

NONLINEAR
PHYSICAL
SCIENCE

Abdul-Majid Wazwaz

Linear and Nonlinear Integral Equations

Methods and Applications

线性与非线性积分方程
方法及应用



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*THIS BOOK IS DEDICATED TO
my wife, our son, and our three daughters
for supporting me in all my endeavors*

Preface

Many remarkable advances have been made in the field of integral equations, but these remarkable developments have remained scattered in a variety of specialized journals. These new ideas and approaches have rarely been brought together in textbook form. If these ideas merely remain in scholarly journals and never get discussed in textbooks, then specialists and students will not be able to benefit from the results of the valuable research achievements.

The explosive growth in industry and technology requires constructive adjustments in mathematics textbooks. The valuable achievements in research work are not found in many of today's textbooks, but they are worthy of addition and study. The technology is moving rapidly, which is pushing for valuable insights into some substantial applications and developed approaches. The mathematics taught in the classroom should come to resemble the mathematics used in varied applications of nonlinear science models and engineering applications. This book was written with these thoughts in mind.

Linear and Nonlinear Integral Equations: Methods and Applications is designed to serve as a text and a reference. The book is designed to be accessible to advanced undergraduate and graduate students as well as a research monograph to researchers in applied mathematics, physical sciences, and engineering. This text differs from other similar texts in a number of ways. First, it explains the classical methods in a comprehensible, non-abstract approach. Furthermore, it introduces and explains the modern developed mathematical methods in such a fashion that shows how the new methods can complement the traditional methods. These approaches further improve the understanding of the material.

The book avoids approaching the subject through the compact and classical methods that make the material difficult to be grasped, especially by students who do not have the background in these abstract concepts. The aim of this book is to offer practical treatment of linear and nonlinear integral equations emphasizing the need to problem solving rather than theorem proving.

The book was developed as a result of many years of experiences in teaching integral equations and conducting research work in this field. The author

has taken account of his teaching experience, research work as well as valuable suggestions received from students and scholars from a wide variety of audience. Numerous examples and exercises, ranging in level from easy to difficult, but consistent with the material, are given in each section to give the reader the knowledge, practice and skill in linear and nonlinear integral equations. There is plenty of material in this text to be covered in two semesters for senior undergraduates and beginning graduates of mathematics, physical science, and engineering.

The content of the book is divided into two distinct parts, and each part is self-contained and practical. Part I contains twelve chapters that handle the linear integral and nonlinear integro-differential equations by using the modern mathematical methods, and some of the powerful traditional methods. Since the book's readership is a diverse and interdisciplinary audience of applied mathematics, physical science, and engineering, attempts are made so that part I presents both analytical and numerical approaches in a clear and systematic fashion to make this book accessible to those who work in these fields.

Part II contains the remaining six chapters devoted to thoroughly examining the nonlinear integral equations and its applications. The potential theory contributed more than any field to give rise to nonlinear integral equations. Mathematical physics models, such as diffraction problems, scattering in quantum mechanics, conformal mapping, and water waves also contributed to the creation of nonlinear integral equations. Because it is not always possible to find exact solutions to problems of physical science that are posed, much work is devoted to obtaining qualitative approximations that highlight the structure of the solution.

Chapter 1 provides the basic definitions and introductory concepts. The Taylor series, Leibnitz rule, and Laplace transform method are presented and reviewed. This discussion will provide the reader with a strong basis to understand the thoroughly-examined material in the following chapters. In Chapter 2, the classifications of integral and integro-differential equations are presented and illustrated. In addition, the linearity and the homogeneity concepts of integral equations are clearly addressed. The conversion process of IVP and BVP to Volterra integral equation and Fredholm integral equation respectively are described. Chapters 3 and 5 deal with the linear Volterra integral equations and the linear Volterra integro-differential equations, of the first and the second kind, respectively. Each kind is approached by a variety of methods that are described in details. Chapters 3 and 5 provide the reader with a comprehensive discussion of both types of equations. The two chapters emphasize the power of the proposed methods in handling these equations. Chapters 4 and 6 are entirely devoted to Fredholm integral equations and Fredholm integro-differential equations, of the first and the second kind, respectively. The ill-posed Fredholm integral equation of the first kind is handled by the powerful method of regularization combined with other methods. The two kinds of equations are approached

