北京邮电大学高等数学双语教学组◎编

高等数学（上）

Advanced Mathematics (I)

北京邮电大学出版社
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内 容 提 要

本书是根据国家教育部非数学专业数学基础课教学指导分委员会制定的工科类本科数学基础课程教学基本要求编写的全英文教材，全书分为上、下两册，此为上册，主要包括函数与极限，一元函数微积分及其应用和无穷级数三部分。本书对基本概念的叙述清晰准确，对基本理论的论述简明易懂，例题习题的选配典型多样，强调基本运算能力的培养及理论的实际应用。本书可作为高等理工科院校非数学类专业本科生的教材，也可供其他专业选用和社会读者阅读。

The aim of this book is to meet the requirement of bilingual teaching of advanced mathematics. This book is divided into two volumes, and the first volume contains functions and limits, calculus of functions of a single variable and infinite series. The selection of the contents is in accordance with the fundamental requirements of teaching issued by the Ministry of Education of China and based on the property of our university. This book may be used as a textbook for undergraduate students in the science and engineering schools whose majors are not mathematics, and may also be suitable to the readers at the same level.

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前　　言

关于高等数学

高等数学(微积分)是一门研究运动和变化的数学,产生于16至17世纪,受当时科学家们在研究力学问题时对相关数学的需要而逐渐发展起来的.高等数学中微分处理的目的是求已知函数的变化率的问题,例如,曲线的斜率,运动物体的速度和加速度等;而积分处理的目的则是在当函数的变化率已知时,如何求原函数的问题,例如,通过物体当前的位置和作用在该物体上的力来预测该物体的未来位置,计算不规则平面区域的面积,计算曲线的长度等。现在,高等数学已经成为高等院校学生尤其是工科学生最重要的数学基础课程之一,学生在这门课程上学习情况的好坏对其后续课程能否顺利学习有着至关重要的影响。

关于本书

虽然国内一些出版社已经影印出版了许多种国外编写的"高等数学"的优秀英文教材,但这些教材的内容都过于简单,不符合教育部给出的"高等数学"本科教学内容的要求,因此并不十分适合我国现有高等院校尤其是重点理工科高等院校的教学实际情况。国内编写的双语"高等数学"教材几乎没有,这些因素促使我们下决心编写一本适合我国重点理工科高等院校特别是我校特色专业学生学习的英文"高等数学"教材,以满足我校乃至全国理工科高等院校对"高等数学"课程的双语教学要求。

本书的所有作者都在我校主讲了多年的双语"高等数学"课程,获得了丰富的教学经验,了解学生在学习双语"高等数学"课程中所面临的问题与困难。与此同时,主要成员都有出国留学或学术访问经历,英文水平良好。本书函数、空间解析几何及微分部分由张文博、王学丽和朱萍三位副教授编写,级数、微分方程及积分部分则由艾文宝教授和袁健华副教授编写,全书由孙洪祥教授审阅校对。本书对高等数学中首次出现的数学术语用英文和中文同时标出,以方便学生学习及在国内教研。此外,本书在内容编排和讲解上适当吸收了欧美国家微积分教材的一些优点。由于作者水平有限,加上时间匆忙,书中出现一些错误在所难免,欢迎感谢读者通过邮箱(jianhuayuan@bupt.edu.cn)指出错误,以便我们及时纠正。

致谢

本书在编写过程中得到北京邮电大学、北京邮电大学理学院和国际学院的教改项目资金支持，作者在此表示衷心感谢。同时也借此机会，感谢所有在本书写作过程中支持和帮助过我们的同事和朋友。

致学生的话

高等数学的学习没有捷径可走，它需要你们付出艰苦的努力。只要你能勤奋学习并持之以恒，定能取得成功。希望你们能喜欢这本书，并预祝你取得成功！

Preface

What is advanced mathematics?

Advanced mathematics that we refer to contains mainly calculus. Calculus is the mathematics of motion and change. It was first invented to meet the mathematical needs of the scientists of the sixteenth and seventeenth centuries, needs that were mainly mechanical in nature. Differential calculus Deals with the problem of calculating rates of change. It enables people to define slopes of curves, to calculate velocities and accelerations of moving bodies, etc.. Integral calculus Deals with the problem of determining a function from information about its rate of change. It enables people to calculate the future location of a body from its present position and a knowledge of the forces acting on it, to find the areas of irregular regions in the plane, to measure the lengths of curves, and so on. Now, advanced mathematics becomes one of the most important courses of the college students in natural science and engineering.

