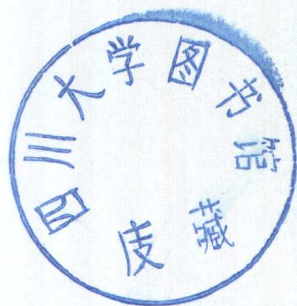


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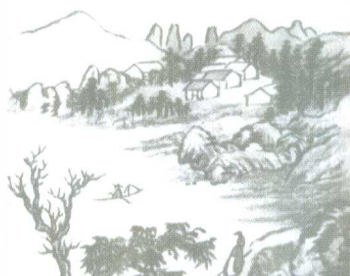


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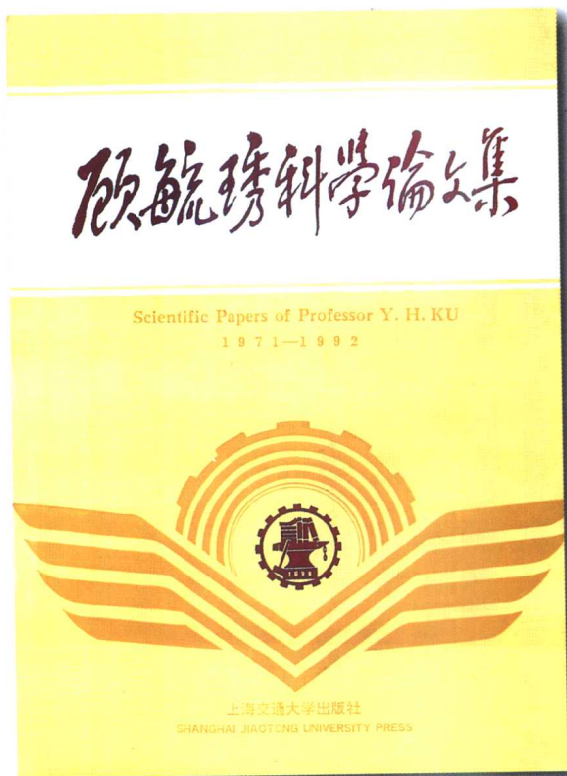
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◎参加全国政协招待海外学者宴会（一九九二年六月）
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顾毓琇科学论文集（四）

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NEW THEOREMS ON ABSOLUTE STABILITY OF NON-AUTONOMOUS NONLINEAR CONTROL SYSTEMS

Y. H. KU and H. T. CHIEH, *The Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pennsylvania*

SUMMARY

This paper gives four new theorems on absolute stability of non-autonomous nonlinear control systems. Theorem I is concerned with the absolute stability of a nonlinear control system with system function $G(s)$, nonlinearity N_1 and a strongly bounded input $r_1(t)$ (such as $e^{-at} \sin bt$). Theorem II is concerned with the absolute stability of a control system with two nonlinearities N_1 and N_2 , and a strongly bounded input $r_1(t)$. Theorem III is concerned with the absolute stability of a control system with system function

$G_1(s)$, which has a pole at the origin, and nonlinearity N_1 , subjected to a monotonically bounded input $r_2(t)$ (such as $a/\ln(bt+c)$). Theorem IV is then concerned with the absolute stability of a control system similar to the system of Theorem III, but with an additional input $r_1(t)$.

Two more theorems are concerned with the absolute stability of autonomous nonlinear systems with one or two nonlinearities.

All theorems are given with proofs in the Appendices.

1. INTRODUCTION

Absolute stability of autonomous control systems have been extensively discussed and important theorems were given by many authors such as Lur'e¹, Letov², Popov³, Kalman⁴, Aizerman and Gantmacher⁵, Rekasius and Gibson⁶, Rekasius⁷, Ibrahim and Rekasius⁸, Brockett⁹, Willems and Brockett¹⁰, Brockett and Willems¹¹, Lasalle¹², Narendra and Neuman¹³, Hahn¹⁴, Lefschetz¹⁵, etc. The theorems for autonomous systems are considered as special cases of nonautonomous systems in this paper.

