理工科核心课程双语规划教材

经典力学讲义

Lectures on Classic Mechanics

钟学富　编著

中国科学技术大学出版社

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内 容 简 介

本书精选经典力学的基本内容，按课堂教学顺序组织成44讲，内容包括坐标系、质点力学、分析力学、有心力场中的运动、非惯性系中的运动、非线性振动、波的传播、刚体力学和流体力学等。各讲内容均衡、简练，公式推导详细，附思考问题，突出重点，减轻阅读困难。

本书重点解决课堂教学的"程序化"（将科学体系变为讲授的时序）问题，可直接用作教师教案，组织课堂讲授；在适当增加内容之后，本书可作为普通大学本科或师范院校物理系经典力学课程的教材；本书还可作为参考阅读资料，帮助提高科技英语水平。

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钟学富，1939年生，四川新都人，毕业于四川大学物理系理论物理专门化。曾任中国科学院半导体研究所副研究员，硕士生导师。1987年赴美为访问学者，在密苏里大学堪萨斯城分校物理系从事固体理论研究。现居美国。

单独与合作发表中英文物理论文约30篇。主要成果包括确立半导体中一类光转化杂质模型，经实验证实并获中国科学院科技成果二等奖；首次提出在晶体场计算中考虑传导电子贡献，此概念被用于修改穆斯堡尔效应中的电场梯度公式。另外在《中国社会科学》、《哲学研究》、《光明日报》、《自然辩证法研究》等刊物发表涉及信息论和物理学的哲学问题的论文约10篇。近年来陆续出版《物理社会学》、《社会系统》、《休闲哲学》等专著，尝试将自组织及相关理论应用于社会研究，发展社会科学的演绎理论。

Preface 序 言

《经典力学讲义》和《电动力学讲义》两本教材源于1992—1998年间本人在密苏里大学堪萨斯城分校(UMKC)物理系任教时的讲稿，程度与国内物理本科的四大力学相当，但我同时还接收邻近专业如化学系的硕士和博士生。由于美国教材大体属于“包罗万象”式，教科书都是厚厚的一本，作为参考书很有查询价值，但绝对不可能在有限的学时内逐节讲授，教授不得不另写教案，以组织课堂教学，不过这却带来一系列问题。首先是讲授(听讲)和教科书之间脱节，教授“跳着讲”，学生课后“跳着读”，影响思维的连贯性。尤其是，为了和教授的讲解衔接，学生必须认真记笔记，但记笔记其实有碍专心听讲：前一个要点还没记下来，后一个又来了。为了解决这些矛盾，本人便将原本仅供自己使用、字迹潦草的教案整理出来，作为讲义发给学生。学生不必笔记，顶多在上面加点批注，省下力气专心听讲，并随时提问。讲义的内容大体上是自相包容的，假如要求不高，听讲之后，只消把不多几页真正看明白，就算达到课程的基本要求，所以学生阅读的负担不重。尤其物理系的课程，几乎处处都要“推公式”，一般教科书反而非常简略。本人斟酌情况多写两行中间步骤，便能省去学生不少时间，这大受数学底子相对薄弱的美国学生欢迎。这些体恤学生的做法，源于自己当年做学生的经验，我的老师就给我们发讲义！本人的做法其实是传承中国早年的教学思想，并将其推广到美国的课堂。

这样的讲义对中国的物理教学有什么用处？首先对教师来说，现在的教科书大多是按章节编写的，这没错，科学是严谨的思想逻辑体系。然而别忘了，教学却是要将这个逻辑结构变为“时序结构”：课，是一堂一堂上的，先讲什么后讲什么，和排程序一模一样。所以我的讲义没有章节，只有第一、第二、……，直到第N讲。这个编排不一定高明，甚至未必合于各校的要求，但我希望教师不要忽略这种叙述方式，而且都要能拿出自己编排的高招，包括问题的切入方式。其次是学生，假如只是为了学英文，那么，每讲内容可能正是一餐的饭量，语法、词汇都是最普通和常见的。而物理系的学生，假如还能从内容叙述和相关公式的推导中得点启发，那就算是意外的收获了。

钟学富

2011年2月18日于美国堪萨斯城

Contents

Lecture 1　Introduction, Kinematics

1.1　Objective of mechanics — motion

Mechanics studies the motion of an object or a system of objects. By motion we mean an object changes its position with respect to another object or the objects in a system change their relative positions. The position in one dimensional case can be simply defined by the distance between the two objects. However, in case of more than one-dimension it includes two factors: distance and direction, which correspond to the concepts of length and angle, respectively.