About this book

Although several excellent textbooks in English language concerned with advanced mathematics that came from abroad have been published in China, their contents do not amply meet the requirements presented by the Ministry of Education for advanced mathematics. So they do not amply accord with the teaching requirements of domestic universities. Furthermore, few textbooks in English language concerned with advanced mathematics, which are written by domestic authors, can be found in locality. All these factors inspire us to write out this book, in order to satisfies the increasing bilingual-teaching demand of universities in China.

All the authors have been teaching this course by bilingual languages for many years. They have much experience in both course-teaching and bilingual-teaching. They know the obstacles encountered by Chinese students in learning this course in English. Moreover, all the authors had experience visiting abroad. The contents of functions, space analytic geometry and differential calculus were jointly written by Associate Professors Wenbo Zhang, Xueli Wang and Ping Zhu, while the contents of series, differential equations and integration calculus jointly by Professor Wenbao Ai and Associate Professor Jianhua Yuan. The whole book was proofread by Professor Hongxiang Sun. Because of the careful checking and proofing by us, the authors believe this book to be almost error-free. For any errors remaining, the authors would be grateful if they were sent to: jhyuan_bupt@yahoo. com. cn.

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The authors wish to thank all the person who have been involved in producing the book. Our special thanks go to the Science School and the International School of BUPT for their financial support on producing the book by teaching-reform grants.

To the students

Learning advanced mathematics requires a lot of hard work and effort on your part. No one else can do this for you, and there are no shortcuts. However, if you work consistently and diligently through this book you will succeed. Enjoy this book, and good luck for you!

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Chapter 0

Preliminary Knowledge

Overview

In this chapter we will introduce some most important concepts which will be widely used in the following context.

Polar coordinate system plays an important role while denoting points on a plane, describing functions and etc., just as the Cartesian coordinate does.

0.1 Polar Coordinate System

In mathematics, the **polar coordinate system**[极坐标系]is a two-dimensional coordinate system in which each point on a plane is determined by an angle and a distance. The polar coordinate system is especially useful in situations where the relationship between two points is most easily expressed in terms of angles and distance; in the more familiar Cartesian or rectangular coordinate system, such a relationship can only be found through trigonometric formulae.

As the coordinate system is two-dimensional, each point is determined by two polar coordinates: the radial coordinate and the angular coordinate see Figure 0.1.1. The radial coordinate (usually denoted as r or ρ) denotes the point's distance from a central point known as the **pole** [极点](equivalent to the origin in the Cartesian system). The angular coordinate (also known as the **polar angle** [极角]or the **azimuth angle** [方位角], and usually denoted by θ or t) denotes the positive or **anticlockwise** [逆时针](counterclockwise) angle required to reach the point from the 0° ray or **polar axis** [极轴]**(which is equivalent to the positive x-axis in the Cartesian coordinate plane).**

0.1.1 Plotting Points with Polar Coordinates

Each point in the polar coordinate system can be described with the two polar coordinates, which are usually called r (or ρ, the radial coordinate) and θ (*the angular coordinate*, *polar angle*, or *azimuth angle*, sometimes represented as φ or t). The r coordinate represents the radial distance from the pole, and the θ coordinate represents the anticlockwise (counterclockwise) angle from the 0° ray (sometimes called the polar axis), which are known as the positive *x-axis* on the Cartesian coordinate plane[1] (See Figure 0.1.2).