by many appropriate algorithms that are illustrated in details. A comprehensive study is introduced where a variety of reliable methods is applied independently and supported by many illustrative examples. Chapter 7 is devoted to studying the Abel's integral equations, generalized Abel's integral equations, and the weakly singular integral equations. The chapter also stresses the significant features of these types of singular equations with full explanations and many illustrative examples and exercises. Chapters 8 and 9 introduce a valuable study on Volterra-Fredholm integral equations and Volterra-Fredholm integro-differential equations respectively in one and two variables. The mixed Volterra-Fredholm integral and the mixed Volterra-Fredholm integro-differential equations in two variables are also examined with illustrative examples. The proposed methods introduce a powerful tool for handling these two types of equations. Examples are provided with a substantial amount of explanation. The reader will find a wealth of well-known models with one and two variables. A detailed and clear explanation of every application is introduced and supported by fully explained examples and exercises of every type.

Chapters 10, 11, and 12 are concerned with the systems of Volterra integral and integro-differential equations, systems of Fredholm integral and integro-differential equations, and systems of singular integral equations and systems of weakly singular integral equations respectively. Systems of integral equations that are important, are handled by using very constructive methods. A discussion of the basic theory and illustrations of the solutions to the systems are presented to introduce the material in a clear and useful fashion. Singular systems in one, two, and three variables are thoroughly investigated. The systems are supported by a variety of useful methods that are well explained and illustrated.

Part II is titled "Nonlinear Integral Equations". Part II of this book gives a self-contained, practical and realistic approach to nonlinear integral equations, where scientists and engineers are paying great attention to the effects caused by the nonlinearity of dynamical equations in nonlinear science. The potential theory contributed more than any field to give rise to nonlinear integral equations. Mathematical physics models, such as diffraction problems, scattering in quantum mechanics, conformal mapping, and water waves also contributed to the creation of nonlinear integral equations. The nonlinearity of these models may give more than one solution and this is the nature of nonlinear problems. Moreover, ill-posed Fredholm integral equations of the first kind may also give more than one solution even if it is linear.

Chapter 13 presents discussions about nonlinear Volterra integral equations and systems of Volterra integral equations, of the first and the second kind. More emphasis on the existence of solutions is proved and emphasized. A variety of methods are employed, introduced and explained in a clear and useful manner. Chapter 14 is devoted to giving a comprehensive study on nonlinear Volterra integro-differential equations and systems of nonlinear Volterra integro-differential equations, of the first and the second kind.

A variety of methods are introduced, and numerous practical examples are explained in a practical way. Chapter 15 investigates thoroughly the existence theorem, bifurcation points and singular points that may arise from nonlinear Fredholm integral equations. The study presents a variety of powerful methods to handle nonlinear Fredholm integral equations of the first and the second kind. Systems of these equations are examined with illustrated examples. Chapter 16 is entirely devoted to studying a family of nonlinear Fredholm integro-differential equations of the second kind and the systems of these equations. The approach we followed is identical to our approach in the previous chapters to make the discussion accessible for interdisciplinary audience. Chapter 17 provides the reader with a comprehensive discussion of the nonlinear singular integral equations, nonlinear weakly singular integral equations, and systems of these equations. Most of these equations are characterized by the singularity behavior where the proposed methods should overcome the difficulty of this singular behavior. The power of the employed methods is confirmed here by determining solutions that may not be unique. Chapter 18 presents a comprehensive study on five scientific applications that we selected because of its wide applicability for several other models. Because it is not always possible to find exact solutions to models of physical sciences, much work is devoted to obtaining qualitative approximations that highlight the structure of the solution. The powerful Padé approximants are used to give insight into the structure of the solution. This chapter closes Part II of this text.

The book concludes with seven useful appendices. Moreover, the book introduces the traditional methods in the same amount of concern to provide the reader with the knowledge needed to make a comparison.

I deeply acknowledge Professor Albert Luo for many helpful discussions, encouragement, and useful remarks. I am also indebted to Ms. Liping Wang, the Publishing Editor of the Higher Education Press for her effective cooperation and important suggestions. The staff of HEP deserve my thanks for their support to this project. I owe them all my deepest thanks.

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The author would highly appreciate any notes concerning any constructive suggestions.

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Contents

Part I Linear Integral Equations

1	Preliminaries	3
1.1	Taylor Series	4
1.2	Ordinary Differential Equations	7
1.2.1	First Order Linear Differential Equations	7
1.2.2	Second Order Linear Differential Equations	9
1.2.3	The Series Solution Method	13
1.3	Leibnitz Rule for Differentiation of Integrals	17
1.4	Reducing Multiple Integrals to Single Integrals	19
1.5	Laplace Transform	22
1.5.1	Properties of Laplace Transforms	23
1.6	Infinite Geometric Series	28
	References	30
2	Introductory Concepts of Integral Equations	33
2.1	Classification of Integral Equations	34
2.1.1	Fredholm Integral Equations	34
2.1.2	Volterra Integral Equations	35
2.1.3	Volterra-Fredholm Integral Equations	35
2.1.4	Singular Integral Equations	36
2.2	Classification of Integro-Differential Equations	37
2.2.1	Fredholm Integro-Differential Equations	38
2.2.2	Volterra Integro-Differential Equations	38
2.2.3	Volterra-Fredholm Integro-Differential Equations	39
2.3	Linearity and Homogeneity	40
2.3.1	Linearity Concept	40
2.3.2	Homogeneity Concept	41
2.4	Origins of Integral Equations	42
2.5	Converting IVP to Volterra Integral Equation	42

2.5.1	Converting Volterra Integral Equation to IVP	47
2.6	Converting BVP to Fredholm Integral Equation	49
2.6.1	Converting Fredholm Integral Equation to BVP	54
2.7	Solution of an Integral Equation	59
	References	63
3	Volterra Integral Equations	65
3.1	Introduction	65
3.2	Volterra Integral Equations of the Second Kind	66
3.2.1	The Adomian Decomposition Method	66
3.2.2	The Modified Decomposition Method	73
3.2.3	The Noise Terms Phenomenon	78
3.2.4	The Variational Iteration Method	82
3.2.5	The Successive Approximations Method	95
3.2.6	The Laplace Transform Method	99
3.2.7	The Series Solution Method	103
3.3	Volterra Integral Equations of the First Kind	108
3.3.1	The Series Solution Method	108
3.3.2	The Laplace Transform Method	111
3.3.3	Conversion to a Volterra Equation of the Second Kind	114
	References	118
4	Fredholm Integral Equations	119
4.1	Introduction	119
4.2	Fredholm Integral Equations of the Second Kind	121
4.2.1	The Adomian Decomposition Method	121
4.2.2	The Modified Decomposition Method	128
4.2.3	The Noise Terms Phenomenon	133
4.2.4	The Variational Iteration Method	136
4.2.5	The Direct Computation Method	141
4.2.6	The Successive Approximations Method	146
4.2.7	The Series Solution Method	151
4.3	Homogeneous Fredholm Integral Equation	154
4.3.1	The Direct Computation Method	155
4.4	Fredholm Integral Equations of the First Kind	159
4.4.1	The Method of Regularization	161
4.4.2	The Homotopy Perturbation Method	166
	References	173
5	Volterra Integro-Differential Equations	175
5.1	Introduction	175
5.2	Volterra Integro-Differential Equations of the Second Kind	176
5.2.1	The Adomian Decomposition Method	176
5.2.2	The Variational Iteration Method	181

5.2.3	The Laplace Transform Method	186
5.2.4	The Series Solution Method	190
5.2.5	Converting Volterra Integro-Differential Equations to Initial Value Problems	195
5.2.6	Converting Volterra Integro-Differential Equation to Volterra Integral Equation	199
5.3	Volterra Integro-Differential Equations of the First Kind	203
5.3.1	Laplace Transform Method	204
5.3.2	The Variational Iteration Method	206
	References	211
6	Fredholm Integro-Differential Equations	213
6.1	Introduction	213
6.2	Fredholm Integro-Differential Equations of the Second Kind	214
6.2.1	The Direct Computation Method	214
6.2.2	The Variational Iteration Method	218
6.2.3	The Adomian Decomposition Method	223
6.2.4	The Series Solution Method	230
	References	234
7	Abel's Integral Equation and Singular Integral Equations	237
7.1	Introduction	237
7.2	Abel's Integral Equation	238
7.2.1	The Laplace Transform Method	239
7.3	The Generalized Abel's Integral Equation	242
7.3.1	The Laplace Transform Method	243
7.3.2	The Main Generalized Abel Equation	245
7.4	The Weakly Singular Volterra Equations	247
7.4.1	The Adomian Decomposition Method	248
7.4.2	The Successive Approximations Method	253
7.4.3	The Laplace Transform Method	257
	References	260
8	Volterra-Fredholm Integral Equations	261
8.1	Introduction	261
8.2	The Volterra-Fredholm Integral Equations	262
8.2.1	The Series Solution Method	262
8.2.2	The Adomian Decomposition Method	266
8.3	The Mixed Volterra-Fredholm Integral Equations	269
8.3.1	The Series Solution Method	270
8.3.2	The Adomian Decomposition Method	273
8.4	The Mixed Volterra-Fredholm Integral Equations in Two Variables	277

8.4.1	The Modified Decomposition Method	278
References	283
9	Volterra-Fredholm Integro-Differential Equations	285
9.1	Introduction	285
9.2	The Volterra-Fredholm Integro-Differential Equation	285
9.2.1	The Series Solution Method	285
9.2.2	The Variational Iteration Method	289
9.3	The Mixed Volterra-Fredholm Integro-Differential Equations	296
9.3.1	The Direct Computation Method	296
9.3.2	The Series Solution Method	300
9.4	The Mixed Volterra-Fredholm Integro-Differential Equations in Two Variables	303
9.4.1	The Modified Decomposition Method	304
References	309
10	Systems of Volterra Integral Equations	311
10.1	Introduction	311
10.2	Systems of Volterra Integral Equations of the Second Kind	312
10.2.1	The Adomian Decomposition Method	312
10.2.2	The Laplace Transform Method	318
10.3	Systems of Volterra Integral Equations of the First Kind ...	323
10.3.1	The Laplace Transform Method	323
10.3.2	Conversion to a Volterra System of the Second Kind	327
10.4	Systems of Volterra Integro-Differential Equations	328
10.4.1	The Variational Iteration Method	329
10.4.2	The Laplace Transform Method	335
References	339
11	Systems of Fredholm Integral Equations	341
11.1	Introduction	341
11.2	Systems of Fredholm Integral Equations	342
11.2.1	The Adomian Decomposition Method	342
11.2.2	The Direct Computation Method	347
11.3	Systems of Fredholm Integro-Differential Equations	352
11.3.1	The Direct Computation Method	353
11.3.2	The Variational Iteration Method	358
References	364

12 Systems of Singular Integral Equations 365

12.1 Introduction 365

12.2 Systems of Generalized Abel Integral Equations 366

12.2.1 Systems of Generalized Abel Integral Equations in Two Unknowns 366

12.2.2 Systems of Generalized Abel Integral Equations in Three Unknowns 370

12.3 Systems of the Weakly Singular Volterra Integral Equations 374

12.3.1 The Laplace Transform Method 374

12.3.2 The Adomian Decomposition Method 378

References 383

Part II Nonlinear Integral Equations

13 Nonlinear Volterra Integral Equations 387

13.1 Introduction 387

13.2 Existence of the Solution for Nonlinear Volterra Integral Equations 388

13.3 Nonlinear Volterra Integral Equations of the Second Kind . . 388

13.3.1 The Successive Approximations Method 389

13.3.2 The Series Solution Method 393

13.3.3 The Adomian Decomposition Method 397

13.4 Nonlinear Volterra Integral Equations of the First Kind 404

13.4.1 The Laplace Transform Method 405

13.4.2 Conversion to a Volterra Equation of the Second Kind 408

13.5 Systems of Nonlinear Volterra Integral Equations 411

13.5.1 Systems of Nonlinear Volterra Integral Equations of the Second Kind 412

13.5.2 Systems of Nonlinear Volterra Integral Equations of the First Kind 417

References 423

14 Nonlinear Volterra Integro-Differential Equations 425

14.1 Introduction 425

14.2 Nonlinear Volterra Integro-Differential Equations of the Second Kind 426

14.2.1 The Combined Laplace Transform-Adomian Decomposition Method 426

14.2.2 The Variational Iteration Method 432

14.2.3 The Series Solution Method 436

14.3 Nonlinear Volterra Integro-Differential Equations of the First Kind 440

14.3.1	The Combined Laplace Transform-Adomian Decomposition Method	440
14.3.2	Conversion to Nonlinear Volterra Equation of the Second Kind	446
14.4	Systems of Nonlinear Volterra Integro-Differential Equations	450
14.4.1	The Variational Iteration Method	451
14.4.2	The Combined Laplace Transform-Adomian Decomposition Method	456
References	465
15	Nonlinear Fredholm Integral Equations	467
15.1	Introduction	467
15.2	Existence of the Solution for Nonlinear Fredholm Integral Equations	468
15.2.1	Bifurcation Points and Singular Points	469
15.3	Nonlinear Fredholm Integral Equations of the Second Kind	469
15.3.1	The Direct Computation Method	470
15.3.2	The Series Solution Method	476
15.3.3	The Adomian Decomposition Method	480
15.3.4	The Successive Approximations Method	485
15.4	Homogeneous Nonlinear Fredholm Integral Equations	490
15.4.1	The Direct Computation Method	490
15.5	Nonlinear Fredholm Integral Equations of the First Kind ...	494
15.5.1	The Method of Regularization	495
15.5.2	The Homotopy Perturbation Method	500
15.6	Systems of Nonlinear Fredholm Integral Equations	505
15.6.1	The Direct Computation Method	506
15.6.2	The Modified Adomian Decomposition Method	510
References	515
16	Nonlinear Fredholm Integro-Differential Equations	517
16.1	Introduction	517
16.2	Nonlinear Fredholm Integro-Differential Equations	518
16.2.1	The Direct Computation Method	518
16.2.2	The Variational Iteration Method	522
16.2.3	The Series Solution Method	526
16.3	Homogeneous Nonlinear Fredholm Integro-Differential Equations	530
16.3.1	The Direct Computation Method	530
16.4	Systems of Nonlinear Fredholm Integro-Differential Equations	535
16.4.1	The Direct Computation Method	535
16.4.2	The Variational Iteration Method	540

References	545
17 Nonlinear Singular Integral Equations	547
17.1 Introduction	547
17.2 Nonlinear Abel's Integral Equation	548
17.2.1 The Laplace Transform Method	549
17.3 The Generalized Nonlinear Abel Equation	552
17.3.1 The Laplace Transform Method	553
17.3.2 The Main Generalized Nonlinear Abel Equation	556
17.4 The Nonlinear Weakly-Singular Volterra Equations	559
17.4.1 The Adomian Decomposition Method	559
17.5 Systems of Nonlinear Weakly-Singular Volterra Integral Equations	562
17.5.1 The Modified Adomian Decomposition Method	563
References	567
18 Applications of Integral Equations	569
18.1 Introduction	569
18.2 Volterra's Population Model	570
18.2.1 The Variational Iteration Method	571
18.2.2 The Series Solution Method	572
18.2.3 The Padé Approximants	573
18.3 Integral Equations with Logarithmic Kernels	574
18.3.1 Second Kind Fredholm Integral Equation with a Logarithmic Kernel	577
18.3.2 First Kind Fredholm Integral Equation with a Logarithmic Kernel	580
18.3.3 Another First Kind Fredholm Integral Equation with a Logarithmic Kernel	583
18.4 The Fresnel Integrals	584
18.5 The Thomas-Fermi Equation	587
18.6 Heat Transfer and Heat Radiation	590
18.6.1 Heat Transfer: Lighthill Singular Integral Equation ..	590
18.6.2 Heat Radiation in a Semi-Infinite Solid	592
References	594
Appendix A Table of Indefinite Integrals	597
A.1 Basic Forms	597
A.2 Trigonometric Forms	597
A.3 Inverse Trigonometric Forms	598
A.4 Exponential and Logarithmic Forms	598
A.5 Hyperbolic Forms	599
A.6 Other Forms	599

Appendix B	Integrals Involving Irrational Algebraic Functions	600
B.1	Integrals Involving $\frac{t^n}{\sqrt{x-t}}$, n is an integer, $n \geq 0$	600
B.2	Integrals Involving $\frac{t^{\frac{n}{2}}}{\sqrt{x-t}}$, n is an odd integer, $n \geq 1$	600
Appendix C	Series Representations	601
C.1	Exponential Functions Series	601
C.2	Trigonometric Functions	601
C.3	Inverse Trigonometric Functions	602
C.4	Hyperbolic Functions	602
C.5	Inverse Hyperbolic Functions	602
C.6	Logarithmic Functions	602
Appendix D	The Error and the Complementary Error Functions	603
D.1	The Error Function	603
D.2	The Complementary Error Function	603
Appendix E	Gamma Function	604
Appendix F	Infinite Series	605
F.1	Numerical Series	605
F.2	Trigonometric Series	605
Appendix G	The Fresnel Integrals	607
G.1	The Fresnel Cosine Integral	607
G.2	The Fresnel Sine Integral	607
Answers	609
Index	637

Part I
Linear Integral Equations