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辽宁教育出版社

◎顾毓琇夫妇参加麻省理工学院毕业五十周年聚会（一九七五年六月）

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顾毓琇科学论文集（二）

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AIEE
Miscellaneous Paper
52-126

February 1952
Litho in USA

TENSOR ANALYSIS OF UNBALANCED THREE-PHASE CIRCUITS

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A Paper recommended by the AIEE Committee on Basic Sciences and approved by the AIEE Technical Program Committee for presentation at the AIEE North Eastern District Meeting, Binghamton, N. Y., April 30-May 2, 1952. Manuscript submitted June 22, 1951; made available for printing February 28, 1952.

PRICE: 30 cents To Members
60 cents To Nonmembers

TENSOR ANALYSIS OF UNBALANCED THREE-PHASE CIRCUITS

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SYNOPSIS

This paper gives a tensor analysis of unbalanced three - phase circuits in terms of all 3 sets of reference frames representing the phase components, the symmetrical components, and the quadrature components. While Krondid pioneer work in bringing the phase components and the symmetrical components into one tensor "group" and analyzing unbalanced faults by "connection tensors", the present paper attempts to add the Clarke components (α, β, 0 components) to the family and correlate all 3 sets of components in the solution of unbalanced faults. Following the Second General-

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ization Postulate of Kron, the concepts of "transformation", "invariance", and "group" are introduced simultaneously. The relations between the 3 sets of reference frames are established for the complete "invariance" of instantaneous power. The invariant transformations form one "transformation tensor" for the "group". The impedance matrices and admittance matrices are then correlated as impedance and admittance tensors. The different unbalanced faults are solved by "connection tensors". It is shown in this paper that in the study of line-to-line fault and double-line-to-ground fault under load, the use of network admittance matrices and connection tensors between the old and new voltages gives simpler results. It is found that not only the fault currents are readily obtained, but also the division of currents between the parallel networks is given simultaneously.

INTRODUCTION

In a recent paper[①] the author showed how tensor analysis can be helpful in organizing and correlating the solutions of problems involving rotating machines, stationary networks, and transmission lines with lumped or distributed constants. The present paper attempts to show how tensor analysis gives a unified view for the solution of unbalanced three-phase circuits in terms of all 3 sets of reference frames for the phase components, the symmetrical or Fortescue components, and the quadrature

① For numbered references, see Bibliography.

or Clarke components.

The tensor analysis of three-phase circuits① correlating the phase components and the symmetrical components was given by Kron.[2] However, as the Clarke components (α, β, 0 components)[3] are becoming more and more important in recent years[4,5], it is deemed worthwhile to examine these components from the tensor point of view and bring them into one "group" with the phase components and the symmetrical components. An additional impetus to such an effort is offered by the correlation of different components in the rotating reference frames[6], known as direct and quadrature components, and forward and backward components in the study of synchronous machines.

Following the Second Generalization Postulate of Kron, the concepts of "transformation", "invarianec" and "group" must be introduced simultaneously. So the paper will start with the establishment of proper tensor transformations between the 3 sets of components so that the instantaneous power is completely "invariant". When these invariant transformations are found, the different sets of components so defined will form one "group". In this way, a matrix equation valid for one particular reference frame is changed to an invariant equation valid for an infinite variety of other reference frames of the same type. It is further possible to correlate the invariant equation for a stationary type of reference frame to another type of reference frame, say the rotating reference frame. So eventually we can organize the 3 sets of stationary reference

① See Chapter 20, reference 2.

frames and the 2 sets of rotating reference frames into one big family.

As these invariant transformations form one "transformation tensor" for the "group", any solution in terms of one set of components will automatically lead to solutions in terms of other sets of components in the family. The knowledge gained from using one set of components can be readily transferred in terms of other sets of components, and thus we may visualize the whole physical problem not only form a general point of view but also form various advantageous points of view.

Since impedance matrices and admittance matrices are correlated by one "transformation tensor" in different forms, they become impedance or admittance tensors for the whole system. For instance, if we know the sequence impedance matrix or the sequence admittance matrix, it is a routine matter to find the component impedance matrix or the component admittance matrix in the reference frame for α, β, 0 components, and vice versa.

In the solution of unbalanced faults, it is further possible to transform the original or primitive system into the new or resultant system by means of "connection tensors." These connection tensors between old and new currents or voltages will transform the impedance or admittance tensor in the old system to a corresponding tensor in the resultant system. In the following examples, it will be found that if we choose a proper and simple connection tensor, the results will be much simpler. For instance, in the study of line-to-line fault or double-line-to-ground fault under load, we have a choice between using the connection tensor for old and new currents, and new voltages. It is found that the use of network admittance matrices and the connection tensor between old and new voltages gives simpler results, It is further shown that not only the total fault currents are obtained, but also the division of currents between the parallel networks is given simultaneously.

It is the purpose of this paper to bring the Clarke components into the tensor "group" and apply the modified components to the crucial tests of different unbalanced faults. In the different examples, it is shown that a solution in one reference frame will automatically lead to a correct solution in any other reference frame in this "group".

THE SYMMETRICAL COMPONENTS

We shall start with the symmetrical components and examine how these components and the phase components form one "group", as the quadrature components are eventually to be brought in as an additional member of the family.

The symmetrical components as defined by Fortescue[7] are related to the phase components as follows:

$$\left.\begin{aligned} i_0 &= \frac{1}{3}\ (i_A + i_B + i_C) \\ i_1 &= \frac{1}{3}\ (i_A + \underline{a} i_B + \underline{a}^2 i_C) \\ i_2 &= \frac{1}{3}\ (i_A + \underline{a}^2 i_B + \underline{a} i_C) \end{aligned}\right\} \ (1a) \qquad \left.\begin{aligned} i_A &= i_0 + i_1 + i_2 \\ i_B &= i_0 + \underline{a}^2 i_1 + \underline{a} i_2 \\ i_C &= i_0 + \underline{a} i_1 + \underline{a}^2 i_2 \end{aligned}\right\} \ (1b)$$

where i_A, i_B, i_C represent phase quantities (see List of Symbols), i_0, i_1, i_2 represent zero-sequence, positive-sequence, and negative-sequence components, $\underline{a} = e^{j2\pi/3}$ and $\underline{a}^2 = e^{-j2\pi/3}$. The transformations are the same for voltages.

The instantaneous power is given by

$$p = v_A i_A + v_B i_B + v_C i_C \tag{2a}$$

$$P = 3\ (v_0 i_0 + v_1 i_2 + v_2 i_1) \tag{2b}$$

This is invariant except for a constant factor 3, and may be termed "partial invariance" of power.①

From the tensor point of view, "complete invariance" is necessary in order to get perfect transformation between the different sets of components. Since i_0, i_1, i_2 and v_0, v_1, v_2 are too small for "complete invariance" of power, it is logical to make the following modifications:

$$\left.\begin{aligned} i_0 &= \frac{1}{\sqrt{3}}\ (i_A + i_B + i_C) \\ i_1 &= \frac{1}{\sqrt{3}}\ (i_A + \underline{a} i_B + \underline{a}^2 i_C) \\ i_2 &= \frac{1}{\sqrt{3}}\ (i_A + \underline{a}^2 i_B + \underline{a} i_C) \end{aligned}\right\} (3a) \qquad \left.\begin{aligned} i_A &= \frac{1}{\sqrt{3}}\ (i_0 + i_1 + i_2) \\ i_B &= \frac{1}{\sqrt{3}}\ (i_0 + \underline{a}^2 i_1 + \underline{a} i_2) \\ i_C &= \frac{1}{3}\ (i_0 + \underline{a} i_1 + \underline{a}^2 i_2) \end{aligned}\right\} (3b)$$

The same transformations hold for voltages. Then we get

$$P = v_0 i_0 + v_1 i_2 + v_2 i_1 \tag{2c}$$

which shows "complete invariance" of instantaneous power.