Since the position is always relative, it is meaningless to say the position of a single object without specifying its spatial relation to another object, so the motion is relative too. There is no "absolute" motion or motion by a single object itself. Rest can be considered as the limiting case of motion — the rate of position change is zero.

Any motion happens and proceeds in space and time. Nothing can go beyond these "containers". In classic mechanics, space and time are independent and both are independent of the processes happened in them. However, Einstein's theory of relativity indicates that this is untrue, space and time are related to each other and in fact affected by the processes happened in them.

Mechanics studies how to describe the motion (kinematics) and why the motion is changed (dynamics). Force is understood as the cause or the reason that any object changes its motion or state. A special topic is called statics which studies the balance of forces, or how an object or a system of objects can stay rest or equilibrium under the influence of more than one forces.

1.2 System of reference

As the position is relative by nature, we must have a reference body to define the position or motion of an object. This is the system of reference. Since an object usually has different spatial relations with different objects, its motion looks different from different systems of reference. Some differences may even have their dynamical origin, however, we shall not discuss this problem in kinematics.

We need a coordinate system to describe the position of an object for a reference body. The most commonly used ones are rectangular, spherical or cylindrical coordinate system. All these are three dimensional. In two-dimensional case, we have plane polar coordinate system. The choice of coordinate system is rather arbitrary in kinematics. We prefer the system in which the motion looks simpler. For example, we choose a rectangular coordinate system for the motion on a straight line, or a polar coordinate system for a circular motion. The transformation from one coordinate system to another is a mathematical event, which will be discussed in the next lecture.

1.3 Position and trajectory

The simplest object is a particle; its dimension is small in comparison with its distance to other objects. In a rectangular coordinate system, the position of a particle is denoted by a vector:

$$\boldsymbol{r} = x_1 \boldsymbol{e}_1 + x_2 \boldsymbol{e}_2 + x_3 \boldsymbol{e}_3 \tag{1.1}$$

where $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ are the basic vectors of the coordinate system. They have a unit length and are normally perpendicular to each other. $x_i\,(i=1, 2, 3)$ are the coordinates of the particle. If the coordinates vary with time t, or x_i (and thus $\boldsymbol{r}$) are the functions of time t, then we have a change in the position of the particle or a motion. This will draw a trajectory (curve) in space:

$$\boldsymbol{r}(t) = x_1(t)\boldsymbol{e}_1 + x_2(t)\boldsymbol{e}_2 + x_3(t)\boldsymbol{e}_3 \tag{1.2}$$

All the information about the motion is included in this trajectory or the function

$\boldsymbol{r}(t)$. It is important to realize that all the functions $x_i(t)$ are continuous and differentiable as the space and time (and thus motion) are continuous by nature. The differential properties of $\boldsymbol{r}(t)$ represent the fundamental quantities of kinematics.

1.4 Velocity

The most fundamental quantity in kinematics is the velocity $\boldsymbol{v}$, which is the first order derivative of $\boldsymbol{r}(t)$ with respect to time t:

$$\boldsymbol{v}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r}(t) = \frac{\mathrm{d}}{\mathrm{d}t}x_1(t)\boldsymbol{e}_1 + \frac{\mathrm{d}}{\mathrm{d}t}x_2(t)\boldsymbol{e}_2 + \frac{\mathrm{d}}{\mathrm{d}t}x_3(t)\boldsymbol{e}_3 \tag{1.3}$$

or, if we write $\boldsymbol{v}$ in component form:

$$\boldsymbol{v}(t) = v_1(t)\boldsymbol{e}_1 + v_2(t)\boldsymbol{e}_2 + v_3(t)\boldsymbol{e}_3 \tag{1.4}$$

where

$$v_i(t) = \frac{\mathrm{d}x_i}{\mathrm{d}t} \quad (i = 1, 2, 3) \tag{1.5}$$

It is important to realize that the velocity $\boldsymbol{v}$ thus defined is belonging to an instant, not a time interval. For the latter we have only the average velocity, which is less powerful as it is not able to describe how the velocity varies in the time interval.

