



国防科学技术大学

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# 声子晶体局域共振带隙机理 及减振特性研究

王 刚 著

国防科技大学出版社

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## 工学博士学位论文

# 声子晶体局域共振带隙 机理及减振特性研究

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# 2007—2008 年国防科技大学 全国优秀博士学位论文及 全国优秀博士学位论文提名论文

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2008 年三篇全国优秀博士学位论文：

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# **Research on the Mechanism and the Vibration Attenuation Characteristic of Locally Resonant Band Gap in Phononic Crystals**

Candidate: **Wang Gang**  
Supervisor: **Prof. Wen Xi-sen**

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# 摘要

声子晶体是具有弹性波带隙的周期性弹性材料或结构,声子晶体的局域共振带隙机理,使较小结构尺寸的声子晶体同样可以具有低频弹性波带隙,这在低频减振降噪方面具有重要的应用前景。本文围绕声子晶体局域共振带隙机理及减振降噪应用研究所迫切需要解决的有关理论和技术问题,通过理论分析、有限元仿真和振动实验研究相结合的方法,对声子晶体的局域共振带隙机理展开了系统深入的研究,对声子晶体局域共振带隙在减振领域的应用进行了有益的探索,主要研究内容包括:

(1)将一维声子晶体带结构计算的集中质量法推广至二维和三维声子晶体。与传统算法相比,集中质量法具有更加广泛的应用场合,算法收敛性好,这为揭示声子晶体的局域共振带隙机理提供了有力工具。

(2)局域共振带隙不仅存在于三组元声子晶体结构中,理论和实验均证实二维二组元声子晶体中同样存在局域共振带隙,深化了以往对局域共振带隙的认识。

(3)通过深入研究声子晶体的局域共振带隙机理,发现并非局域振子的所有共振模式都能导致局域共振带隙的产生。研究表明,声子晶体局域共振带隙产生的关键因素是:声子晶体中局域振子的振动必须能够与基体中长波行波发生相互耦合作用。

(4)声子晶体局域共振带隙截止频率所对应的振动模式为振子与基体的反相共振模式。通过建立相应的简化模型,以及完善具有局域共振带隙的二维三组元和三维三组元声子晶体的简化模型的参数计算方法,更加准确地、直观地揭示了声子晶体局域共振带隙起止频率的影响因素,为局域共振带隙的简化设计提供有效方法。

(5)声子晶体局域共振带隙有效衰减的决定因素分别为振子/基体间的等效刚度比和等效质量比,而其中等效刚度比是关键因素。

(6)将声子晶体的局域共振带隙机理应用于梁板类结构设计中,提出利用局域共振带隙特性抑制梁板类结构弯曲振动传播的思想。构造了相应的梁板类结构声子晶体,

通过理论分析、有限元仿真和振动实验研究了其弯曲弹性波局域共振带隙的减振特性。

总之,本文在低频、小尺寸减振降噪应用需求的牵引下,通过理论分析、有限元仿真和振动实验研究,深化了以往对局域共振带隙的认识,回答了局域共振带隙产生的关键因素、带隙频率及带隙有效衰减的影响因素等制约其低频减振降噪应用的相关理论和技术问题。本文的研究成果,对于推动声子晶体这一新概念和新原理的减振降噪应用具有重要的理论意义和工程价值。

**关键词:** 声子晶体;弹性波带隙;局域共振;集中质量;减振;梁板结构

# ABSTRACT

Phononic crystals are the periodic elastic materials or structures with elastic wave band gaps, which was proposed twelve years ago. The locally resonant band gap mechanism of phononic crystals has a much shorter history of only five years. The new mechanism can exhibit low frequency band gaps with structures of small size. Therefore, it has many important potential applications in the decreasing of low-frequency vibration and noise. In this thesis, the scientific problems according to the mechanism of locally resonant band gap are studied with theoretical and experimental methods deeply and systematically, and meaningful attempts are done in the application of locally resonant band gaps on vibration attenuation. Our research works include:

(1) The lumped-mass method in the calculation of band structure of one-dimensional phononic crystals is extended into two and three dimensions. It has been concluded that, by comparing with other algorithms, the applicability of the lumped-mass method is wider, and the convergence of it is better. Thus a powerful facility is provided for the research of band gap mechanism of the phononic crystals.

(2) The existence of the locally resonant band gap in the two-dimensional binary phononic crystals has been proposed and proved theoretically and experimentally. Thus the original view that the locally resonant band gap exists only in ternary systems is changed.

(3) The key factor that influences the formation of a locally resonant band gap corresponding to a resonant mode in the phononic crystals is found and proved, which can be stated as: the interaction between the vibration of the localized oscillator in the resonant mode and the long-wavelength elastic wave in the hosting media must exist.

(4) The vibration mode corresponding to the upper edge of the locally resonant band gaps is proposed. Simple analog models of the two- and three-dimensional ternary phononic crystals with locally resonant band gap are improved. Such models can be used to describe the vibration modes on the edges of the lowest band gap and estimate the frequencies of it correctly. With the analog models, we can get the analytical expressions of the factors that influence the frequencies on the edges of the locally resonant band gap.

(5) With the analytical study on the locally resonant band gap of one-dimensional phononic

crystal, the equivalent stiffness ratio and mass ratio are discovered as the deciding factors of the attenuation in the band gap. It is also shown that the equivalent stiffness ratio is the key factor. As the two- and three- dimensional phononic crystals with locally resonant band gaps are all equivalent to corresponding one-dimensional models, this conclusion is also valid for them.

(6) The locally resonant band gap mechanism of phononic crystals is introduced in the design of beam and plate structures, where the band gap property is used to restrain the flexural vibration in them. Further theoretical and experimental studies proved the feasibility of it and illustrated the vibration attenuation properties of the locally resonant flexural wave band gap.

In summary, drawn by the application to the decreasing of low-frequency vibration and noise with small structures, the locally resonant band gap mechanism and its capability in vibration attenuation are studied therotically and experimentally in this thesis. The algorithm problems restricting the study on the locally resonant band gap of phononic crystal are solved. Several important therotical and technical questions about the formation, the frequency and the attenuation of locally resonant band gap that restricting the application of it in low-frequency vibration/noise attenuation are answered. These research results are meaningful for the application of the new concept and principles of the phononic crystals in the control of vibration and noise.

**Keyword:** Phononic crystals; Elastic wave band gap; Locally resonant; Lumped-mass; Vibration attenuation; Beam and plate structures

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