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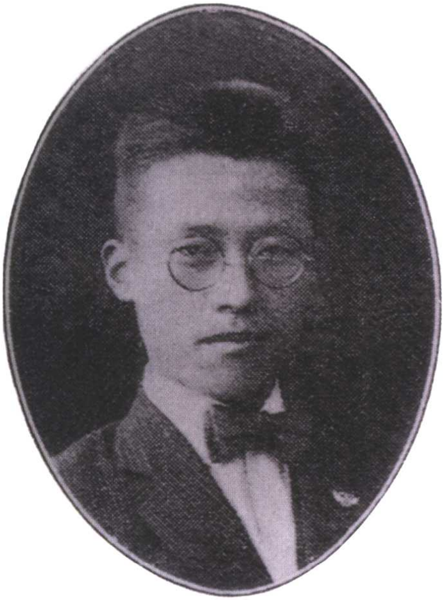

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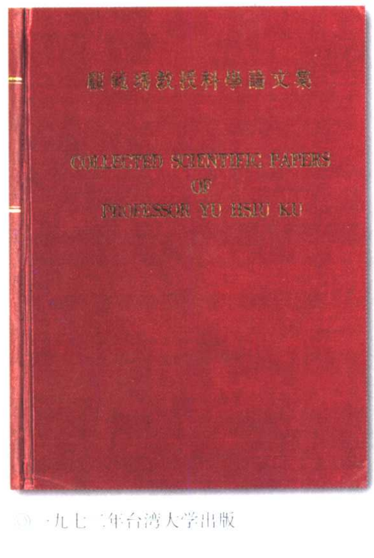

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THEORY OF NONLINEAR CONTROL

Y. H. KU

INTRODUCTION

Nature is non-linear

Since nature embodies a manifold of beautiful phenomena, it cannot be limited by a linear relationship between elementary quantities. Thus a simple pendulum as discovered by Galileo represents a simple harmonic motion, and yet the equation of a pendulum is a second-order *non-linear* differential equation. Kepler's laws of planetary motion are non-linear. Not only Newton's law of gravitation is an inverse square law, but also Coulomb's law of force between electric charges and the simliar law of force between magnetic poles are inverse square laws. Van der Pol's equation representing an electronic oscillator is non-linear and gives rise to a limit cycle. Transistors and magnetic amplifiers are both non-linear devices.

Feedback control systems

Truxal[1] pointed out: 'The situations of primary interest to the designer of feedback systems are those in which he is using feedback for a definite purpose'. Feedback is used to overcome limitations of the physical components to effect a specific change in the characteristics of the system. Wiener[2] has borrowed the term *cybernetics* from Ampere to mean the sciences of 'control and communication in the animal and the machine'. His recent book[3] has examined the role of non-linear processes in physics, mathematics, electrical engineering, physiology and communication theory.

THREE STAGES IN THE STUDY OF PHYSICAL SYSTEMS

Development of linear system analysis and synthesis

The classical method of solving linear differential equations is well known and furnishes a great urge to the idealization of actual physical systems as linear. Heaviside's operational calculus was developed by Carson, Bush, Berg, Cohen, Wagner, and others in the years 1926-1942. From 1942 on, Laplace transforms[4] have been largely used in the study of feedback control systems[5]. From 1947 on, control system *synthesis*[1] rigorously calls for a logical procedure for the transition from specifications to system. At the same time Fourier transforms and integrals have been developed for the analysis and synthesis of networks.

Recognition and analysis of non-linear systems

Recognition of non-linear systems as they are indeed has had a long and glorious history. We shall roughly divide the methods of attack into four categories: (a) perturbation and iteration methods; (b) isocline method and other graphical methods; (c) linearization methods; and (d) phase-space method, and the use of analogue and digital computers.

The perturbation method was developed by Poincare[6] and Lindsteht[7] primarily for astronomical problems. Poincare and Liapounov[8] published their treatises in 1892-93. Lord Ray-leigh's *Theory of Sound*[9] was published in 1893-96; in it non-linear problems were discussed. The isocline method is well known and was used by Van der Pol to find the limit cycle. Other graphical methods are developed by Lienard[10], Ku[11], Hsia[12], Paynter[13] and Buland[14]. The third category includes the work of Krylov and Bogoliubov[15], and the recent work of Goldfarb[16], Tustin[17], Oppelt[18], Kochenburger[19], Johnson[20] and others. Krylov. and Bogoliubov's method of equivalent linearization and principle of harmonic balaice are made known to the world at larget through Minorsky[21] and Lefschetz[15]. According to Andronow and Chaikin[22]: 'We may consider the q_i, q_i' as coordinates of a space S of $2n$ dimensions called the *phase space*', where q_i denote n positional coordinates and q_i' their velocities in a dynamical system. The phase-space method has been developed and applied to control systems by Ku[23-30] and others.