Theorems on L_2 -stability of nonautonomous control systems were given by I. W. Sandbury¹⁶ and C. A. Desoer¹⁷. In these theorems, L_2 -stability was considered for systems with a single nonlinearity and a transfer function having poles with negative real parts. This paper gives theorems with proofs on absolute stability for non-autonomous systems with two nonlinearities (one in the forward branch and another in feedback branch) and a transfer function having additional poles on the imaginary

axis, subjected to a strongly bounded input. Moreover, if the transfer function contains a pole at the origin, similar theorems can be given with a single nonlinearity but subjected to a strongly bounded input plus a monotonically bounded input.

Illustrative examples with digital computer verifications are given for the theorems.

2. DEFINITIONS

(a) A piecewise continuous function $r_1(t)$ is said to be strongly bounded if

$$|r_1(t)| \leq \mu_1, \quad \int_0^\infty |r_1(t)| dt \leq \mu_2 \text{ and} \\ \int_0^\infty |\dot{r}_1(t)| dt \leq \mu_3, \text{ where } \mu_1, \mu_2, \text{ and } \mu_3 \text{ are}$$

non-negative constants which can be replaced by a single constant μ in the above inequalities if $\mu = \max[\mu_1, \mu_2, \mu_3]$ is assumed.

(b) A continuous function $r_2(t)$ is said to be monotonically bounded if

$|r_2(t)| \leq \mu$, $|\dot{r}_2(t)| \leq \mu$, and $\dot{r}_2(t)$ and $r_2(t)$ are monotonic.

(c) A real single-valued and continuous function $f_1(e)$ is said to be in the sector

$[k_1^*, k_2^*]$ if it satisfies the condition

$$k_1^* \leq f_1(e)/e \leq k_2^*$$

where the equalities take place, at most, finite number of times in any

finite interval, and that $f_1(e) = 0$ only at $e = 0$ when $k_1^* = 0$. The function $f_1(e)$ is shown in Fig.1.

(d) A real single-valued and continuous function $f_2(c)$ is said to be in the sector $[k_1, k_2]$ if it satisfies the condition^①

$$k_1 \leq f_2(c) / c \leq k_2$$

The function $f_2(c)$ is shown in Fig.2.

(e) The control system subjected to the input $r(t)$ as shown by Fig.3. is said to be absolutely stable^② in the sector $[k_1^*, k_2^*]$ if there exist two positive numbers α and β such that for all $f_1(e)$ in $[k_1^*, k_2^*]$ and all $t > 0$

$$|c(t)| \leq \alpha\mu + \beta|c(0)|$$

and

$$\lim_{t \rightarrow \infty} c(t) = 0$$

where μ is the same as defined in (a) and (b).

(f) The control system subjected to the input $r(t)$ as shown by Fig.4. is said to be absolutely stable in sectors $[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exist two positive numbers α and β such that for all $f_1(e)$ in $[k_1^*, k_2^*]$, $f_2(c)$ in $[k_1, k_2]$ and all $t \geq 0$

$$|c(t)| \leq \alpha\mu + \beta|c(0)|$$

and

$$\lim_{t \rightarrow \infty} c(t) = 0$$

① This condition was given by Aizerman and Gantmacher in reference 5.

② Definition of absolute stability for autonomous systems was given by Aizerman and Gantmacher in reference 5.

where μ is the same as defined in (a) and (b).

3. THEOREMS

Theorem I

The non-autonomous control system (Fig.3) subjected to a strongly bounded input $r_1(t)$ is absolutely stable in the sector $[k_1^*, k_2^*]$ if there exists a non-negative q such that the expression $1/k_2^* H + (1 + sq) G(s)$ is positive real, where $k_2^* > k_1^* \geq 0$ and $H > 0$.

