文 Ⅳ. ①O411. 1 ②O174. 6

中国版本图书馆 CIP 数据核字（2014）第 292848 号

责任编辑 曹晓勇 **装帧设计** 文道思

出版发行	中国宇航出版社		
社址	北京市阜成路 8 号 **邮编** 100830 (010)68768548	版次	2014 年 12 月第 1 版 2014 年 12 月第 1 次印刷
网址	www.caphbook.com	规格	787 × 1092
发行部	(010)68371900 (010)88530478(传真) (010)68768541 (010)68767294(传真)	开本	1/16
		印张	29. 5
零售店	读者服务部 北京宇航文苑 (010)68371105 (010)62529336	字数	710 千字
		书号	ISBN 978 - 7 - 5159 - 0850 - 2
承印	北京画中画印刷有限公司	定价	158. 00 元

本书如有印装质量问题，可与发行部联系调换

ABSTRACT

The classical analytical method of mathematical physics is described briefly in the book. The method of separation of complex variable and the real principle in mathematical physics are suggested which can be used to solve the problem of partial deferential equations, which have odd order cross derivatives and the background of anisotropic physics, in Cartesian, skew, cylindrical and spherical coordinates. The series of complex special functions which can be used to solve anisotropic mathematical physics equation are developed. These complex special functions in cylindrical coordinates include that complex cylindrical polynomial, complex cylindrical-annular function, modified complex cylindrical polynomial, complex spherical cylindrical polynomial, modified complex spherical cylindrical polynomial, complex cylinder function, complex cylindrical surface function and so on. It is noted that many complex special functions are orthogonal with particular weight over particular domain and these corresponding complex cylindrical functions expansion theorems are also suggested. In solving the problem of isotropic physics equation these complex special functions in cylindrical coordinates can be reduced to the corresponding Bessel functions respectively. The complex special functions in spherical coordinates include that complex spherical polynomial and complex spherical function, associated complex spherical function, complex spherical zonal function, associated complex spherical zonal function, associated complex sphere function and so on. Some corresponding theorems of complex spherical functions expansions are also suggested. The author conducted a systematic study of some complex special functions. By these complex special functions, many analytical solutions for the problems of anisotropic physics are presented in the book and some numerical results are presented.

The series of complex cylindrical function transforms are also developed and several complex function transforms are presented. The basic properties and tables of complex cylindrical function transform are also presented. For the problem of isotropic physics, the complex cylindrical function transform is reduce to Hankel Transform. The anisotropic wave equations are solved by the method of complex variables and complex special functions.

This book can be used as textbook or reference for senior or graduate students, for faculty members in engineering mechanics, applied physics and mathematics.

Preface

Many of the problems facing physicists, engineers, and applied mathematicians involve difficulties as governing partial differential equations which have odd order cross derivatives with respective to multiply spatial variables. The classical method of mathematical physics which is based on the method of separation of variable, failed in analyzing these partial differential equations. In solving partial differential equations for the mechanical response of the laminated plate, the author and Professor Yang guangsong have developed a new complex series method (NCST) . NCST succeeds in solving the boundary value problem of various partial deferential equations with constant coefficients. Many results can be found in the book of "a new type complex series method for composite structure mechanics and mathematical physics " by the author. In further investigation, the new complex series method is evolved into the method of separation of complex variable in this book and the real principle in mathematical physics is suggested.

The partial differential equations in cylindrical coordinates and spherical coordinates have always variable coefficients . Thus, another difficulty arose in solving the boundary value problem of partial deferential equations, which have odd order cross derivatives, in cylindrical coordinates and spherical coordinates. By the method of separation of complex variable, the author developed the series of complex special functions to solve these mathematical physics equations, which have always the background of anisotropic physics. These complex special functions include two series, one kind is associated with cylindrical coordinates and another kind is associated with spherical coordinates . Several ordinary differential equations are suggested. These complex special functions suggested newly in cylindrical coordinates include that the complex cylindrical polynomial, complex cylindrical-annular function, the modified complex cylindrical polynomial, complex spherical cylindrical polynomial, modified complex spherical cylindrical polynomial, complex cylinder function, complex cylindrical surface function and so on. It is noted that many special functions are orthogonal with particular weight over particular domain and these corresponding theorems of complex cylindrical functions expansions are suggested in this book. In solving the problem of isotropic physics equation these complex special functions in cylindrical coordinates can be reduced to the corresponding Bessel functions respectively.

