Geir E. Dullerud
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TEXTS IN APPLIED MATHEMATICS

A Course in Robust Control Theory

A Convex Approach

鲁棒控制理论教程

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A Course in Robust Control Theory

A Convex Approach

With 36 Illustrations

Springer

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To our Families

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)*.

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

Robust control theory has been one of the most active areas of mainstream systems theory since the late 1970s. This research activity has been at the confluence of dynamical systems theory, functional analysis, matrix analysis, numerical methods, complexity theory, and control engineering applications. Remarkably, the discipline has exhibited deep theoretical aspects of interest to pure and applied mathematicians, and at the same time has remained in close connection with tangible engineering applications. By now this research effort has produced a rather extensive set of approaches using a wide variety of mathematical techniques, and applications of robust control theory are spreading to diverse areas such as aerospace systems, chemical processes, power networks, and control of fluids. Some major milestones of the theory were developed in the 1980s, and by now have been reported in a number of textbooks. However, during the 1990s the theory has seen major advances and achieved a new maturity, hinging around the notion of convexity. This emphasis is two-fold. On one hand, a close, and mutually beneficial interaction has developed between control theory and the active area of convex optimization; in particular the methods of convex programming have proven relevant to a wide class of control problems. Simultaneously a new understanding has developed on the computational complexity implications of uncertainty modeling; in particular it has become clear that one must go beyond a time invariant structure to describe uncertainty in terms amenable to convex robustness analysis.

Our broad goal in this book is to give a graduate-level course on robust control theory that emphasizes these new developments, but at the same time conveys the main principles and ubiquitous tools at the heart of the subject. We thus aim at a coherent and unified introduction to robust control which starts at the "beginning" of basic systems theory, covers some of the more established landscape of the field, and leads up to the more recent themes and to topics of current research. The effect of this intention on the book is that as it progresses the new topics are increasingly emphasized, whereas more conventional techniques appear less frequently. While many aspects of the vast literature have been left out, we have selected those that we believe are central and most effectively form a launching point for further study of the field. We therefore hope the book will be of value to graduate students planning to do research in the area, to mathematical researchers and computer scientists wishing to learn about robust control theory, and engineers interested in the basis of advanced control techniques.

The text is written to comprise a two-quarter or two-semester graduate course in applied mathematics or engineering. Alternatively, for students with background in state space methods, a serious approach at a significant portion of the material can be achieved in one semester. The material has been successfully taught in this capacity during the past few years by the authors at Caltech, University of Waterloo, University of Illinois, and UCLA. Students are assumed to have some familiarity with linear algebra, and otherwise only advanced calculus and basic complex analysis are strictly required. The presentation style assumes, however, a mathematically inclined reader, since we focus on a complete theoretical foundation rather than on application examples.

After a conceptual introduction in Chapter 0, the next three chapters cover basic tools to be used throughout the course. Chapter 1 reviews some basic elements of linear algebra and matrix theory, including the Jordan form and the singular value decomposition. It then presents some elementary concepts of convex analysis in finite dimensional space, and ends by introducing the important concept of a linear matrix inequality (LMI). Chapter 2 provides a compact, yet thorough treatment of state space systems theory, including controllability and observability, eigenvalue assignment and minimal realizations. In Chapter 3 we introduce the functional analysis and operator theory used in the sequel, beginning with normed and inner product spaces, and following with operators on Hilbert space. The focus of the text is on L_2 signal norms, so a number of associated function spaces are presented, including the H_2 and H_∞ spaces which are related to the concepts of time invariance and causality in systems.

Chapter 4 revisits the topic of open-loop systems and realizations by using the newly introduced concept of a norm. Controllability and observability can now be quantified by means of gramians, and balanced realizations are introduced. Hankel operators and singular values are then discussed, leading up to model reduction using balanced truncation.

In Chapters 5 through 7 three feedback design problems are discussed: stabilization, H_2 optimization, and H_∞ control. We take advantage of this progression to present three main tools for controller synthesis: the parametrization of stabilizing controllers via coprime factorizations, Riccati equation techniques for optimization, and the most recent LMI techniques. The latter play a relatively minor role in Chapters 5 and 6, but are essentially the exclusive tool employed in Chapter 7. In this way we endeavor to exhibit the power of the new methods while simultaneously providing an in depth covering of important, more classical techniques.

In Chapters 8 and 9 we discuss uncertain system models and techniques for analysis and synthesis in the presence of uncertainty. Chapter 8, which focuses exclusively on analysis, adopts an abstract operator viewpoint which leads to the study of structural properties of uncertainty and the corresponding robustness analysis tools, based on generalizations of the Small gain theorem. The more recent convex necessary and sufficient conditions which apply to uncertainty that is not time invariant are discussed first, and are then followed by the presentation of the more classical structured singular value methods. The operator perspective allows one to develop the theory in isolation from some technical details which arise when dealing with potentially unstable systems; these are later tackled in Chapter 9, and subsequently techniques for robust controller synthesis are presented.

The final two chapters are devoted to the presentation of four topics of recent research, in a more descriptive manner. Chapter 10 covers extensions of robustness analysis. First the method of integral quadratic constraints (IQCs) is discussed, which provides generalized uncertainty descriptions and is closely tied to the tools of Chapter 8. Secondly, the analysis of robust H_2 performance is covered, which aims at reconciling the desirable performance characteristics of the H_2 norm with the advantages of L_2-induced norms to measure uncertainty. Chapter 11 provides some generalizations with an emphasis on synthesis. The first is a consideration of multidimensional state space techniques, aimed at uncertain system realization, distributed systems, and parameter-varying systems. Linear parameter-varying control synthesis and multidimensional model reduction are discussed. Finally, time-varying systems are studied using an operator theoretic framework, which shows how these systems can be treated formally as though they were time invariant, leading to analogs of many of the earlier results of the book in a more general setting.

The core chapters of the book are accompanied by exercises, ranging from simple application examples to more significant extensions of the exposed theory.

We are indebted to a number of people as we complete this book. Much of what we report here we have learned from our Ph.D. advisors, Keith Glover and John Doyle, who have also had a considerable influence on our careers and our views of research and control. John also provided significant encouragement for this project, and has tried out our draft in recent courses at Caltech. Raff D'Andrea at Cornell also used early drafts and provided very useful feedback. Bruce Francis provided his notes on introductory state space theory, which were used as a basis for the succinct treatment in Chapter 2. We also thank Carolyn Beck and Sanjay Lall for their in depth comments on the manuscript, and the anonymous reviewers for valuable suggestions. This text was fine-tuned with the help of many students who studied from earlier versions and pointed out numerous mistakes; we particularly thank Carol Pirie and Federico Najson for very thorough revisions. In addition, we are grateful to Carolyn Jackson for her assistance in typing a preliminary version of the manuscript.

Finally, we wish to thank our wives, Carolyn and Malena for their support and patience, and our children, Natalie, Rafael and Fernando for the time taken away from them during this effort.

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November, 1999

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