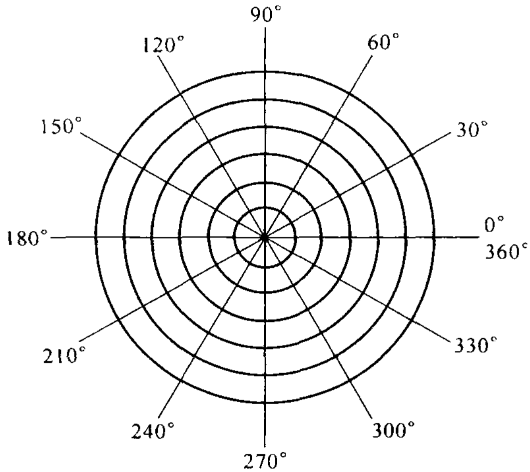

Figure 0.1.1

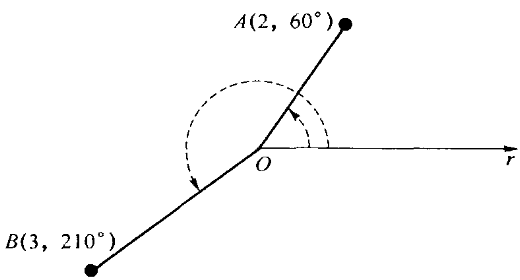

Figure 0.1.2

0.1.2 Converting between Polar and Cartesian Coordinates

The two polar coordinates r and θ can be converted to the Cartesian coordinates x and y by using the trigonometric functions sine and cosine:

$$x=r\cos\theta,$$
$$y=r\sin\theta.$$

while the two Cartesian coordinates x and y can be converted to polar coordinate r by $r=\sqrt{x^2+y^2}$ (by a simple application of the *Pythagorean theorem*).

To determine the angular coordinate θ, the following two ideas must be considered:

- For $r=0$, θ can be set to any real value;
- For $r\neq 0$, to get a unique representation for θ, it must be limited to an interval of size 2π. Conventional choices for such an interval are $[0,2\pi)$ and $(-\pi,\pi]$.

To obtain θ in the interval $[0,2\pi)$, the following may be used (arctan denotes the inverse of the tangent function):

$$\theta=\begin{cases}\arctan\left(\frac{y}{x}\right), & \text{if } x>0 \text{ and } y\geqslant 0\\ \arctan\left(\frac{y}{x}\right)+2\pi, & \text{if } x>0 \text{ and } y<0\\ \arctan\left(\frac{y}{x}\right)+\pi, & \text{if } x<0\\ \frac{\pi}{2}, & \text{if } x=0 \text{ and } y>0\\ \frac{3\pi}{2}, & \text{if } x=0 \text{ and } y<0\end{cases}$$

The readers can also give out the formulas to obtain θ in the interval $(-\pi,\pi]$.

Example 0.1.1 (Circle with center at $(0,0)$ and radius a). The usual way we express a circle with center at $(0,0)$ and radius $a>0$ in Cartesian coordinates is the set of all points which satisfies the following equation

$$\sqrt{x^2+y^2}=a \text{ or } x^2+y^2=a^2 \quad (a>0).$$

This expression can be very simple while we use polar coordinates. In fact, since the circle is a set contains all points which have the same distance to the point $(0,0)$, if we take the origin of the Cartesian coordinated as our pole and x-axis as our polar axis, then the circle can be expressed as

$$r(\theta)=a \ (a>0).$$

The general equation of a circle with a center (r_0,θ_0) and radius $a>0$ is

$$r^2-2rr_0\cos(\theta-\theta_0)+r_0^2=a^2.$$

Example 0.1.2 (Line). *Radial* lines (those running through the pole) are represented by the equation

$$\theta=\theta_0,$$

where θ_0 is the angle of elevation of the line; that is, $\theta_0 = \arctan m$ where m is the slope of the line in the Cartesian coordinate system. The none-radial line that crosses the radial line $\theta = \theta_0$ perpendicularly at the point (r_0, θ_0) has the equation

$$r(\theta) = r_0 \sec(\theta - \theta_0).$$

Example 0.1.3 (Polar rose). A polar rose is a famous mathematical curve that looks like a petalled flower, and that can be expressed as a simple polar equation,

$$r(\theta) = a \cos(k\theta + \theta_0)$$

for any constant θ_0 (including 0). If k is an integer, these equations will produce a k-petalled rose if k is odd, or a $2k$-petalled rose if k is even. If k is rational but not an integer, a rose-like shape may form but with overlapping petals. Note that these equations never define a rose with 2, 6, 10, 14, etc. petals. The variable a represents the length of the petals of the rose (See Figure 0.1.3).