Those complex special functions suggested newly in spherical coordinates include that complex spherical polynomials and complex spherical function, associated complex spherical functions, complex spherical zonal function, associated complex spherical zonal function, associated complex sphere function and so on. Some corresponding theorems of complex spherical functions expansions are also suggested in the book. Influenced by the monumental work of Bessel and Legendre, the author conducted a systematic study of some complex special functions. By these complex special functions, many analytical solutions for the problems of anisotropic physics are presented in this book and some numerical results are presented.

Furthmore, the author developed the series of complex cylindrical function transforms and several complex function transforms are presented in this book. The basic properties and tables of complex cylindrical function transforms are also presented in this book. For the problem of isotropic physics, the complex cylindrical function transform is reduce to Hankel Transform. The anthor also used the method of separation of complex varlables and complex speaical functions to solve some anisotropic wave equations in the book and some analytical solutions are presented.

The book involves the problem of anisotropic physics . The partialdifferential equations of anisotropic physics are different from those of isotropic physics and the corresponding theories are different from those of isotropic physics. Therefore, the name of book presents the notation of anisotropic mathematical physics. These special functions which are suggested in the book are in complex form and may be called as complex special functions. In some sense, the methods of anisotropic mathematical physics may be associated with the complex special functions and the methods of isotropic mathematical physics associated with the classical special functions.

In hundreds ago the corresponding studies on the classical mathematical physics and special functions were carried out by mathematicians by hand computation! Those were really the ground-breaking work. The study on the anisotropic mathematical physics and complex special functions lasted several years since 1991. The author finished these works with the help of computer. Since the research results are so many that the author decided to present the results in this book. Due to short of time, the author is afraid of some possible errors in the book and welcomes the readers to correct.

Ph. D Zhang Chengzong

2014. 10. 20

List of Symbols

i	$\mathrm{i}^2=-1$
i,j,k,l,m,n	integer
x,y,z	Cartesian coordinates
r,θ,z	Cylindrical coordinates
r,θ,φ	Spherical coordinates
t	Time
α	$=\dfrac{a}{b}$ or the thermal diffusivity
h ,δ	Thickness of the shell or the plate
β	Skew angle between an oblique of the skew plate and the x-axis or the angle between normal axis with maximum conductivity and the direction in particular coordinate in heat conduction problem
q	Distributed transverse load or heat source per unit volume
ρ	Mass density
C_p	Special heat
ρ	Density
W	the transverse displacement
T	Temperature
$\mathrm{J}_p(x)$	Bessel function of the first kind of order p
$\mathrm{Y}_p(x)$	Bessel function of the second kind of order p
$Z_{ip}(x)$	Complex cylindrical polynomial of the first kind of order ip
$Z_{ip}(x,\theta)$	Complex cylindrical function of the first kind
$Y_{ip}(x)$	Complex cylindrical polynomial of the second kind of order ip
$H_{ip}^{(1)}(x)$	The first complex cylindrical polynomial of the third kind of order ip
$H_{ip}^{(2)}(x)$	The second complex cylindrical polynomial of the third kind of order ip
$S_{ip_n}(x)$	Complex cylindrical-annular polynomial