Obviously, velocity is a vector, as can be seen from Fig. 1.1. The direction of $\boldsymbol{v}$ is always along the tangent of the trajectory at each point. The magnitude of $\boldsymbol{v}$ is called the speed, which is calculated by the general rule for the norm of a vector:

$$v = |\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} \tag{1.6}$$

1.5 Acceleration

As the velocity $\boldsymbol{v}(t)$ is also a function of time t, its time rate can be defined as the acceleration $\boldsymbol{a}$:

$$\boldsymbol{a}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{v}(t) = \frac{\mathrm{d}}{\mathrm{d}t}v_1(t)\boldsymbol{e}_1 + \frac{\mathrm{d}}{\mathrm{d}t}v_2(t)\boldsymbol{e}_2 + \frac{\mathrm{d}}{\mathrm{d}t}v_3(t)\boldsymbol{e}_3 \tag{1.7}$$

which is the second order derivative of the trajectory $\boldsymbol{r}(t)$:

$$\boldsymbol{a}(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\boldsymbol{r}(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}x_1(t)\boldsymbol{e}_1 + \frac{\mathrm{d}^2}{\mathrm{d}t^2}x_2(t)\boldsymbol{e}_2 + \frac{\mathrm{d}^2}{\mathrm{d}t^2}x_3(t)\boldsymbol{e}_3 \quad (1.8)$$

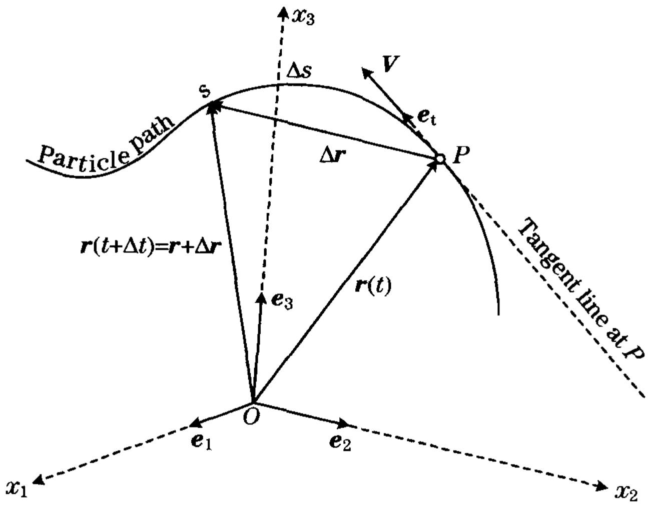

Fig. 1.1 The definition of velocity.

The magnitude of an acceleration can also be calculated by the general rule for the norm of a vector, i.e.,

$$a = |\boldsymbol{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} \quad (1.9)$$

where

$$a_i = \frac{\mathrm{d}v_i}{\mathrm{d}t} = \frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} \quad (i = 1, 2, 3) \quad (1.10)$$

However, the direction of acceleration needs some explanation. Since the velocity is always in the tangential direction of the trajectory, we can rewrite it as

$$\boldsymbol{v} = v\boldsymbol{e}_t \quad (1.11)$$

where v is the speed and $\boldsymbol{e}_t$ is the unit vector in the tangential direction. Note that in this case both factors (v and $\boldsymbol{e}_t$) may vary with time t. We then perform the differentiation on (1.11) to get the acceleration:

$$\boldsymbol{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\boldsymbol{e}_t + v\frac{\mathrm{d}\boldsymbol{e}_t}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}t}\boldsymbol{e}_t + v\frac{\mathrm{d}\boldsymbol{e}_t}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}t}\boldsymbol{e}_t + v^2\frac{\mathrm{d}\boldsymbol{e}_t}{\mathrm{d}s} = \frac{\mathrm{d}v}{\mathrm{d}t}\boldsymbol{e}_t + \frac{v^2}{\rho}\boldsymbol{e}_n \quad (1.12)$$