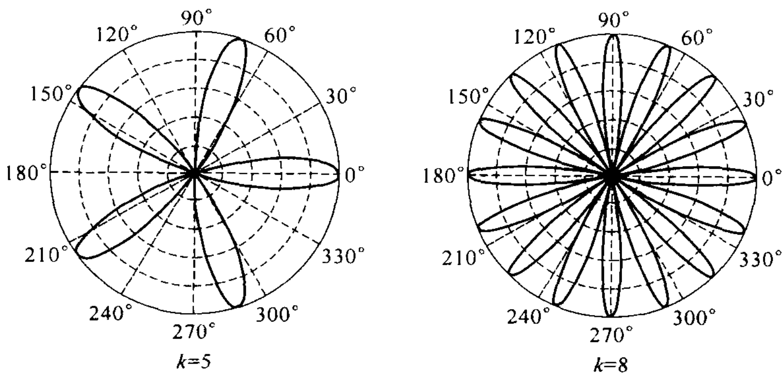

Figure 0.1.3

▶**Reading Material:** *History of polar system coordinate*

The concepts of angle and radius were already used by ancient people of the 1st millennium BCE [①]. The astronomer Hipparchus (190-120 BCE) created a table of chord functions giving the length of the chord for each angle, and there are references to his using polar coordinates in establishing stellar positions. On *Spirals*, Archimedes described the Archimedean spiral, a function whose radius depends on the angle. The Greek work, however, did not extend to a full coordinate system.

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① The *Common Era*, also known as the *Current Era* or the *Christian Era*, abbreviated *CE*, is the period of time beginning with year 1 of the Gregorian calendar. Earlier years are abbreviated *BCE*, and described as "before the Common/Current/Christian Era". The abbreviations are sometimes written with small capital letters, or with periods (e.g., "bce" or "C. E.").

There are various accounts of the introduction of polar coordinates as part of a formal coordinate system. The full history of the subject was described in Harvard professor Julian Lowell *Coolidge's Origin of Polar Coordinates*[3]. Grégoire de Saint-Vincent and Bonaventura Cavalieri independently introduced the concepts in the mid-seventeenth century. Saint-Vincent wrote about them privately in 1625 and published his work in 1647, while Cavalieri published his work in 1635 with a corrected version appearing in 1653. Cavalieri first used polar coordinates to solve a problem relating to the area within an Archimedean spiral. Blaise Pascal subsequently used polar coordinates to calculate the length of parabolic arcs.

In *Method of Fluxions* (written in 1671, published in 1736), Sir Isaac Newton examined the transformations between polar coordinates, which he referred to as the "Seventh Manner; For Spirals", and nine other coordinate systems[4]. In the journal *Acta Eruditorum* (1691), Jakob Bernoulli used a system with a point on a line, called the *pole* and *polar axis* respectively. Coordinates were specified by the distance from the pole and the angle from the polar axis. Bernoulli's work extended to finding the radius of curvature of curves expressed in these coordinates.

The actual term *polar coordinates* has been attributed to Gregorio Fontana and was used by 18th-century Italian writers. The term appeared in English in George Peacock's 1816 translation of Lacroix's *Differential and Integral Calculus*[5]. Alexis Clairaut was the first to think of polar coordinates in three dimensions, and Leonhard Euler was the first to actually develop them[3]. ◀

▶**Reading Material**: *The applications of polar coordinate system*

Polar coordinates are two-dimensional and thus they can be used only in the place where point positions lie on a single two-dimensional plane. They are most appropriate in any context where the phenomenon being considered is inherently tied to direction and length from a center point. For instance, the examples above show how elementary polar equations suffce to define curves — such as the Polar rose — whose equation in the Cartesian coordinate system would be much more intricate. Moreover, many physical systems — such as those concerned with bodies moving around a central point or with phenomena originating from a central point — are simpler and more intuitive to model using polar coordinates. The initial motivation for the introduction of the polar system was the study of circular and orbital motion.

Position and navigation

Polar coordinates are used often in navigation, as the destination or direction of travel can be given as an angle and distance from the object being considered. For instance, air-