$S_{ip_n}(x)e^{in\theta}$	Complex cylindrical-annular function
$I_p(x)$	Modified Bessel function of the first kind of order p
$K_p(x)$	Modified Bessel function of the second kind of order p
$\hat{Z}_{ip}(x)$	Modified complex cylindrical polynomial of the first kind of order ip
$K_{ip}(x)$	Modified complex cylindrical polynomial of the second kind of order ip
$\hat{H}_{ip}^{(1)}(x)$	The first modified complex cylindrical polynomial of the third kind of order ip
$\hat{H}_{ip}^{(2)}(x)$	The second modified complex cylindrical polynomial of the third kind
$C_{l,n,k,j}(r,\theta,z)$	Complex cylinder function
$i_p(x)$	Modified spherical Bessel function of the first kind of order p
$k_p(x)$	Modified spherical Bessel function of the first kind of order p
$\hat{C}_{ip}(x)$	Modified complex spherical cylindrical polynomial of order ip
$k_{ip}(x)$	Modified complex spherical cylindrical polynomial of the second kind of order ip
$j_p(x)$	Spherical Bessel function of the first kind of order p
$y_p(x)$	Spherical Bessel function of the second kind of order p
$C_{ip}(x)$	Complex spherical cylindrical polynomial of the first kind of order ip
$C_{ip}(x,\theta)$	Complex spherical cylindrical function
$y_{ip}(x)$	Complex spherical cylindrical polynomial of the second kind of order ip
$h_{ip}^{(1)}(x)$	The first complex spherical cylindrical polynomial of the third kind of order ip
$h_{ip}^{(2)}(x)$	The second complex spherical cylindrical polynomial of the third kind of order ip
$h_{ip}^{(1)}(x,\theta)$	The first complex spherical cylindrical function of the third kind

$h_{ip}^{(2)}(x,\theta)$	The second complex spherical cylindrical function of the third kind
$B_{ip_n}(x)$	Spherical cylindrical-annular polynomial
$B_{ip_n}(x)e^{in\theta}$	Complex spherical cylindrical-annular function
$M_{is}(z,\theta)$	The general complex cylindrical surface function
$O_{s_{n,m}}(z)$	Complex cylindrical surface polynomial of the first kind
$O_{s_{n,m}}(z,\theta)$	Complex cylindrical surface function of the first kind
$\hat{O}_{s_{n,m}}(z)$	Parametric complex cylindrical surface polynomial of the first kind
$\hat{O}_{s_{n,m}}(z,\theta)$	Parametric complex cylindrical surface function of the first kind
$P_n(x)$	Legendre function of the first kind
$Q_n(x)$	Legendre function of the second kind
$Z_{is}(x,\varphi)$	Complex spherical function
$\Omega_{m,n}^{(1)}(x)$	Complex spherical function of the first kind
$\Omega_{m,n}^{(2)}(x)$	Complex spherical function of the second kind
$Z_{is}(x,\varphi,\lambda)$	Parametric complex spherical function
$\Omega_{m,n}^{(1)}(x,\lambda)$	Parametric complex spherical function of the first kind
$\Omega_{m,n}^{(2)}(x,\lambda)$	Parametric complex spherical function of the second kind
$\hat{Z}_{is}(\theta,\varphi)$	Associated complex spherical function
$\hat{\Omega}_{m,n}^{(1)}(x)$	Associated complex spherical function of the first kind
$\hat{\Omega}_{m,n}^{(2)}(x)$	Associated complex spherical function of the second kind
$\hat{Z}_{is}(\theta,\varphi,\lambda)$	Parametric associated complex spherical function
$\hat{\Omega}_{m,n}^{(1)}(x,\lambda)$	Parametric associated complex spherical function of the first kind
$\hat{\Omega}_{m,n}^{(2)}(x,\lambda)$	Parametric associated complex spherical function of the second kind
$P_l^{[m]}(x)$	The derivative of order m of Legendre function of the first kind of l degree
$Q_l^{[m]}(x)$	The derivative of order m of Legendre function of the second kind of l degree
$Z_{m,n}^{(h)}(x,\lambda_{m,n}^{0})e^{im\varphi}$	Complex spherical zonal function
$\Omega_{m,n,k}(r,x,\varphi)$	Complex sphere function