Introduction of the new concept of non-linear control

The new concept of non-linear control can be stated as follows[30]:

A feedback system can be controlled either linearly or non-linearly. If the system has inherent non-linear characteristics, it is possible to introduce linear compensating networks to improve the system performance.

If optimum system performance desired in a control system cannot be obtained by a combination of linear transfer functions, it is most desirable to introduce non-linearities into the control system. If the system has inherent non-linear characteristics, linear compensation and non-linear control can both be introduced. In short, the newest aspect of feedback control is the development of the theory of non-linear control.

While a physical system with a given input has to be analysed to know its response or synthesized to yield the desired output, a general non-linear system with random inputs offers even greater challenge for analysis and synthesis. The recent development of adaptive systems makes the study of a linear systems even more important than before.

LINEARIZATION TECHNIQUES

Development of the describing function method

Use of describing function representation[19] of non-linear elements substitutes *a quasi-linear* system whose parameters change slowly for one containing rapidly varying relationships. The new system with slowly changing parameters can be thought of in terms of the poles and zeros of its response functions. As amplitude and frequency shift, the poles and zeros of the hypothetical system move about in the complex plane. One can then tell whether the hypothetical system is stable or unstable, or has periodic oscillation. However, as both Johnson[20] and Nichols[31] pointed out, the low-frequency divergent limit cycle prediction must be disregarded. At the Heidelberg Conference 1956, West[32] discussed the use of frequency response analysis in non-linear control systems and Klotter[33] suggested the Hamilton describing function in contrast to Kochenburger's Fourier describing function.

Other linearizing techniques

We shall briefly mention three other linearizing techniques: (*a*) quasi-linearization; (*b*) equivalent linearization; and (*c*) piece-wise linearization.

The quasi-linearization technique developed by Chen[34] can be used for transient study of non-linear feedback control systems. Assuming that the non-linearity is piece-wise linear, the operating point of the non-linear element is expected to shift during the transient over several linear ranges. A quick evaluation of the actual transient response in the first linear range through linear servo techniques determines the test function for the non-linearity in the first two linear ranges. The representation thus obtained permits a quick evaluation of the transient response in the second linear range, and so on.

Bass[35] proposed the equivalent linearization for a type of difference-differential equation where the time-lag in the nonlinear function is taken into account by a time-lead in the variable, say, $x(t)$ and its derivatives. The steps are: (*a*) insert $x(t) = a\varepsilon^{j\nu t}$ in the modified non-linear equation to obtain the formal instantaneous characteristic polynomial; (*b*) replace complex numbers in the arguments of all non-linear terms by their real parts, and weight each of these terms by a factor of 2; (*c*) divide the polynomial by $\varepsilon^{j\nu t}$; (*d*) multiply it by $d(\nu t)$ and average over a period (from 0 to 2π); and (*e*) separate this averaged polynomial into real and imaginary parts and find roots a, ν of the resultant pair of real equations. Then one can study whether each solution (a_k , v_k) is stable or unstable. Stern[36] has developed piece-wise linear network analysis and synthesis with the view of bridging the gap between the work of Wiener[3], Zadeh[37], and others. A systematic formulation of the problem leads to a suitable symbolism and an algebra of inequalities[38]. This formulation helps to broaden the scope of diode network synthesis in particular and of piece-wise linear synthesis in general. Booton's quasi-linearization method[39] for analysis of non-linear systems with random inputs gives the statistical characteristics of the response in terms of its autocoorelation and the cross-correlation between the input and the response.

THE PHASE-SPACE METHOD AND RELATED CONCEPTS

The phase-space method as a general formulation of the physical system

A phase space of $2n$ dimensions is in general associated with the behaviour of the dynamical system. By choosing a system of coordinates we do not necessarily have to solve the problem graphically. However, the usual association of the 'isocline method' with the solution of a phase-plane equation suggests a close relationship between the phase portrait and the analytic solution.

In discussing Chen's paper[34], Kalman said:

It should be strongly emphasized here that the phase-plane, or rather phase space, is not just a name for a clever method directed toward solving particular problems, but it is the most fundamental concept of the entire theory of ordinary differential equations (linear or non-linear, time-dependent or time-invariant) at the present time. This is because the phase-space is a way of representing (sometimes only in an abstract sense) all possible solutions of a differential equation……

Method of simultaneous phase space equations

Ku[26] developed the method of simultaneous phase-space equations (see section 10-6, reference 38, p. 234). For a system of three degrees of freedom, the phase-space is characterized by six coordinates-three positional coordinates q_i^1 and three velocity corrdinates q_i'. The simultaneous phase-space equations are of the form:

$$\frac{dq_i'}{dq_i} = \frac{q_i''}{q_i'} \tag{1}$$

where q_i'' denotes the second derivative of q_i with respect to time. The method of simultaneous phase-space equations is highly powerful[28,29] in the analysis of feedback control systems with non-linearity either in the forward branch or in the feed-back branch or in both. In a non-linear control system shown in *Figure* 1 (see section 12-9, reference 30, p.