where ρ is the curvature radius of the trajectory at the point, $\boldsymbol{e}_n$ is the unit vector along the normal direction at the same point, $\boldsymbol{e}_n$ and $\boldsymbol{e}_t$ are perpendicular to each other. In deriving (1.12), we have made use of the following facts or geometrical relations:

$$v = \frac{ds}{dt}; \quad \frac{d\boldsymbol{e}_t}{ds} = \frac{1}{ds}\frac{de_t}{ds}\boldsymbol{e}_n = \frac{1}{ds}\frac{ds}{\rho}\boldsymbol{e}_n$$

The second formula can be obtained from the additional diagram.

The formula (1.12) indicates that the acceleration is decomposed into two components. One is along the tangential direction, which determines the change in speed. The other is in the normal direction, which is responsible for the change in the direction of the velocity. (see Fig. 1.2). If the second component vanishes, the motion will be along a straight line. In comparison with a circular motion, we call the second component the centripetal acceleration.

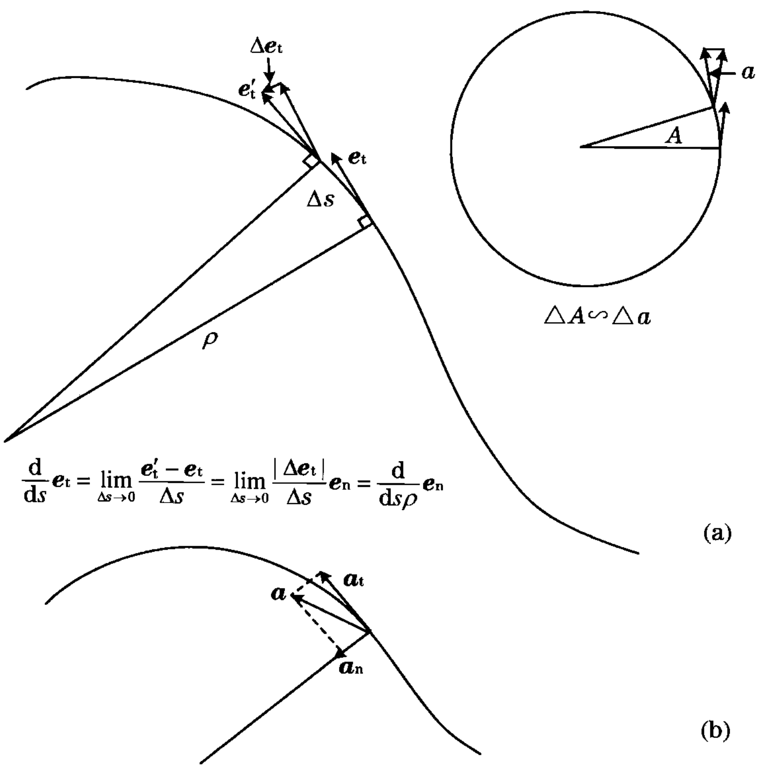

Fig. 1.2 Understanding acceleration.

1.6 Angular velocity and angular acceleration for axial rotation

A general rotation in space is rather complicated and can only be our future topic. Here we confine us to the case of axial rotation, i.e., the rotation about an axis which is fixed in space. In this case, the position of an object is defined by the angle θ, measured from a given basic line in the plane perpendicular to the axis. The rotation is then described by the angular velocity, i.e., the derivative of the angle θ with respect to time t:

$$\omega = \frac{d\theta}{dt} \tag{1.13}$$

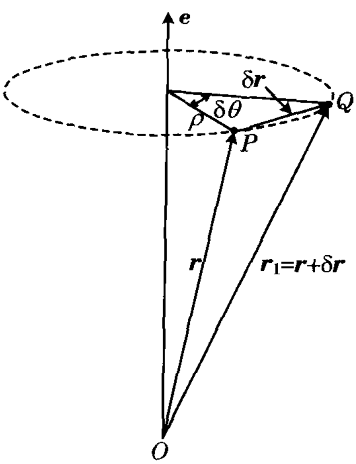

Fig. 1.3 Infinitesimal rotation.