$\hat{Z}{}^{(h)}_{m,n}(x,\lambda^0_{m,n})\mathrm{e}^{im\varphi}$	Associated complex spherical zonal function
$\overset{\wedge}{\Omega}_{m,n,k}(r,x,\varphi)$	Associated complex sphere function
$H(\xi)=\int_0^\infty xf(x)\mathrm{J}_\nu(\xi x)\mathrm{d}x$	Hankel transform of order $\nu\geqslant 0$ of $f(x)$
$Z(\alpha)=\int_0^\infty xf(x)\left[Z_{ip}(\alpha x)\right]^*\mathrm{d}x$	Complex cylindrical function integral transform of $f(x)$
$F_z(\alpha_j)=\int_0^R rf(r)\left[Z_{ip}(\alpha_j\frac{r}{R})\right]^*\mathrm{d}r$	Finite complex cylindrical function integral transform of $f(r)$ of order ip
$H(\alpha)=\int_0^\infty x^2f(x)\left[C_{ip-\frac{1}{2}}(\alpha x)\right]^*\mathrm{d}x$	Complex spherical-cylindrical function integral transform of $f(x)$ of order $(ip-\frac{1}{2})$
$H_z(\alpha_j)=\int_0^R r^2f(r)\left[C_{ip-\frac{1}{2}}(\alpha_j\frac{r}{R})\right]^*\mathrm{d}r$	Finite complex cylindrical function integral transform of $f(r)$ of order ip
$G(\beta)=\int_0^\infty xf(x)\left[\hat{Z}_{ip}(\beta x)\right]^*\mathrm{d}x$	Complex cylindrical function integral transform of $f(x)$ of order ip
$G_z(\beta_j)=\int_0^R rf(r)\left[\hat{Z}_{ip}(\beta_j\frac{r}{R})\right]^*\mathrm{d}r$	Finite modified complex cylindrical function integral transform of $f(r)$ of order ip
$Y(\beta)=\int_0^\infty x^2f(x)\left[\hat{C}_{ip-\frac{1}{2}}(\beta x)\right]^*\mathrm{d}x$	Modified complex spherical cylindrical function integral transform of $f(x)$ of order ip
$Y_z(\beta_j)=\int_0^R r^2f(r)\left[\hat{C}_{ip-\frac{1}{2}}(\beta_j\frac{r}{R})\right]^*\mathrm{d}r$	Finite modified complex spherical-cylindrical function integral transform of $f(r)$ of order $(ip-\frac{1}{2})$
$Z(\alpha,n)$	Two-dimensional complex cylindrical function integral transform of order $ip\times n$ of $f(r,\theta)$
$Q(\alpha,n)$	Two-dimensional complex spherical-cylindrical function integral transform of order $(ip-\frac{1}{2})\times n$ of $f(r,\theta)$
$X(\beta,n)$	Two-dimensional modified complex cylindrical function integral transform of order $ip\times n$ of $f(r,\theta)$
$Y(\beta,n)$	Two-dimensional modified complex spherical-cylindrical function integral transform of order $(ip-\frac{1}{2})$

	$\times n$ of $f(r,\theta)$
$C(\alpha,n,\omega)$	Three-dimensional complex cylindrical function integral transform of order $ip\times n\times\omega$ of $f(r,\theta,z)$
$Z_{s_1}(x), Z_{s_2}(x), Z_{s_3}(x), Z_{s_4}(x)$	The first, second, third and fourth spherical polynomials, respectively.
$Z_{s_1}(x,\lambda), Z_{s_2}(x,\lambda), Z_{s_3}(x,\lambda), Z_{s_4}(x,\lambda)$	The first, second, third and fourth parametric spherical polynomials, respectively
$\hat{Z}_{s_1}(x), \hat{Z}_{s_2}(x), \hat{Z}_{s_3}(x), \hat{Z}_{s_4}(x)$	The first, second, third and fourth associated spherical polynomials, respectively
$\hat{Z}_{s_1}(x,\lambda), \hat{Z}_{s_2}(x,\lambda), \hat{Z}_{s_3}(x,\lambda), \hat{Z}_{s_4}(x,\lambda)$	The first, second, third and fourth parametric associated spherical polynomials, respectively

Contents