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# CALCULUS

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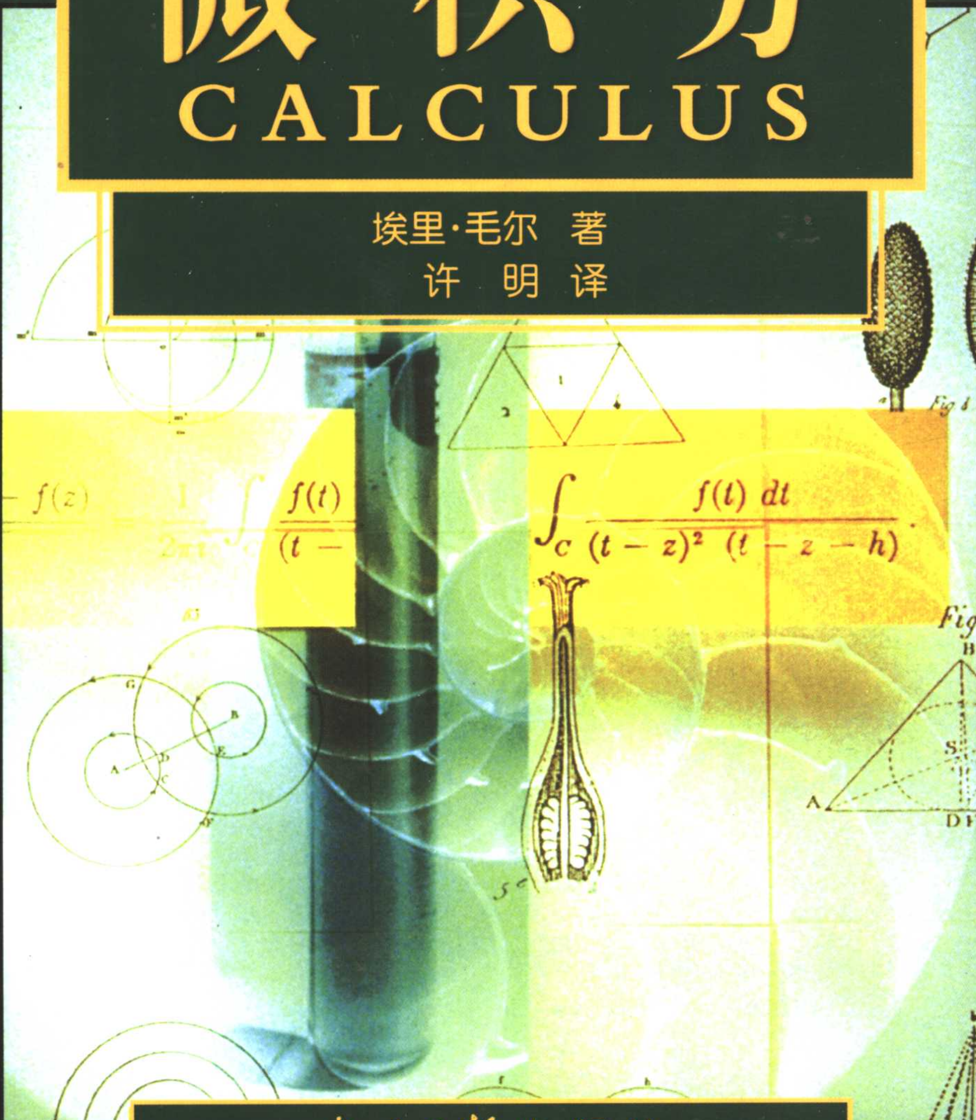

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# THE FACTS ON FILE CALCULUS HANDBOOK

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Facts On File, Inc.

**图书在版编目(CIP)数据**

科学分类手册·微积分 /(美)毛尔著;许明译.
北京:光明日报出版社,2004

ISBN 7-80145-793-5

Ⅰ.科… Ⅱ.①毛…②许… Ⅲ.①自然科学-词典-英、汉
②微积分-词典-英、汉 Ⅳ.N61

中国版本图书馆 CIP 数据核字(2003)第 119526 号



**微积分**

光明日报出版社 (北京永安路 106 号)

全国新华书店经销 合肥锐达印务有限责任公司制版、印刷

开本 880×1230 1/32 印张 147.5 字数 2800 千字

2004 年 第 1 版 2004 年 第 1 次印刷

ISBN 7-80145-793-5/G 定价:234.00 元(全 10 册)

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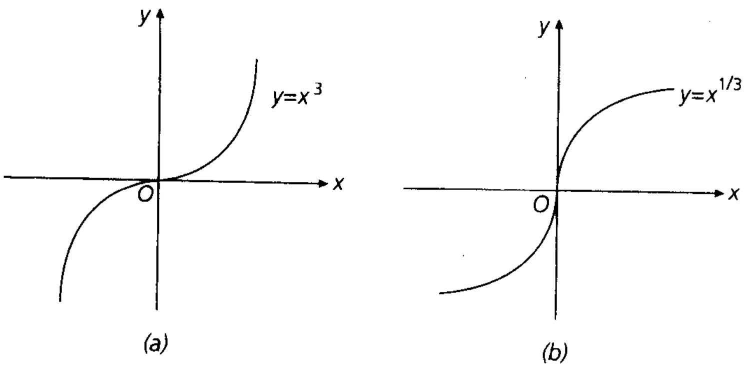
# GLOSSARY

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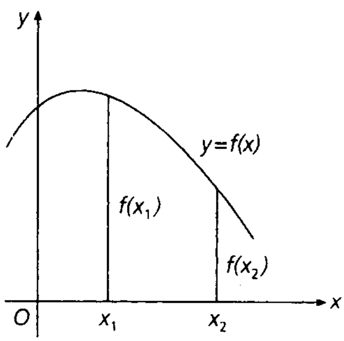


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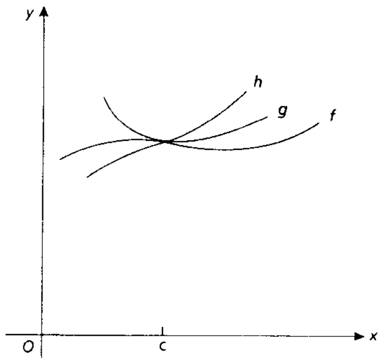


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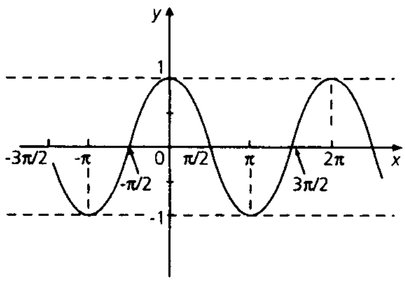
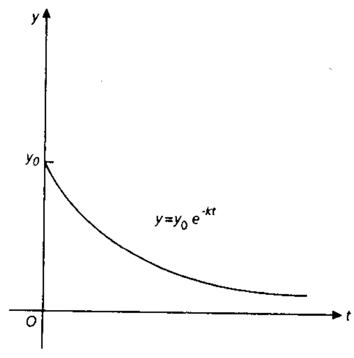



余弦函数 y=cos x




指数式衰减

**abscissa** The first number of an ordered pair (x, y); also called the *x*-coordinate.

**absolute convergence** *See* CONVERGENCE, ABSOLUTE.

**absolute error** *See* ERROR, ABSOLUTE.

**absolute maximum** *See* MAXIMUM, ABSOLUTE.

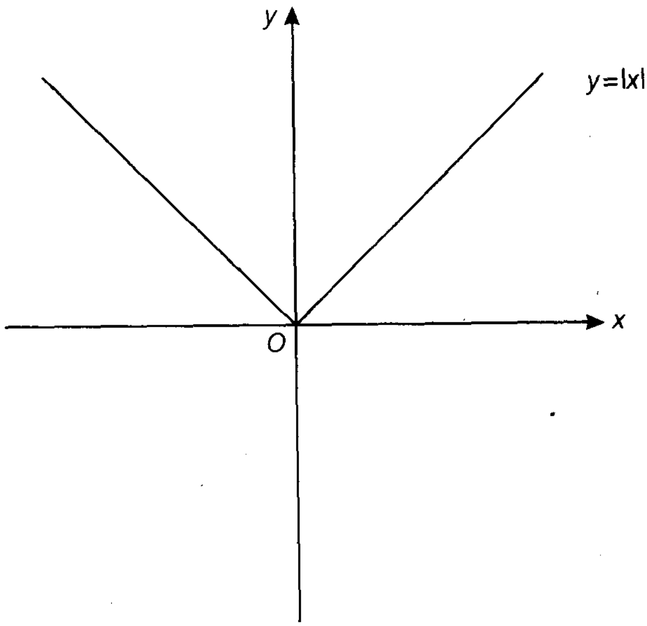
**absolute minimum** *See* MINIMUM, ABSOLUTE.

**absolute value** The absolute value of a real number *x*, denoted $|x|$, is the number "without its sign." More precisely, $|x| = x$ if $x \geq 0$, and $|x| = -x$ if $x < 0$. Thus $|5| = 5$, $|0| = 0$, and $|-5| = -(-5) = 5$. Geometrically, $|x|$ is the distance of the point *x* from the origin *O* on the number line.

*See also* TRIANGLE INEQUALITY.

**absolute-value function** The function $y = f(x) = |x|$. Its domain is all real numbers, and its range all nonnegative numbers.

**acceleration** The rate of change of velocity with respect to time. If an object moves along the *x*-axis, its position is a function of time, $x = x(t)$. Then its velocity is $v = dx/dt$, and its acceleration is $a = dv/dt = d(dx/dt)/dt = d^2x/dt^2$, where d/dt denotes differentiation with respect to time.




**Absolute–value funtion**

**addition of functions** The sum of two functions $f$ and $g$, written $f + g$. That is to say, $(f + g)(x) = f(x) + g(x)$. For example, if $f(x) = 2x + 1$ and $g(x) = 3x - 2$, then $(f + g)(x) = (2x + 1) + (3x - 2) = 5x - 1$. A similar definition holds for the difference of $f$ and $g$, written $f - g$.

**additive properties of integrals**

1. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$. In abbreviated form, $\int_a^c + \int_c^b = \int_a^b$.

Note: Usually $c$ is a point in the interval $[a, b]$, that is, $a \leq c \leq b$. The rule, however, holds for any point $c$ at which the integral exists, regardless of its relation relative to $a$ and $b$.

2. $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$, with a similar rule for the difference $f(x) - g(x)$. The same rule also applies for indefinite integrals (antiderivatives).

**algebraic functions** The class of functions that can be obtained from a finite number of applications of the algebraic operations addition, subtraction, multiplication, division, and root extraction to the variable $x$. This includes all polynomials and rational functions (ratios of polynomials) and any finite number of root extractions of them; for example, $\sqrt{x + \sqrt[3]{x}}$.

**algebraic number** A zero of a polynomial function $f(x)$ with integer coefficients (that is, a solution of the equation $f(x) = 0$). All rational numbers are algebraic, because if $x = a/b$, where $a$ and $b$ are two integers with $b \neq 0$, then $x$ is the solution of the linear equation $bx - a = 0$. Other examples are $\sqrt{2}$ (the positive solution of the quadratic equation $x^2 - 2 = 0$) and $\sqrt[3]{1 + \sqrt{2}}$ (a solution of the sixth-degree polynomial equation $x^6 - 2x^3 - 1 = 0$). The imaginary number $i = \sqrt{-1}$ is also algebraic, because it is the solution of the equation $x^2 + 1 = 0$ (note that in all the examples given, all coefficients are integers).

*See also* TRANSCENDENTAL NUMBER.

**alternating *p*-series** *See* *p*-SERIES, ALTERNATING.

**alternating series** *See* SERIES, ALTERNATING.

**amplitude** One-half the width of a sine or cosine graph. If the graph has the equation $y = a \sin(bx + c)$, then the amplitude is $|a|$, and similarly for $y = a \cos(bx + c)$.

**analysis** The branch of mathematics dealing with continuity and limits. Besides the differential and integral calculus, analysis includes differential equations, functions of a complex variable, operations research, and many more areas of modern mathematics.

*See also* DISCRETE MATHEMATICS.

**analytic geometry** The algebraic study of curves, based on the fact that the position of any point in the plane can be given by an ordered pair of numbers (coordinates), written (x, y). Also known as *coordinate geometry,* it was invented by Pierre de Fermat and René Descartes in the first half of the 17th century. It can be extended to three-dimensional space, where a point $P$ is given by the three coordinates $x$, $y$, and $z$, written (x, y, z).

**angle** A measure of the amount of rotation from one line to another line in the same plane.

Between lines: If the lines are given by the equations $y = m_1x + b_1$ and $y = m_2x + b_2$, the angle between them—provided neither of the lines is vertical—is given by the formula $\phi = \tan^{-1} (m_2 - m_1)/(1 + m_1m_2)$. For example, the angle between the lines $y = 2x + 1$ and $y = 3x + 2$ is $\phi = \tan^{-1} (3 - 2)/(1 + 3 \cdot 2) = \tan^{-1} 1/7 \approx 8.13$ degrees.

Between two curves: The angle between their tangent lines at the point of intersection.

Of inclination of a line to the $x$-axis: The angle $\phi = \tan^{-1} m$, where $m$ is the slope of the line. Because the tangent function is periodic, we limit the range of $\phi$ to $0 \leq \phi \leq \pi$.

*See also* SLOPE.

**angular velocity** Let a line through the origin rotate with respect to the $x$-axis through an angle $\theta$, measured in radians in a counterclockwise sense. The angle of rotation is thought of as continuously varying with time (as the hands of a clock), though not necessarily at a constant rate. Thus $\theta$ is a function of the time, $\theta = f(t)$. The *angular velocity,* denoted by the Greek letter $\omega$ (omega), is the derivative of this function: $\omega = d\theta/dt = f'(t)$. The units of $\omega$ are radians per second (or radians per minute).

**annuity** A series of equal payments at regular time intervals that a person either pays to a bank to repay a loan, or receives from the bank for a previously-deposited investment.

**antiderivative** The antiderivative of a function f(x) is a function F(x) whose derivative is f(x); that is, $F'(x) = f(x)$. For example, an antiderivative of $5x^2$ is $5x^3/3$, because $(5x^3/3)' = 5x^2$. Another antiderivative of $5x^2$ is $5x^3/3 + 7$, and in fact $5x^3/3 + C$, where $C$ is an arbitrary constant.

The antiderivative of f(x) is also called an *indefinite integral* and is denoted by $\int f(x)\,dx$; thus $\int 5x^2\,dx = 5x^3/3 + C$.
*See also* INTEGRAL, INDEFINITE.

**approximation** A number that is close, but not equal, to another number whose value is being sought. For example, the numbers 1.4, 1.41, 1.414, and 1.4142 are all approximations to $\sqrt{2}$, increasing progressively in accuracy. The word also refers to the *procedure* by which we arrive at the approximated number. Usually such a procedure allows one to approximate the number being sought to any desired accuracy. Associated with any approximation is an estimate of the *error* involved in replacing the true number by its approximated value.
*See also* ERROR; LINEAR APPROXIMATION

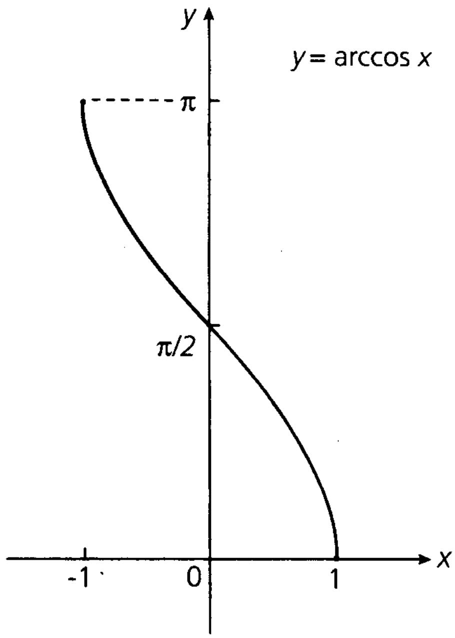
**Archimedes, spiral of (linear spiral)** A curve whose polar equation is $r = a\theta$, where *a* is a constant. The grooves of a vinyl disk have the shape of this spiral.

**arc length** The length of a segment of a curve. For example, the length of an arc of a circle with radius *r* and angular width $\theta$ (measured in radians) is $r\theta$. Except for a few simple curves, finding the arc length involves calculating a definite integral.

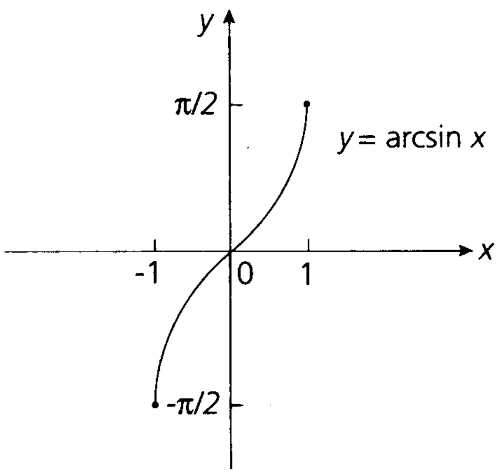
**arccosine function** The inverse of the cosine function, written arccos x or $\cos^{-1}x$. Because the cosine function is periodic, its domain must be restricted in order to have an inverse; the restricted domain is the interval $[0, \pi]$. We thus have the following definition: $y = \arccos x$ if and only if $x = \cos y$, where $0 \le y \le \pi$ and $-1 \le x \le 1$. The domain of arccos x is $[-1, 1]$, and its range $[0, \pi]$. Its derivative is $d/dx \arccos x = -1/\sqrt{1 - x^2}$.

**arcsine function** The inverse of the sine function, written arcsin x or $\sin^{-1}x$. Because the sine function is periodic, its domain must be restricted in order to have an inverse; the restricted domain is the interval $[-\pi/2, \pi/2]$. We thus have the following definition: $y = \arcsin x$ if and only if $x = \sin y$, where $-\pi/2 \le y \le \pi/2$ and $-1 \le x \le 1$. The domain of arcsin x is $[-1, 1]$, and its range $[-\pi/2, \pi/2]$. Its derivative is $d/dx \arcsin x = 1/\sqrt{1 - x^2}$.

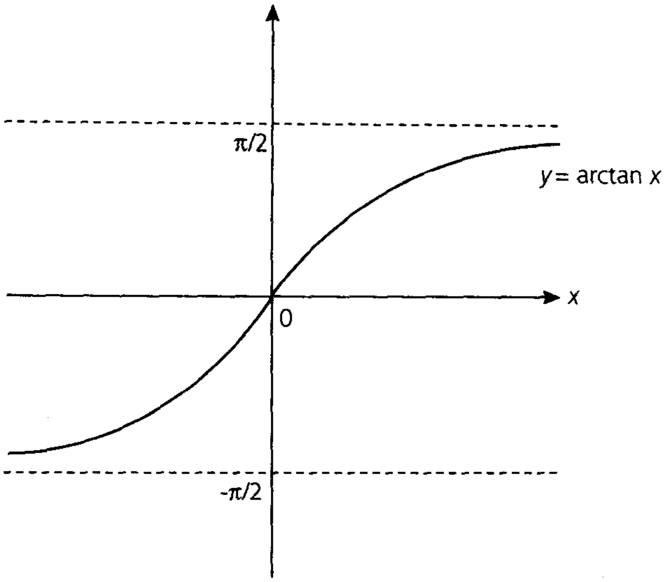
**arctangent function** The inverse of the tangent function, written arctan x or $\tan^{-1}x$. Because the tangent function is periodic, its domain must be restricted in order to have an inverse; the restricted domain is the open interval $(-\pi/2, \pi/2)$. We thus have the following definition: $y = \arctan x$ if and only if $x = \tan y$, where $-\pi/2 < y < \pi/2$. The domain of arctan x is all real numbers, that is, $(-\infty, \infty)$; its




**Arccosine function**




**Arcsine function**




**Arctangent function**

range is $(-\pi/2, \pi/2)$, and the lines $y = \pi/2$ and $y = -\pi/2$ are horizontal asymptotes to its graph. Its derivative is $d/dx \arctan x = 1/(1 + x^2)$.

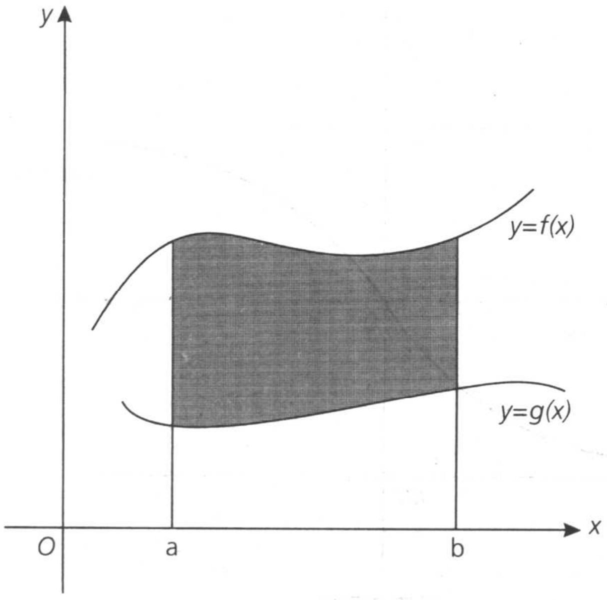
**area** Loosely speaking, a measure of the amount of two-dimensional space, or surface, bounded by a closed curve. Except for a few simple curves, finding the area involves calculating a definite integral.

**area between two curves** The definite integral $\int_a^b [f(x) - g(x)]\, dx$, where f(x) and g(x) represent the "upper" and "lower" curves, respectively, and *a* and *b* are the lower and upper limits of the interval under consideration.

**area function** The definite integral $\int_a^x f(t)\, dt$, considered as a function of the upper limit *x;* that is, we think of t = a as a fixed point and t = x as a variable point, and consider the area under the graph of y = f(x) as a function of *x*. The letter *t* is a "dummy variable," used so as not confuse it with the upper limit of integration *x*.

*See also* FUNDAMENTAL THEOREM OF CALCULUS.

**area in polar coordinates** The definite integral $\frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\, d\theta$, where $r = f(\theta)$ is the polar equation of the curve, and $\alpha$ and $\beta$ are the lower and upper angular limits of the region under consideration.




**Area between two curves**

**area of surface of revolution** The definite integral $2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\, dx$, where y = f(x) is the equation of a curve that revolves about the *x*-axis, and *a* and *b* are the lower and upper limits of the interval under consideration. If the graph revolves about the *y*-axis, we write its equation as x = g(y), and the area is $2\pi \int_c^d g(y)\sqrt{1 + [g'(y)]^2}\, dy$.

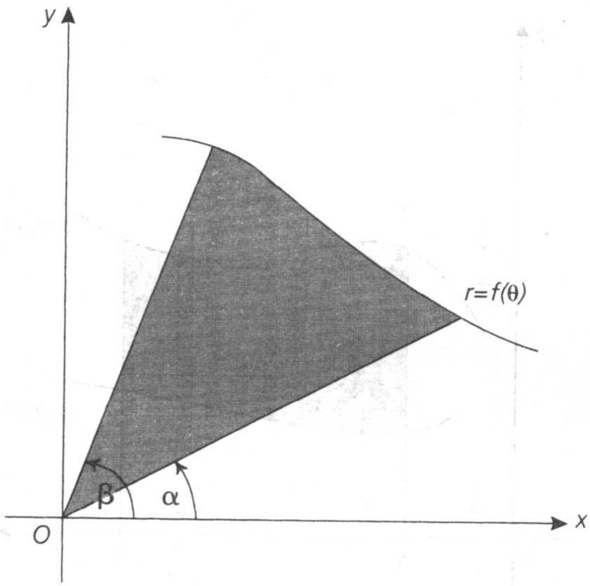
*See also* SOLID OF REVOLUTION.

**area under a curve** Let $f(x) \geq 0$ on the closed interval [a, b]. The area under the graph of f(x) between x = a and x = b is the definite integral $\int_a^b f(x)\, dx$. If $f(x) \leq 0$ on [a, b], we replace f(x) by $|f(x)|$.

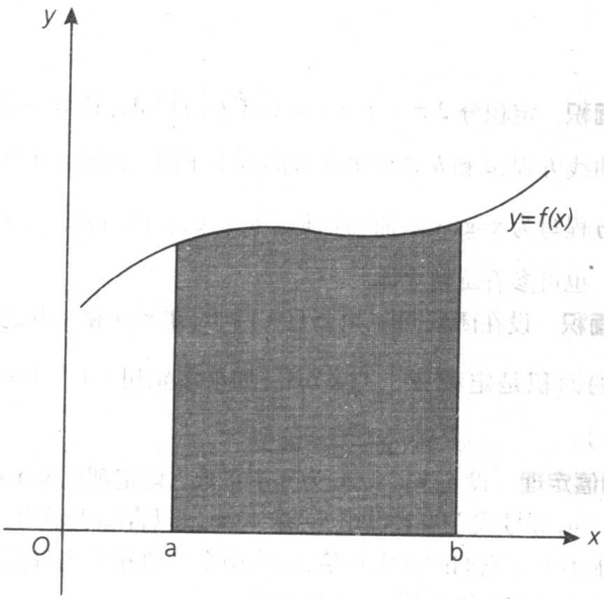
**Arithmetic-Geometric Mean Theorem** Let $a_1, a_2, \ldots, a_n$ be *n* positive numbers. The theorem says that $\sqrt[n]{a_1 a_2 \ldots a_n} \leq (a_1 + a_2 + \ldots + a_n)/n$, with equality if, and only if, $a_1 = a_2 = \ldots = a_n$. In words: the geometric mean of *n* positive numbers is never greater than their arithmetic mean, and the two means are equal if, and only if, the numbers are equal.

*See also* ARITHMETIC MEAN; GEOMETRIC MEAN.

**arithmetic mean** of *n* real numbers $a_1, a_2, \ldots, a_n$ is the expression $(a_1 + a_2 + \ldots + a_n)/n = \frac{1}{n}\sum_{i=1}^{n} a_i$. This is also called the *average* of




**Area in polar coordinates**



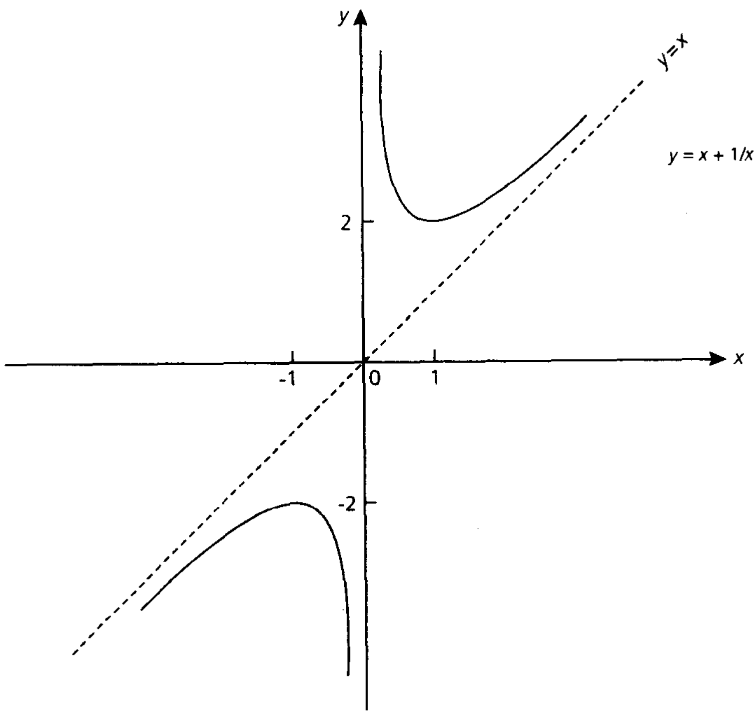

**Area under a curve**

the $n$ numbers. For example, the arithmetic mean of the numbers 1, 2, –5, and 7 is (1 + 2 + (–5) + 7)/4 = 5/4 = 1.25.

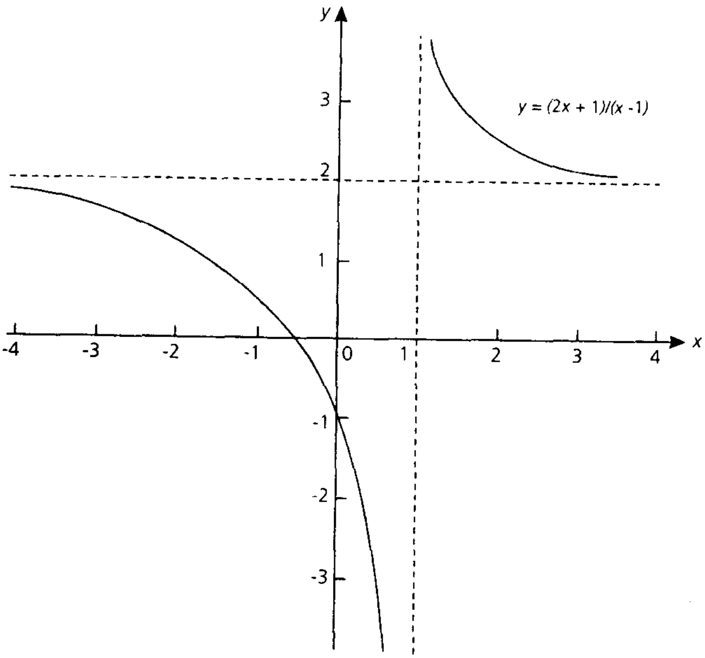
**asymptote** (from the Greek *asymptotus,* not meeting) A straight line to which the graph of a function y = f(x) gets closer and closer as $x$ approaches a specific value $c$ on the $x$-axis, or as $x \to \infty$ or $-\infty$.

Horizontal: A function has a horizontal asymptote if its graph approaches the horizontal line y = c as $x \to \infty$ or $x \to -\infty$. For example, the function y = (2x + 1)/(x – 1) has the horizontal asymptote y = 2.

Slant: A function has a slant asymptote if its graph approaches a line that is neither horizontal nor vertical. This usually happens when the degree of the numerator of a rational function is greater by 1 than the degree of the denominator. For example, the function




**Slant asymptote of y=x+1/x**




**Asymptotes of y=(2x+1)/(x−1)**

$y = (x^2 + 1)/x = x + 1/x$ has the slant asymptote $y = x$, because as $x \to \pm\infty$, $1/x$ approaches 0.

Vertical: A function has a vertical asymptote if its graph approaches the vertical line $x = a$ as $x \to a$. For example, the function $y = (2x + 1)/(x - 1)$ has the vertical asymptote $x = 1$.

**average** Of *n* numbers: Let the numbers be $x_1, x_2, \ldots, x_n$. Their average is the expression $(x_1 + x_2 + \ldots + x_n)/n = \frac{1}{n}\sum_{i=1}^{n} x_i$. Also called the *arithmetic mean* of the numbers.

Of a function: Let the function be $y = f(x)$. Its average over the interval [a, b] is the definite integral $\frac{1}{b-a}\int_a^b f(x)dx$. For example, the average of $y = x^2$ over [1, 2] is $\frac{1}{2-1}\int_1^2 x^2 dx = 7/3$.

**average cost function** A concept in economics. If the cost function of producing and selling *x* units of a commodity is C(x), the average cost per unit is C(x)/x, and is itself a function of *x*. It is measured in dollars per unit.

**average rate of change** *See* RATE OF CHANGE, AVERAGE.

**average velocity** Let a particle move along the *x*-axis. Its position at time *t* is a function of *t*, so we write x = x(t) (we are using here the same letter for the dependent variable as for the function itself). The average velocity of the particle over the time interval $[t_1, t_2]$ is the difference quotient $v = \frac{x_2 - x_1}{t_2 - t_1}$.

**base of logarithms** A positive number $b \neq 1$ such that $b^x = y$. We then write $x = \log_b y$.

**binomial series** The infinite series $(1 + x)^r = 1 + rx + [r(r-1)/2!]x^2 + [r(r-1)(r-2)]/3!]x^3 + \ldots = \sum_{k=0}^{\infty} \binom{r}{k} x^k$, where *r* is any real number and $-1 < x < 1$. This series is the TAYLOR SERIES for the function $(1 + x)^r$; the symbol $\binom{r}{k} = \frac{r!}{k!(r-k)!}$ denotes the binomial coefficients. In the special case when *r* is a nonnegative integer, the series terminates after r + 1 terms and is thus a finite progression.

*See also* BINOMIAL THEOREM.

**Binomial Theorem** The statement that $(a + b)^n = a^n + na^{n-1}b + [n(n-1)/2!] a^{n-2}b^2 + [n(n-1)(n-2)/3!] a^{n-3}b^3 + \ldots + nab^{n-1} + b^n$. The *k*th term (k = 0, 1, 2, . . ., n) in this expansion is $[n(n-1)(n-2)\ldots(n-k+1)/k!]a^{n-k}b^k$, where k! (read "*k* factorial") is $1 \cdot 2 \cdot 3 \cdot \ldots \cdot k$ (by definition, 0! = 1). The coefficients of this expansion are called the *binomial coefficients* and written as $\binom{n}{k}$ or $^nC_k$. As an example, $(a + b)^4 = a^4 + 4a^3b + [4(4-1)/2!]a^2b^2 + [4(4-1)(4-2)/3!]ab^3 + [(4(4-1)(4-2)(4-3)/4!]b^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. Note that the expansion is the same whether read from right to left or from left to right.

**bounds** A number *M* is an *upper bound* of a sequence of numbers $a_1, a_2, \ldots, a_n$, if $a_i \leq M$ for all *i*. A number *N* is a *lower bound* if $a_i \geq N$ for all *i*. For example, the sequence 1/2, 2/3, 3/4, . . ., n/(n + 1) has an upper bound 1 and a lower bound 0. Of course, any number $M' > M$ is also an upper bound, and any number $N' < N$ is also a lower bound of the same sequence; thus upper and lower bounds are not unique.

**Boyle's Law (Boyle-Mariotte Law)** A law in physics that relates the pressure *P* and volume *V* of a gas in a closed container held at constant temperature. The law says that under these circumstances,