Theorem II

The non-autonomous control system (Fig.4) subjected to a strongly bounded input $r_1(t)$ is absolutely stable in sectors $[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exists a non-negative q such that the expression $1/k_2 k_2^* + (1 + sq) G(s)$ is positive real, where $k_2^* > k_1^* \geq 0$, $k_2 \geq k_1 > 0$ and $f'_2(c) > 0$.

Theorem III

The non-autonomous control system (Fig.3) subjected to a monotonically bounded input $r_2(t)$ is absolutely stable in the sector $[k_1^*, k_2^*]$ if there exists a non-negative q such that the expression $1/k_2^* H + (1 + sq) G_1(s)$ is positive real, where $k_2^* > k_1^* > 0$, $H > 0$, $\lim_{t \rightarrow \infty} r_2(t) = 0$ and, $G(s) = G_1(s)$ which has a pole at the origin.

Theorem IV

The non-autonomous control system (Fig.3) subjected to a monotonically bounded input plus a strongly bounded input $[r(t) = r_1(t) + r_2(t)]$ is absolutely stable in the sector $[k_1^*, k_2^*]$ if there exists a

non-negative q such that the expression $1/k_2^* H + (1 + sq) G_1(s)$ is positive real, where $k_2^* > k_1^* \geq 0$, $H > 0$, $\lim_{t \rightarrow \infty} r(t) = 0$ and $G(s) = G_1(s)$ which has a pole at the origin.

Theorem V

The autonomous control system (Fig.4. with $r(t) \equiv 0$) is absolutely stable in sectors $[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exist a nonnegative q and a $\lambda \leq k_1 k_1^*$ such that the expression $1/(k_2 k_2^* - \lambda) + (1 + sq) G(s, \lambda)$ is positive real, where either k_1 or k_1^* is nonnegative and that $G(s, \lambda) = G(s) / [1 + \lambda G(s)]$.

Theorem VI

The autonomous control system (Fig.4. with $r(t) \equiv 0$) is absolutely stable in sectors $[k_1^*, k_2^*]$ and $[k_1, k_2]$ if there exist a real q and a $\lambda < k_1 k_1^*$ such that for all ω , $1/(k_2 k_2^* - \lambda) + \operatorname{Re} [(1 + j\omega q) G(j\omega, \lambda)] \geq \delta > 0$, where either k_1 or k_1^* is non-negative and $G(s, \lambda)$ has no poles with positive real parts, with any poles on the imaginary axis being simple and having positive residues.

Corollary

If q can be set to zero, then all conditions being imposed on $\dot{r}_1(t)$, $\dot{r}_2(t)$ and $f'(c)$ in Theorem I through IV can be removed.

4. ILLUSTRATIVE EXAMPLES

Example 1

Given a control system shown in Fig.3. with $r_1(t) = 100e^{-0.1t}$

$\sin t$, $f_1(e) = e^3 - 5e^2 + 7e$, $H = 0.5$ and $G(s) = (s+1)/(s^2 + s + 1) + s/(s^2 + 1)$. In order to apply the theorems, we make the following investigations.

$$(1) \|r_1(t)\| \leq 86 = \mu_1,$$

$$\int_0^\infty \|r_1(t)\| dt = (100/1.01) \coth(0.1\pi/2) = 638 = \mu_2$$

and $\int_0^\infty \|\dot{r}_1(t)\| dt \simeq 100 \operatorname{csch} 0.1\pi/2 = 637$. Hence $r_1(t)$ is strongly bounded with

$$\mu = \max[86, 638, 637] = 638.$$

$$(2) k_1^* = \min[e^2 - 5e + 7] = 0.75 \text{ and}$$

$$k_2^* = \max[e^2 - 5e + 7] = \infty.$$

(3) For $q=0$ and $k_2^*H = \infty$, we have

$$1/k_2^*H + \operatorname{Re}[(1+j\omega q)G(j\omega)] = [(1-\omega^2)^2 + \omega^2]^{-1} \geq 0.$$

This system is absolutely stable according to Theorem I. The computer results of the output $c(t)$ and the error $e(t)$ are shown in Fig.5.

Example 2

Given a control system shown in Fig.4 with $r_1(t) = 100e^{-0.2t}\sin t$, $f_1(e) = 0.5e(1 + 0.9\cos 10e)$,

$$f_2(c) = c + 0.9(1 - \cos c)$$

$$\text{and } G(s) = (s^2 - s - 1)/(s^3 + 3s^2 + 4s + 2).$$

(1) $r_1(t)$ is strongly bounded.

$$(2) \min[f_1(e)/e] = 0.05 = k_1^*$$

$$\max[f_1(e)/e] = 0.95 = k_2^*,$$

$$\min[f_2(c)/c] = 0.345 = k_1 \text{ and}$$

$$\max[f_2(c)/c] = 1.655 = k_2$$

(3) $k_2 k_2^* = 1.57$, take 1.6. It can be shown that for $0 \leq q \leq 0.5$, $1/k_2 k_2^* + (1 + sq) G(s)$ is positive real.

Hence the system is absolutely stable according to Theorem II. The computer results are shown in Fig.6.

Example 3

Given a control system shown in Fig.3. with $r_2(t) = 10(1+t)^{-1/n}$, $n > 0$, $f_1(e) = e^3 - 5e^2 + 7e$, $H = 2$ and $G(s) = G_1(s)$

$$= \frac{1}{s} - \frac{s^2 + s - 1}{s^3 + 3s^2 + s + 1}.$$

(1) $r_2(t)$ is a decreasing function of t and $\dot{r}_2(t) = -10/n(1+t)^{1+1/n}$ is an increasing function. Hence $r(t)$ is monotonically bounded.

(2) $k_1^* = 0.475$ and $k_2^* = \infty$.

(3) For $k_2^* = \infty$ and $q \geq 3$, $1/k_2^* H + (1 + sq) G_1(s)$ is positive real.

The system is absolutely stable according to Theorem III. The computer results for $n = 2$ are shown in Fig.7.

Example 4

Given a control system shown in Fig.3. with $r(t) = r_1(t) + r_2(t)$ where

$$r_1(t) = 10u(t) - 5u(t-2) - 12.5u(t-4) + 10u(t-6) - 2.5u(t-8) \text{ and } r_2(t) = 10/\ln(100t+2),$$

$$f_1(e) = 0.5e(1 + 0.9\cos 10e), H = 1.6 \text{ and}$$

$$G(s) = G_1(s) = 1/s + (s^2 - s - 1) / (s^3 + 3s^2 + 4s + 2).$$

(1) $r_1(t)$ is strongly bounded and $r_2(t)$ is monotonically bounded.

$$(2) k_2 k_2^* = 1.52.$$

(3) For $0.25 \leq q \leq 0.5$, $1/k_2 k_2^* + (1 + sq) G_1(s)$ is positive real.

The system is absolutely stable according to Theorem IV. The computer results are shown in Fig.8.

Example 5

Given a control system shown in Fig.4. with $r(t) \equiv 0$, $f_1(e) = 0.2(e \cos e) - 0.75e$,

$$f_2(c) = c \text{ and } G(s) = (2s^3 + 4s^2 + 2s + 2) / (s^4 + 4s^3 + 6s^2 + 4s + 3).$$

(1) $r(t) \equiv 0$ implies that $\mu = 0$.

(2) $\min[f_1(e)/e] = -0.95$ and $\max[f_1(e)/e] = -0.55$, $k_1 = 1$, $k_2 = 1$. That is $k_1 k_1^* = -0.95$ and $k_2 k_2^* = -0.55$.

(3) Assuming $\lambda = -1$, we have $G(s, \lambda) = (s+2)/(s^2 + 2s+1) + s/(s^2+1)$.

It can be shown that $1/(k_2 k_2^* - \lambda) + (1 + sq) G(s, \lambda)$