The modified set of transformations satisfy the following conditions as required by tensor analysis:

(1) The current and voltage transfomations are the same.

(2) The instantaneous power is completely invariant.

① The same discussion applies to f, b, o components. $P = 3\ (v_o i_o + v_f v_b + v_b i_f)$ corresponds to eq. (2b).

(3) The conjugate transpose of the inverse matrix is equal to the transformation matrix.

These criteria for tensor transformations were first discussed by Kron[8] in 1935 and formed the basis of Concordia components.[9]

THE QUADRATURE COMPONENTS

The Clarke components[8] are related to the phase components as follows:

$$\left.\begin{aligned} i_o &= \frac{1}{3}\ (i_A + i_B + i_C) \\ i_\beta &= \frac{1}{3}\ (2i_A - i_B - i_C) \\ i_\beta &= \frac{1}{3}\ (i_B - i_C) \end{aligned}\right\}\ (4a) \qquad \left.\begin{aligned} i_A &= i_o + i \\ i_B &= i_o - \frac{1}{2} i_\alpha + \frac{\sqrt{3}}{2} i_\beta \\ i_C &= i_o - \frac{1}{2} i_\alpha - \frac{\sqrt{3}}{2} i_\beta \end{aligned}\right\}\ (4b)$$

where i_α and i_β are known as aplha and beta components. The same transformations hold for the voltages. The relation between these quadrature components and the symmetrical components as defined in eq. (1) is hence

$$\left.\begin{aligned} j_\alpha &= i_1 + i_2 \\ j_\beta &= i_1 - i_2 \end{aligned}\right\}\ (5a) \qquad \left.\begin{aligned} i_1 &= \frac{1}{2}\ (i_\alpha + ji_\beta) \\ i_2 &= \frac{1}{2}\ (i_\alpha - ji_\beta) \end{aligned}\right\}\ (5b)$$

The same transformations hold for voltages.

Substituting these quadrature components into the power equation, there is

$$P = v_A i_A + v_B i_B + v_C i_C = 3\ (v_0 i_0) + (3/2)\ (v_\alpha i_\alpha + v_\beta i_\beta) \quad (6a)$$

Obviously, the power expression is not invariant.① This was pointed out by Boyajian in the discussion of Kimback's paper[10] as follows: "The Clarke matrices have the distinction of being identical for currents and voltages. This is secured, however, at the expense of the vector power equation, because the current and voltage transformation matrices are not the tensorial inverse of each other."

This is perhaps the reason why two other sets of matrices were suggested: Kimbark components (x, y, z components) giving "partial invariance" of power and Boyajian components (α, β, o components) giving "complete invariance" of power. However, in the two new sets of transformations, the matrices are not identical for currents and voltages.

In examining the Clarke components, one finds that the sum $(v_\alpha i_\alpha + v_\beta i_\beta)$ is too big to be consistent with the product $(v_o i_o)$. If i_α, i_β, v_α, or v_β is each reduced by a factor $1\sqrt{/2}$, there is

$$P = 3\ (v_o i_o + v_\alpha i_\alpha + v_\beta i_\beta) \tag{6b}$$

This is similar to eq. (2b) with the same constant factor 3. It takes a further step similar to eq. (3a) to reduce eq. (6b) to

$$P = v_o i_o + v_\alpha i_\alpha + v_\beta i_\beta \tag{6c}$$

Such modified components satisfy the criteria of tensor transformations as given by Kron in reference 8 and may be called Concordia components as they were first given by Concordia in reference 9.

① The same discussion applies to d, q, o components. The power equation gives $P = 3\ (vi) + (3/2)\ (v_d + i_d + v_q i_q)$.