Angular velocity can also be considered as a vector, its direction is along the axis and the sign is usually defined by the right hand rule, depending on whether the rotation is clock-wise or counterclockwise. From Fig. 1.3, it is easy to see $\delta \boldsymbol{r} = \delta\theta \times \boldsymbol{r}$, then we have the relation between angular and linear velocities:

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r} \tag{1.14}$$

Similarly, we can define the angular acceleration by performing differentiation on the angular velocity. For an axial rotation, the angular acceleration is written as

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \tag{1.15}$$

The quantity describes how the angular velocity is increased or decreased.

1.7 Addition of velocity and acceleration

Both velocity and acceleration can be taken as ordinary vector. Thus all operations in vector algebra, especially the addition, apply to them. However, we need to understand the corresponding physical situation to the operation.

The sum of two velocities is another velocity:

$$\boldsymbol{v}_3 = \boldsymbol{v}_1 + \boldsymbol{v}_2 \tag{1.16}$$

In physics, this means that if we measure the velocity of the object 1 from the object 2 to be $\boldsymbol{v}_1$, and the object 2 has a velocity $\boldsymbol{v}_2$ with respect to the object 3, then the velocity of object 1 with respect to the object 3 is $\boldsymbol{v}_3$, or in some cases if a complicated motion can be decomposed into some simpler motions, we also need (1.16).

For the acceleration, we have a similar expression:

$$\boldsymbol{a}_3 = \boldsymbol{a}_1 + \boldsymbol{a}_2 \tag{1.17}$$

However, the physical situation corresponding to (1.17) may include no other object. It simply represents a combined action of two forces.

Questions

(1) How many forms of motion (phenomena) do you know in nature? Why the position change is the simplest one?

(2) Compare the average velocity as defined in lower level physics course with the velocity defined in this lecture. Which one is more powerful? Why?

(3) Why both velocity and acceleration are called the differential properties of trajectory?

(4) How to understand the trajectory contains all information about the motion?

(5) Can you decompose an acceleration into tangential and normal components? Can you make any rough estimate to the acceleration of an object if you know the trajectory?

(6) What is the physical situation corresponding to the subtraction of one velocity from another?

Lecture 2 Various Coordinate Systems, Coordinate Transformation

2.1 Basic concept of coordinate transformation

In mechanics, it is common to change from one reference body to another or to make another choice of coordinate system for the same reference body. For example, we may need to change from a Cartesian system to a spherical system or from one polar system to another. In both cases, we need to perform coordinate transformation. However, the latter is only a mathematical event, while the former may bring about some physical consequence, which shall not be studied in kinematics. Thus the coordinate transformation in kinematics is basically same as in mathematics, except that the object to be transformed may have its physical interpretation, say, velocity or acceleration.

2.2 General linear transformation

In Cartesian system, a vector $\boldsymbol{r}$ is represented by a linear combination of the basic vectors $\boldsymbol{e}_i$. It is then expected that a general coordinate transformation, including both axial rotation and origin shift, should be linear. Since the basic vectors can also be taken as ordinary vectors, but they are much simpler, it is then natural to derive the formulas of coordinate transformation from the relation between the new and old basic vectors. This relation, or the transformation of basic vectors, is described by the equation:

$$\boldsymbol{e}_j{}' = \sum_{i=1}^{3} \alpha_{ji}\boldsymbol{e}_i \quad (j = 1, 2, 3) \tag{2.1}$$

where the primed and unprimed $\boldsymbol{e}$'s are the basic vectors in new and old coordinate systems. Obviously, the set of coefficients α_{ji} specifies the transformation, which can also be cast into a matrix form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{2.2}$$

If there is an origin shift, we need to add the coordinate b_j of the new origin in old coordinate system to the corresponding equation in (2.1). The vector $\boldsymbol{b}$ represents the position of new origin in old system. For a general vector $\boldsymbol{r}$, we have

$$\boldsymbol{r}' = \boldsymbol{A}\boldsymbol{r} + \boldsymbol{b} \tag{2.3}$$

or in component form:

$$x_j' = \sum_{i=1}^{3} a_{ji}x_i + b_j \quad (j = 1, 2, 3) \tag{2.4}$$

2.3 Orthogonality of transformation matrix

Since the basic vectors keep their mutual orthogonality under any rotation, i.e., their scalar products can be written in a Kronecker symbol:

$$\boldsymbol{e}_i' \cdot \boldsymbol{e}_j' = \delta_{ij} = \begin{cases} 0; & \text{if } i \neq j \\ 1; & \text{if } i = j \end{cases} \tag{2.5}$$

