NONLINEAR PHYSICAL SCIENCE

Abdul-Majid Wazwaz

Linear and Nonlinear Integral Equations

Methods and Applications

线性与非线性积分方程

方法及应用

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Author
Abdul-Majid Wazwaz
Saint Xavier University
Chicago, IL 60655, USA
Email: wazwaz@sxu.edu

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THIS BOOK IS DEDICATED TO
my wife, our son, and our three daughters
for supporting me in all my endeavors

Preface

Many remarkable advances have been made in the field of integral equations, but these remarkable developments have remained scattered in a variety of specialized journals. These new ideas and approaches have rarely been brought together in textbook form. If these ideas merely remain in scholarly journals and never get discussed in textbooks, then specialists and students will not be able to benefit from the results of the valuable research achievements.

The explosive growth in industry and technology requires constructive adjustments in mathematics textbooks. The valuable achievements in research work are not found in many of today's textbooks, but they are worthy of addition and study. The technology is moving rapidly, which is pushing for valuable insights into some substantial applications and developed approaches. The mathematics taught in the classroom should come to resemble the mathematics used in varied applications of nonlinear science models and engineering applications. This book was written with these thoughts in mind.

Linear and Nonlinear Integral Equations: Methods and Applications is designed to serve as a text and a reference. The book is designed to be accessible to advanced undergraduate and graduate students as well as a research monograph to researchers in applied mathematics, physical sciences, and engineering. This text differs from other similar texts in a number of ways. First, it explains the classical methods in a comprehensible, non-abstract approach. Furthermore, it introduces and explains the modern developed mathematical methods in such a fashion that shows how the new methods can complement the traditional methods. These approaches further improve the understanding of the material.

The book avoids approaching the subject through the compact and classical methods that make the material difficult to be grasped, especially by students who do not have the background in these abstract concepts. The aim of this book is to offer practical treatment of linear and nonlinear integral equations emphasizing the need to problem solving rather than theorem proving.

The book was developed as a result of many years of experiences in teaching integral equations and conducting research work in this field. The author has taken account of his teaching experience, research work as well as valuable suggestions received from students and scholars from a wide variety of audience. Numerous examples and exercises, ranging in level from easy to difficult, but consistent with the material, are given in each section to give the reader the knowledge, practice and skill in linear and nonlinear integral equations. There is plenty of material in this text to be covered in two semesters for senior undergraduates and beginning graduates of mathematics, physical science, and engineering.

The content of the book is divided into two distinct parts, and each part is self-contained and practical. Part I contains twelve chapters that handle the linear integral and nonlinear integro-differential equations by using the modern mathematical methods, and some of the powerful traditional methods. Since the book's readership is a diverse and interdisciplinary audience of applied mathematics, physical science, and engineering, attempts are made so that part I presents both analytical and numerical approaches in a clear and systematic fashion to make this book accessible to those who work in these fields.

Part II contains the remaining six chapters devoted to thoroughly examining the nonlinear integral equations and its applications. The potential theory contributed more than any field to give rise to nonlinear integral equations. Mathematical physics models, such as diffraction problems, scattering in quantum mechanics, conformal mapping, and water waves also contributed to the creation of nonlinear integral equations. Because it is not always possible to find exact solutions to problems of physical science that are posed, much work is devoted to obtaining qualitative approximations that highlight the structure of the solution.

Chapter 1 provides the basic definitions and introductory concepts. The Taylor series, Leibnitz rule, and Laplace transform method are presented and reviewed. This discussion will provide the reader with a strong basis to understand the thoroughly-examined material in the following chapters. In Chapter 2, the classifications of integral and integro-differential equations are presented and illustrated. In addition, the linearity and the homogeneity concepts of integral equations are clearly addressed. The conversion process of IVP and BVP to Volterra integral equation and Fredholm integral equation respectively are described. Chapters 3 and 5 deal with the linear Volterra integral equations and the linear Volterra integro-differential equations, of the first and the second kind, respectively. Each kind is approached by a variety of methods that are described in details. Chapters 3 and 5 provide the reader with a comprehensive discussion of both types of equations. The two chapters emphasize the power of the proposed methods in handling these equations. Chapters 4 and 6 are entirely devoted to Fredholm integral equations and Fredholm integro-differential equations, of the first and the second kind, respectively. The ill-posed Fredholm integral equation of the first kind is handled by the powerful method of regularization combined with other methods. The two kinds of equations are approached by many appropriate algorithms that are illustrated in details. A comprehensive study is introduced where a variety of reliable methods is applied independently and supported by many illustrative examples. Chapter 7 is devoted to studying the Abel's integral equations, generalized Abel's integral equations, and the weakly singular integral equations. The chapter also stresses the significant features of these types of singular equations with full explanations and many illustrative examples and exercises. Chapters 8 and 9 introduce a valuable study on Volterra-Fredholm integral equations and Volterra-Fredholm integro-differential equations respectively in one and two variables. The mixed Volterra-Fredholm integral and the mixed Volterra-Fredholm integro-differential equations in two variables are also examined with illustrative examples. The proposed methods introduce a powerful tool for handling these two types of equations. Examples are provided with a substantial amount of explanation. The reader will find a wealth of well-known models with one and two variables. A detailed and clear explanation of every application is introduced and supported by fully explained examples and exercises of every type.

Chapters 10, 11, and 12 are concerned with the systems of Volterra integral and integro-differential equations, systems of Fredholm integral and integro-differential equations, and systems of singular integral equations and systems of weakly singular integral equations respectively. Systems of integral equations that are important, are handled by using very constructive methods. A discussion of the basic theory and illustrations of the solutions to the systems are presented to introduce the material in a clear and useful fashion. Singular systems in one, two, and three variables are thoroughly investigated. The systems are supported by a variety of useful methods that are well explained and illustrated.

Part II is titled "Nonlinear Integral Equations". Part II of this book gives a self-contained, practical and realistic approach to nonlinear integral equations, where scientists and engineers are paying great attention to the effects caused by the nonlinearity of dynamical equations in nonlinear science. The potential theory contributed more than any field to give rise to nonlinear integral equations. Mathematical physics models, such as diffraction problems, scattering in quantum mechanics, conformal mapping, and water waves also contributed to the creation of nonlinear integral equations. The nonlinearity of these models may give more than one solution and this is the nature of nonlinear problems. Moreover, ill-posed Fredholm integral equations of the first kind may also give more than one solution even if it is linear.

Chapter 13 presents discussions about nonlinear Volterra integral equations and systems of Volterra integral equations, of the first and the second kind. More emphasis on the existence of solutions is proved and emphasized. A variety of methods are employed, introduced and explained in a clear and useful manner. Chapter 14 is devoted to giving a comprehensive study on nonlinear Volterra integro-differential equations and systems of nonlinear Volterra integro-differential equations, of the first and the second kind.

A variety of methods are introduced, and numerous practical examples are explained in a practical way. Chapter 15 investigates thoroughly the existence theorem, bifurcation points and singular points that may arise from nonlinear Fredholm integral equations. The study presents a variety of powerful methods to handle nonlinear Fredholm integral equations of the first and the second kind. Systems of these equations are examined with illustrated examples. Chapter 16 is entirely devoted to studying a family of nonlinear Fredholm integro-differential equations of the second kind and the systems of these equations. The approach we followed is identical to our approach in the previous chapters to make the discussion accessible for interdisciplinary audience. Chapter 17 provides the reader with a comprehensive discussion of the nonlinear singular integral equations, nonlinear weakly singular integral equations, and systems of these equations. Most of these equations are characterized by the singularity behavior where the proposed methods should overcome the difficulty of this singular behavior. The power of the employed methods is confirmed here by determining solutions that may not be unique. Chapter 18 presents a comprehensive study on five scientific applications that we selected because of its wide applicability for several other models. Because it is not always possible to find exact solutions to models of physical sciences, much work is devoted to obtaining qualitative approximations that highlight the structure of the solution. The powerful Padé approximants are used to give insight into the structure of the solution. This chapter closes Part II of this text.

The book concludes with seven useful appendices. Moreover, the book introduces the traditional methods in the same amount of concern to provide the reader with the knowledge needed to make a comparison.

I deeply acknowledge Professor Albert Luo for many helpful discussions, encouragement, and useful remarks. I am also indebted to Ms. Liping Wang, the Publishing Editor of the Higher Education Press for her effective cooperation and important suggestions. The staff of HEP deserve my thanks for their support to this project. I owe them all my deepest thanks.

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I am deeply indebted to my wife, my son and my daughters who provided me with their continued encouragement, patience and support during the long days of preparing this book.

The author would highly appreciate any notes concerning any constructive suggestions.

Abdul-Majid Wazwaz
Saint Xavier University
Chicago, IL 60655
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Linear Integral Equations