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Wenjing Ding

自激振动

理论、范例及研究方法

Self-Excited Vibration

Theory, Paradigms, and Research Methods

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With 228 figures

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内 容 简 介

本书试图揭示一切自激振动共同的形成机制，同时建立分析研究它的统一程序，从而形成这门横向分支学科的理论体系。全书共分 11 章，全面论述自激振动及其系统的本质特征；介绍分析研究自激振动数学模型应用的各种数学方法；介绍工程中典型的和重要的自激振动——从建立数学模型开始，通过分析研究，揭示其成因和影响因素，并指出有效的控制方法；通过归纳分析许多具体自激振动现象的实践经验，总结出自激振动现象的共同的成因机制和统一的建模分析程序。

本书可作为力学教师和相关专业研究生的教学和科研参考书，也可作为各类工程（如航天航空、军工、机械、车辆、化工、土建）技术人员的研究参考书。

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Preface

In nature and engineering, there is a type of sustained periodic motion of dynamic systems not subjected to any outside alternating effect. It is caused by the interaction among the inside elements of the system, and is consequently named as self-excited vibration. This type of periodic motion is omnipresent. For example, the steady pulse in the heart-blood-vessel system of a human is a complicated self-excited vibration system. So, the continuation of life relies entirely on the sustained periodic motion. However, in engineering, only a few of the self-excited vibrations are corresponding to the normal working state of the system, in contrast, most of them always are the harmful disturbances against the normal working states of the system. Thus, quite often they are undesired. Whether or not they are desirable, a thorough understanding of their motion regulation is beneficial in optimizing the design of systems. In the first half of the last century, many self-excited vibrations in engineering have been studied and complied into the well-known monographs. For example, in 'Mechanical Vibrations' edited by J. P. Den Hartog and 'Nonlinear Vibrations in Mechanical and Electrical System' edited by J. J. Stoker, the materials in these monographs manifest main results that can be acquired at that time. The theory of self-excited vibrations has experienced significant progress in the last decades. In particular, nonlinear dynamics, founded by H. Poincare in the 19th century, has greatly advanced with the rapid development of the calculation technique and the important contributions of many scientists; the Hopf bifurcation theory is extensively used to analyze various self-excited vibrations, and theoretical analyses became more rigorous and more comprehensive consequently. However, in engineering, the seat of self-excited vibrations is ever-changing, whose governing equations generally belong to different types of differential equations. Therefore, the results of the analyses appear to be dispersive and their common the features of the self-excited vibrations have not been fully extracted. This book is devoted to integrating the recent development in the theory of self-excited vibration with modern dynamics and control theory. The main goal is to advance the research about the common behavior and their features in various engineering fields.

The first chapter briefly explains the main features of self-excited vibrations and the remainder of the book is divided into three parts: the first, which consists of chapters 2 to 5, describes a variety of qualitative and quantitative methods; and the second is concerned with the detailed analyses of several types of self-excited vibration in various engineering fields. The analysis results may be used to improve the effectiveness of the practical design. Furthermore, a cross-fertilization of ideas will evolve from the different self-excited vibration phenomena so that the common excitation mechanism in different self-excited vibration systems and the effective analysis techniques are summarized; the last part, namely, chapter 11, provides a workable modeling routine for analyzing the unclear self-excited vibration phenomenon. In combination with the first part, this part constitutes a set of research techniques to study all self-excited vibrations in mechanical systems.

The author also wishes to express his appreciation to Professor Shouwen Yu for his recommendation on this book's creation. In addition, the author wishes to appreciate the valuable suggestions from Professor Haiyan Hu. Special thanks are due to Review Ready Co. for patiently reviewing an early version and making valuable suggestions. The author would like to thank Dr. Shichao Fan for producing the computer-generated plots and equations. Last but not least, the author thanks his daughter, Jinghua Ding, for extensive review and conscientious typing of the manuscript.

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Contents

Chapter 1 Introduction

Abstract: This chapter discusses three subjects associated with self-excited vibration. The main contents are divided into four sections: the first is devoted to explain several features of self-excited vibration by comparing it with other sustained periodic motions; the second indicates the mutual conversion between forced vibration and self-excited vibration; the third surveys excitation mechanisms leading to self-excited vibration; and the fourth is concerned with a classification of self-excited vibration systems, which depends on the type of the differential equations of the motions. In addition, the outline of this book is given at the end of this chapter.

Keywords: *periodic motions, self-excited vibration, nonlinear systems, excitation mechanisms, clasification*

1.1 Main Features of Self-Excited Vibration

In nature and engineering, there are four types of sustained periodic motions, namely, the natural vibration in conservative systems, the forced vibration caused by the periodic excitation, the parametric vibration in the nonautonomous system with periodic parameters, and the self-excited vibration in autonomous systems.

1.1.1 Natural Vibration in Conservative Systems

The natural vibration in conservative systems takes place in the absence of external excitation. It is well known that the sum of the kinetic energy and the potential energy in conservative systems remains constant, but the energy interconversion leads to periodic variation of the generalized coordinates and the generlized velocities of the system.

A simple pendulum is a paradigm of conservative systems, and we can find general features of the natural vibration in nonlinear conservative systems by analyzing its sway motion with large amplitude. According to the conservation theorem of energy[1], we have

$$T + U = E, \tag{1.1}$$

where T is the kinetic energy of the simple pendulum, U is the potential energy,

E is the total energy, which is an arbitrary constant. Obviously, the kinetic and potential energies of the simple pendulum can be expressed as the following,

$$T = \frac{1}{2}ml^2\dot{\theta}^2, \quad U = mgl(1-\cos\theta),$$

where m is the mass of the simple pendulum, g is the gravitational acceleration, l is the length of the pendulum, θ is the deviation angle, and $\dot{\theta}$ is the angular velocity. If let $\theta = \theta_0$, at the highest point of the motion, then,

$$T(\theta_0) = 0, \quad U(\theta_0) = E = mgl(1-\cos\theta_0).$$

Using the trigonometric identity, we obtain

$$U = 2mgl\sin^2(\theta/2)$$

and

$$E = 2mgl\sin^2(\theta_0/2).$$

Expressing the kinetic energy as the difference between the total energy and the potential energy yields

$$\frac{1}{2}ml^2\dot{\theta}^2 = 2mgl[\sin^2(\theta_0/2) - \sin^2(\theta/2)]$$

or

$$\dot{\theta} = 2\sqrt{\frac{g}{l}}[\sin^2(\theta_0/2) - \sin^2(\theta/2)]^{\frac{1}{2}}, \tag{1.2}$$

from which we have

$$\mathrm{d}t = \frac{1}{2}\sqrt{\frac{l}{g}}[\sin^2(\theta_0/2) - \sin^2(\theta/2)]^{-\frac{1}{2}}\mathrm{d}\theta.$$

This equation may be integrated to get the analytical expression of the period τ. Actually, the motion is symmetrical, and the integral over θ from $\theta = 0$ to $\theta = \theta_0$ yields $\tau/4$. Hence,

$$\tau = 2\sqrt{\frac{l}{g}}\int_0^{\theta_0}[\sin^2(\theta_0/2) - \sin^2(\theta/2)]^{-\frac{1}{2}}\mathrm{d}\theta,$$

which is an elliptic integral of the first kind. This may be seen more clearly by making the substitutions

$$z = \frac{\sin(\theta/2)}{\sin(\theta_0/2)}, \quad k = \sin(\theta_0/2).$$

This yields

$$dz = \frac{\cos(\theta/2)}{2\sin(\theta_0/2)}d\theta = \frac{(1-k^2z^2)^{\frac{1}{2}}}{2k}d\theta,$$

from which we have

$$\tau = 4\sqrt{\frac{l}{g}}\int_0^1[(1-z^2)(1-k^2z^2)^{-\frac{1}{2}}]dz. \tag{1.3}$$