This imposes a limitation on the coefficient matrix $\boldsymbol{A}$. From (2.5) and (2.1), we have

$$\begin{aligned} \left(\sum_{k=1}^{3} a_{ki}\boldsymbol{e}_k\right)\cdot\left(\sum_{l=1}^{3} a_{jl}\boldsymbol{e}_l\right) &= \sum_{k=1}^{3}\sum_{l=1}^{3} a_{ki}a_{jl}(\boldsymbol{e}_k \cdot \boldsymbol{e}_l) \\ &= \sum_{k=1}^{3}\sum_{l=1}^{3} a_{ki}a_{jl}\delta_{kl} = \sum_{k=1}^{3} a_{ki}a_{jk} = \delta_{ij} \end{aligned} \tag{2.6}$$

This is the condition of orthogonality, so the matrix $\boldsymbol{A}$ is orthogonal.

It can be shown that the scalar product of two vectors remains unchanged under an orthogonal transformation, i.e.,

$$\boldsymbol{a}' \cdot \boldsymbol{b}' = \boldsymbol{a} \cdot \boldsymbol{b} \tag{2.7}$$

The proof of (2.7) is straightforward if (2.6) is used to simplify the scalar product of primed vectors on the left hand side.

2.4 Velocity in plane polar coordinate system

In a plane polar coordinate system, the two basic vectors are $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ (Fig. 2.1), which are both unit vectors in the direction of increasing the corresponding coordinate and perpendicular to each other. They are related to the basic vectors of Cartesian coordinate system by

$$\left.\begin{aligned} \boldsymbol{e}_r &= \cos\theta\,\boldsymbol{e}_1 + \sin\theta\,\boldsymbol{e}_2 \\ \boldsymbol{e}_\theta &= -\sin\theta\,\boldsymbol{e}_1 + \cos\theta\,\boldsymbol{e}_2 \end{aligned}\right\} \tag{2.8}$$

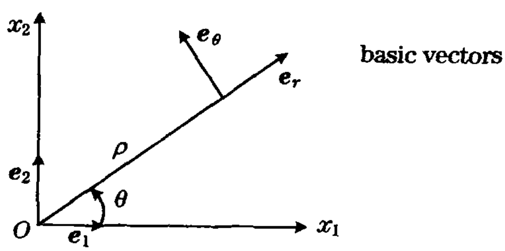

Fig. 2.1 Plane polar system.

The general vector $\boldsymbol{r}$ is written as:

$$\boldsymbol{r} = r\boldsymbol{e}_r \tag{2.9}$$

where $\boldsymbol{e}_r$ is the unit vector in the radial direction. The derivative of $\boldsymbol{r}$ with respect to time t is:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}t}\boldsymbol{e}_r + r\frac{\mathrm{d}\boldsymbol{e}_r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}t}\boldsymbol{e}_r + r\frac{\mathrm{d}\theta}{\mathrm{d}t}\boldsymbol{e}_\theta \tag{2.10}$$

where in the last step, the derivative of $\boldsymbol{e}_r$ is calculated from (2.8) as following:

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{e}_r}{\mathrm{d}t} &= -\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\boldsymbol{e}_1 + \cos\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\boldsymbol{e}_2 \\ &= \frac{\mathrm{d}\theta}{\mathrm{d}t}(-\sin\theta\,\boldsymbol{e}_1 + \cos\theta\,\boldsymbol{e}_2) = \frac{\mathrm{d}\theta}{\mathrm{d}t}\boldsymbol{e}_\theta \end{aligned}$$

The two terms of (2.10) represent the radial and angular velocities, respectively.

The acceleration is the derivative of $\boldsymbol{v}$. It can be calculated from (2.10) as following: