# 顧毓琇全集

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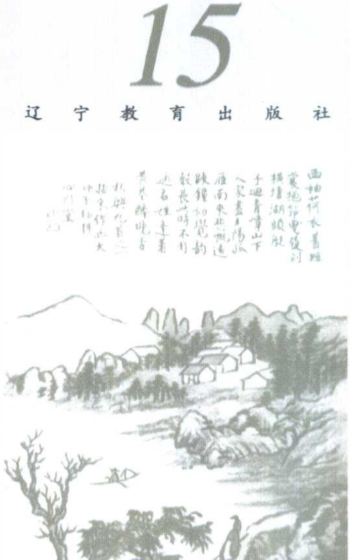

◎参加全国政协招待海外学者宴会（一九九二年六月）
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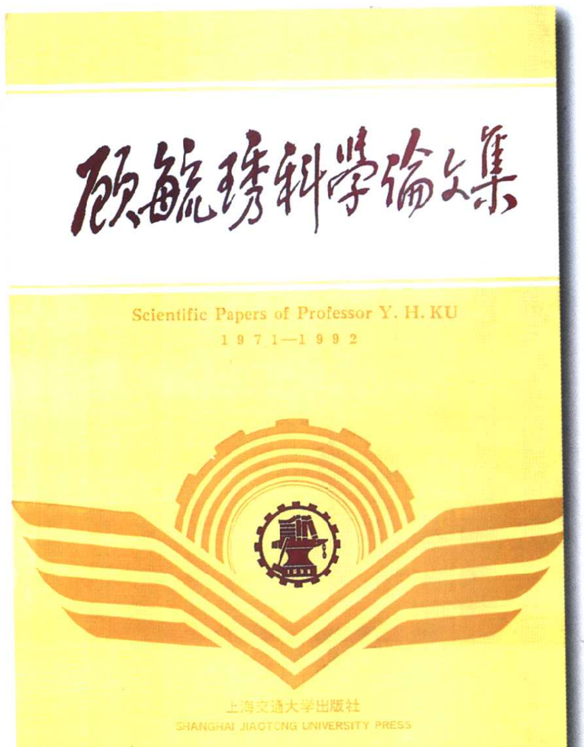


◎江泽民主席题写书名　上海交通大学出版社一九九二年版

# 顾毓琇科学论文集（四）

# CONTENTS

# NEW THEOREMS ON ABSOLUTE STABILITY OF NON-AUTONOMOUS NONLINEAR CONTROL SYSTEMS

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**SUMMARY**

**This paper gives four new theorems on absolute stability of non-autonomous nonlinear control systems. Theorem I is concerned with the absolute stability of a nonlinear control system with system function G (s), nonlinearity $N_1$ and a strongly bounded inpt $r_1$ (t) (such as $e^{-at}$ sin bt). Theorem Ⅱ is concerned with the absolute stability of a control system with two nonlinearities $N_1$ and $N_2$, and a strongly bounded input $r_1$ (t). Theorem Ⅲ is concerned with the absolute stability of a control system with system function**

**$G_1$ (s), which has a pole at the origin, and nonlinearity $N_1$, subjected to a monotonically bounded input $r_2$ (t) (such as a/ln (bt + c)). Theorem Ⅳ is then concerned with the absolute stability of a control system similar to the system of Theorem Ⅲ, but with an additional input $r_1$ (t).**

**Two more theorems are concerned with the absolute stability of autonomous nonlinear systems with one or two nonlinearities.**

**All theorems are given with proofs in the Appendices.**

## 1. INTRODUCTION

Absolute stability of autonomous control systems have been extensively discussed and important theorems were given by many authors such as Lur'e[1], Letov[2], Popov[3], Kalman[4], Aizerman and Gantmacher[5], Rekasius and Gibson[6], Rekasius[7], Ibrahim and Rekasius[8], Brockett[9], Willems and Brockett[10], Brockett and Willems[11], Lasalle[12], Narendra and Neuman[13], Hahn[14], Lefschetz[15], etc. The theorems for autonomous systems are considered as special cases of nonautonomous systems in this paper.