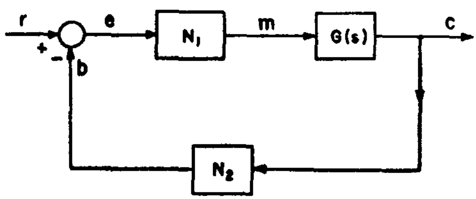

Fig. 1. Block diagram of a non-linear control system.

303), the two variables are e and c, corresponding to the positional coordinates q_1 and q_2. The other two coordinates are e' and c' corresponding to q_1' and q_2'. The simultaneous solution of equation 1 calls for numerical techniques or the help of computers.

Phase-space approach to the design of saturating servomechanisms

Hopkin[40] used the phase-space approach in the compensatin of saturating servomechanisms. An anticipator was developed to control the application and reversal of the manipulated variable. Recently Hopkin and Iwama[41] reported about the design of a predictor-type air-frame controller by phase-space analysis. The predictor was set up to handle the case where the controlled system is of third order with one highly oscillatory mode. A control system of the optimum relay type was obtained. Kalman[42,43] made a phase-plane analysis of auto-matic control systems with non-linear gain elements and then developed a method for the design of second and higher order saturating servomechanisms. His method is based on linear transformations in the phase space, corresponding with the partial-fraction expansion of transfer functions to separate natural frequencies, and makes use of the root-locus method for the qualitative study of closed loop stability.

McDonald[44], Hopkin[40] and others used the phase-plane technique to optimize performance of servomechanisms. Bogner and Kazda[45] extended McDonald's method to obtain unique switching criteria for contactor servomechanisms of order greater than two. Their analysis makes use of the particular phase space in which servo error is plotted against its higher derivatives. It is of interest to note that this phase space was used in the author's method of high-order phase space equation[23-25,27] (see section

10-5 of reference 30, p. 231). This phase space is a consequence of Cauchy's Theorem of Existence for the solution of a differential equation, as a differential equation of the *nth* order is equivalent to a system of n equations of the first order. Let the *three* phase-space coordinates be $u_1 = e$, $u_2 = e'$, *and* $u_3 = e''$. Bogner and Kazda made use of the *principal coordinate* phase-space, where the new coordinates are: $v_1 = u_1 - u_3$, $v_2 = u_1 + u_3$ and $v_3 = u_2$. Rose[46] used a more complicated switching rule. Chang[47] investigated the optimum open-circuit switching criteria for a high-order contactor servo. The optimum switching theory was further generalized by Hung and Chang[48].

NEW ANALYTIC TECHNIQUES AND NEW TRANSFORMS

Reversion method and iteration method

Pipes, reversion method[49] is based on the algebraic procedure used in reverting power series. It can be compared with the method of Picard. Whereas this method was used to analyse a series circuit with a non-linear inductor, Pipes introduced the minimum-mean-square-error method in the analysis of a series circuit with a non-linear capacitor[50]. It may be noted that the author used the acceleration-plane method to analyse a series circuit with both non-linear inductance and non-linear canacitance[51]. Other methods were discussed by Pipes[52-54]. An iterative method for non-linear vibrations was given by Roberson.[55]

Number series transformation method

The number series transformation method was introduced by Tustin in 1947[56]. Madwed[57-58] made further developments and showed that it is possible to take the Fourier and Laplace transforms of the number series transformations of functions.

Harmonic analysis and periodic solutions

Following the principle of *harmonic balance*, Lewis[59] suggested the use of polynomial functions for carrying out the harmonic analysis. In discussing this paper, Stour remarked: 'In addition to the suggested applications to nonlinear differential equations, modulation and distortion, it should be useful for the calculation of "describing functions" employed in non-linear feed-back system analysis,' Gillies[60-61] used the power series in the study of non-linear circuits and discussed the periodic solution of a modified Van der Pol equation in which there is a relatively large term kx in the damping coefficient besides the presence of a forcing function.

Step-by-step method

We shall mention as examples the methods developed by Stout[62], Naumov[63] and Dietz[64]. Referring *Figure* 2, where $y(t)$ is a non-linear function of $x(t)$ and can be expressed as $y = N(x)$, we have

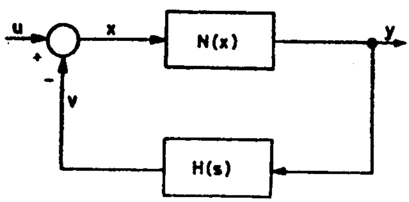

Fig.2. Block diagram of a system with one non-linear element.

$$x(t) = u(t) - v(t) = u(t) - \int_0^t h(t-\tau)N[x(\tau)]d\tau \qquad (2)$$