In order to get vibratory motion, let $\theta_0 < \pi$, or equivalently, let $\sin(\theta_0/2) = k < 1$. In this case, the integral in the Eq. (1.3) can be evaluated by expanding $(1-k^2z^2)^{-\frac{1}{2}}$ in a power series[1]:

$$(1-k^2z^2)^{-\frac{1}{2}} = 1 + \frac{k^2z^2}{2} + \frac{3k^4z^4}{8} + \cdots. \tag{1.4}$$

Then, the expression for the period becomes

$$\tau = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{k^2}{4} + \frac{9k^4}{64} + \cdots\right). \tag{1.5}$$

If k is not too large, i.e., θ_0 is less than $\pi/2$, the expansion converges rapidly, then, $k \cong \left(\frac{\theta_0}{2} - \frac{\theta_0^3}{48}\right)$, and the result is corrected to the fourth order, which is

$$\tau \cong 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{1}{3072}\theta_0^4 - \cdots\right). \tag{1.6}$$

Therefore, we see that although the simple pendulum is not isochronous and the solution of the differential Eq. (1.2), which describes the motion of the simple pendulum, cannot be a harmonic function, it is nearly so for small amplitude of vibration.

1.1.2 Forced Vibration under Periodic Excitations

Vibrations that take place as a result of external excitation are called forced vibrations. When the excitation is a periodic oscillatory, the system is forced to vibrate at the excitation frequency. If the frequency of the excitation coincides with one of the natural frequencies of the system, the condition of resonance is encountered. In the simplest case of forced vibration, an external driving force varying harmonically with time is applied to a linear system with single degree of freedom. The differential equation of such a motion in the form of a mass-damper-spring model is[2]:

$$m\ddot{x} + c\dot{x} + kx = mA\cos\Omega t,$$

where m is the mass, c is the coefficient of viscous damping, k is the spring stiffness, A is the magnitude parameter, and Ω is the frequency of the harmonic excitation.

The previous equation can be written as follows:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\Omega t\,, \tag{1.7}$$

where $\beta = c/2m$ is the damping parameter and $\omega_0 = \sqrt{k/m}$ is the natural frequency in the absence of damping.

The solution for Eq. (1.7) consists of two parts: a complementary function $x_c(t)$, which is the solution of Eq. (1.7) with the right-hand side equal to zero, as the homogeneous differential equation, plus a particular solution $x_p(t)$, which reproduces the right-hand side. The complementary solution is clearly the general solution of the homogeneous differential equation, and it can be written as follows[2]:

$$x_c(t) = \mathrm{e}^{-\beta t}\left[A_1\exp\left(\sqrt{\beta^2 - \omega_0^2}\,t\right) + A_2\exp\left(\sqrt{\beta^2 - \omega_0^2}\,t\right)\right].$$

For the particular solution, we try

$$x_p(t) = D\cos(\Omega t - \delta)\,.$$

Substituting $x_p(t)$ in Eq. (1.7) and expanding $\cos(\Omega t - \delta)$ and $\sin(\Omega t - \delta)$, we obtain

$$\left\{A - D\left[(\omega_0^2 - \Omega^2)\cos\delta + 2\Omega\beta\sin\delta\right]\right\}\cos\Omega t$$
$$-\left\{D\left[(\omega_0^2 - \Omega^2)\sin\delta - 2\Omega\beta\cos\delta\right]\right\}\sin\Omega t = 0$$

Since $\sin\Omega t$ and $\cos\Omega t$ are linearly independent functions, this equation can be satisfied in general only if the coefficient of each term vanishes identically. From the $\sin\Omega t$ term, we have

$$\tan\delta = \frac{2\Omega\beta}{\omega_0^2 - \Omega^2},$$

so that

$$\sin\delta = \frac{2\Omega\beta}{\sqrt{(\omega_0^2 - \Omega^2)^2 + 4\Omega^2\beta^2}}, \quad \cos\delta = \frac{\omega_0^2 - \Omega^2}{\sqrt{(\omega_0^2 - \Omega^2)^2 + 4\Omega^2\beta^2}}.$$