Theorems on $L_2$-stability of nonautonomous control systems were given by I. W. Sandbury[16] and C. A. Desoer[17]。 In these theorems, $L_2$-stability was considered for systems with a single nonlinearity and a transfer function having poles with negative real parts. This paper gives theorems with proofs on absolute stability for non-autonomous systems with two nonlinearities (one in the forward branch and another in feedback branch) and a transfer function having additional poles on the imaginary axis, subjected to a strongly bounded input. Moreover, if the transfer function contains a pole at the origin, similar theorems can be given with a single nonlinearity but subjected to a strongly bounded input plus a monotonically bounded input.

Illustrative examples with digital computer verifications are given for the theorems.

## 2. DEFINITIONS

(a) A piecewise continuous function $r_1(t)$ is said to be strongly bounded if

$$|r_1(t)| \leq \mu_1, \quad \int_0^\infty |r_1(t)|dt \leq \mu_2 \text{ and}$$

$$\int_0^\infty |\dot{r}_1(t)|dt \leq \mu_3, \text{ where } \mu_1, \mu_2, \text{ and } \mu_3 \text{ are}$$

non-negative constants which can be replaced by a single constant $\mu$ in the above inequalities if $\mu = \max[\mu_1, \mu_2, \mu_3]$ is assumed.

(b) A continuous function $r_2(t)$ is said to be monotonically bounded if

$|r_2(t)| \leq \mu$, $|\dot{r}_2(t)| \leq \mu$, and $\dot{r}_2(t)$ and $r_2(t)$ are monotonic.

(c) A real single-valued and continuous function $f_1(e)$ is said to be in the sector

$[k_1^*, k_2^*]$ if it satisfies the condition

$$k_1^* \leq f_1(e)/e \leq k_2^*$$

where the equalities take place, at most, finite number of times in any finite interval, and that $f_1(e)=0$ only at $e=0$ when $k_1^*=0$. The function $f_1(e)$ is shown in Fig.1.

(d) A real single-valued and continuous function $f_2(c)$ is said to be in the sector $[k_1, k_2]$ if it satisfies the condition①

$k_1 \leq f_2(c)/c \leq k_2$

The function $f_2(c)$ is shown in Fig.2.

(e) The control system subjected to the input $r(t)$ as shown by Fig.3. is said to be absolutely stable② in the sector $[k_1^*, k_2^*]$ if there exist two positive numbers $\alpha$ and $\beta$ such that for all $f_1(e)$ in $[k_1^*, k_2^*]$ and all $t>0$

$$|c(t)| \leq \alpha\mu + \beta|c(0)|$$

and

$$\lim_{t\to\infty} c(t) = 0$$

where $\mu$ is the same as defined in (a) and (b).

(f) The control system subjected to the input $r(t)$ as shown by Fig.4. is said to be absolutely stable in sectors $[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exist two positive numbers $\alpha$ and $\beta$ such that for all $f_1(e)$ in $[k_1^*, k_2^*]$, $f_2(c)$ in $[k_1, k_2]$ and all $t\geq 0$

$$|c(t)| \leq \alpha\mu + \beta|c(0)|$$

and

$$\lim_{t\to\infty} c(t) = 0$$

① This condition was given by Aizerman and Gantmacher in reference 5.

② Definition of absolute stability for autonomous systems was given by Aizerman and Gantmacher in reference 5.

where $\mu$ is the same as defined in (a) and (b).

## 3. THEOREMS

Theorem Ⅰ

The non-autonomous control system (Fig.3) subjected to a strongly bounded input $r_1$ (t) is absolutely stable in the sector$[k_1^*, k_2^*]$ if there exists a non-negative q such that the expression $1/k_2^* H + (1 + sq) G(s)$ is positive real, where $k_2^* > k_1^* \geq 0$ and $H > 0$.

Theorem Ⅱ

The non-autonomous control system (Fig.4) subjected to a strongly bounded input $r_1$ (t) is absolutely stable in sectors$[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exists a non-negative q such that the expression $1/k_2 k_2^* + (1 + sq) G(s)$ is positive real, where $k_2^* > k_1^* \geq 0$, $k_2 \geq k_1 > 0$ and $f'_2(c) > 0$.

Theorem Ⅲ

The non-antonomous control system (Fig.3) subjected to a monotonically bounded input $r_2$ (t) is absolutely stable in the sector$[k_1^*, k_2^*]$ if there exists a non-negative q such that the expression $1/k_2^* H + (1 + sq) G_1(s)$ is positive real, where $k_2^* > k_1^* > 0$, $H > 0$, $\lim_{t \to \infty} r_2(t) = 0$ and, $G(s) = G_1(s)$ which has a pole at the origin.

Tneorem Ⅳ

The non-antonomous control system (Fig.3) subjected to a monotonically bounded input plus a strongly bounded input$[r(t) = r_1(t) + r_2(t)]$ is absolutely stable in the sector$[k_1^*, k_2^*]$ if there exists a non-negative q such that the expression $1/k_2^* H + (1+sq) G_1(s)$ is positive real, where $k_2^* > k_1^* \geq 0$, $H > 0$, $\lim_{t\to\infty} r(t) = 0$ and $G(s) = G_1(s)$ which has a pole at the origin.

Theorem Ⅴ

The autonomous control system (Fig.4. with $r(t) \equiv 0$) is absolutely stable in sectors $[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exist a nonnegative q and a $\lambda \leq k_1 k_1^*$ such that the expression $1/(k_2 k_2^* - \lambda) + (1+sq) G(s, \lambda)$ is positive real, where either $k_1$ or $k_1^*$ is nonnegative and that $G(s, \lambda) = G(s)/[1+\lambda G(s)]$.

Theorem Ⅵ

The autonomous control system (Fig.4. with $r(t) \equiv 0$) is absolutely stable in sectors $[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exist a real q and a $\lambda < k_1 k_1^*$ such that for all $\omega$, $1/(k_2 k_2^* - \lambda) + \mathrm{Re}[(1+j\omega q) G(j\omega, \lambda)] \geq \delta > 0$, where either $k_1$ or $k_1^*$ is non-negative and $G(s, \lambda)$ has no poles with positive real parts, with any poles on the imaginary axis being simple and having positive residues.

Corollary

If q can be set to zero, then all conditions being imposed on $\dot{r}_1(t)$, $\dot{r}_2(t)$ and $f'(c)$ in Theorem Ⅰ through Ⅳ can be removed.

## 4. ILLUSTRATIVE EXAMPLES

### Example 1

Given a control system shown in Fig.3. with $r_1(t) = 100e^{-0.1t}$

sint, $f_1(e) = e^3 - 5e^2 + 7e$, $H = 0.5$ and $G(s) = (s+1)/(s^2+s+1) + s/(s^2+1)$. In order to apply the theorems, we make the following investigations.

(1) $|r_1(t)| \leq 86 = \mu_1$,

$\int_0^\infty |r_1(t)| dt = (100/1.01) \coth(0.1\pi/2) = 638 = \mu_2$

and $\int_0^\infty |\dot{r}_1(t)| dt \simeq 100 \operatorname{csch} 0.1\pi/2 = 637$. Hence $r_1(t)$ is strongly bounded with

$$\mu = \max[86, 638, 637] = 638.$$

(2) $k_1^* = \min[e^2 - 5e + 7] = 0.75$ and

$k_2^* = \max[e^2 - 5e + 7] = \infty$.

(3) For $q = 0$ and $k_2^* H = \infty$, we have

$1/k_2^* H + \mathrm{Re}[(1 + j\omega q) G(j\omega)] = [(1-\omega^2)^2 + \omega^2]^{-1} \geq 0$.

This system is absolutely stable according to Theorem Ⅰ. The computer results of the output c (t) and the error e (t) are shown in Fig.5.

## Example 2

Given a control system shown in Fig.4 with $r_1(t) = 100e^{-0.2t}\sin t$,

$f_1(e) = 0.5e(1 + 0.9\cos 10e)$,

$f_2(c) = c + 0.9(1 - \cos c)$

and $G(s) = (s^2 - s - 1)/(s^3 + 3s^2 + 4s + 2)$.

(1) $r_1(t)$ is strongly bounded.

(2) $\min[f_1(e)/e] = 0.05 = k_1^*$

$\max[f_1(e)/e] = 0.95 = k_2^*$,

$\min[f_2(c)/c] = 0.345 = k_1$ and

$\max[f_2(c)/c] = 1.655 = k_2$

(3) $k_2 k_2^* = 1.57$, take 1.6. It can be shown that for $0 \leq q \leq 0.5$, $1/k_2 k_2^* + (1 + sq) G(s)$ is positive real.

Hence the system is absolutely stable according to Theorem Ⅱ. The computer results are shown in Fig.6.

## Example 3

Given a control system shown in Fig.3. with $r_2(t) = 10(1 + t)^{-1/n}$, $n > 0$, $f_1(e) = e^3 - 5e^2 + 7e$, $H = 2$ and $G(s) = G_1(s)$

$$= \frac{1}{s} - \frac{s^2 + s - 1}{s^3 + 3s^2 + s + 1}.$$

(1) $r_2(t)$ is a decreasing function of t and $\dot{r}_2(t) = -10/n(1 + t)^{1+1/n}$ is an increasing function. Hence $r(t)$ is monotonically bounded.

(2) $k_1^* = 0475$ and $k_2^* = \infty$.

(3) For $k_2^* = \infty$ and $q \geq 3$, $1/k_2^* H + (1 + sq) G_1(s)$ is positive real.

The system is absolutely stable according to Theorem Ⅲ. The computer results for $n = 2$ are shown in Fig.7.

## Example 4

Given a control system shown in Fig.3. with $r(t) = r_1(t) + r_2(t)$ where

$r_1(t) = 10u(t) - 5u(t-2) - 12.5u(t-4)$
$+ 10u(t-6) - 2.5u(t-8)$ and $r_2(t) = 10/\ln(100t+2)$,
$f_1(e) = 0.5e(1+0.9\cos 10e)$, $H = 1.6$ and
$G(s) = G_1(s) = 1/s + (s^2 - s - 1)/(s^3 + 3s^2 + 4s + 2)$.

(1) $r_1(t)$ is strongly bounded and $r_2(t)$ is monotonically bounded.

(2) $k_2 k_2^* = 1.52$.

(3) For $0.25 \leq q \leq 0.5$, $1/k_2 k_2^* + (1 + sq) G_1(s)$ is positive real.

The system is absolutely stable according to Theorem Ⅳ. The computer results are shown in Fig.8.

## Example 5

Given a control system shown in Fig.4. with $r(t) \equiv 0$, $f_1(e) = 0.2(e \cos e) - 0.75e$,

$f_2(c) = c$ and $G(s) = (2s^3 + 4s^2 + 2s + 2)/(s^4 + 4s^3 + 6s^2 + 4s + 3)$.

(1) $r(t) \equiv 0$ implies that $\mu = 0$.

(2) $\min[f_1(e)/e] = -0.95$ and $\max[f_1(e)/e] = -0.55$, $k_1 = 1$, $k_2 = 1$. That is $k_1 k_1^* = -0.95$ and $k_2 k_2^* = -0.55$.

(3) Assuming $\lambda = -1$, we have $G(s, \lambda) = (s+2)/(s^2 + 2s + 1) + s/(s^2 + 1)$.

It can be shown that $1/(k_2 k_2^* - \lambda) + (1 + sq) G(s, \lambda)$