

国外电子信息精品著作(影印版)

# 计算机辅助建模技术

**Computer Aided Simulations**

(China Edition)

**Lanka Udawatta  
Buddhika Jayasekara**



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## 内 容 简 介

计算机辅助建模技术一书从概念到程序上阐述了从基础系统到高级复杂系统中基于 Matlab 语言的仿真模拟技术。本书包含计算机辅助建模的设计介绍、物理系统建模仿真、计算机仿真建模实例、高级仿真以及仿真结果的方法性描述等内容,使用了大量的 Matlab 实例。本书对从事系统仿真领域的相关人员有很好的参考价值。

Lanka Udawatta, Buddhika Jayasekara

COMPUTER AIDED SIMULATIONS

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要使我国的信息技术更好地发展起来，需要科学工作者和工程技术人员付出艰辛的努力。此外，我们要从客观上为科学工作者和工程技术人员创造更有利于发展的环境，加强对信息技术的支持与投资力度，其中也包括与信息技术相关的图书出版工作。

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此次科学出版社在广泛征求专家意见的基础上，经过反复论证、仔细遴选，共引进了接近 30 本外版书，大体上可以分为两类，第一类是基础理论著作，第二类是工程应用方面的著作。所有的著作都涉及信息领域的最新成果，大多数是 2005 年后出版的，力求“层次高、内

容新、参考性强”。在内容和形式上都体现了科学出版社一贯奉行的严谨作风。

当然，这批书只能涵盖信息科学技术的一部分，所以这项工作还应该继续下去。对于一些读者面较广、观点新颖、国内缺乏的好书还应该翻译成中文出版，这有利于知识更好更快地传播。同时，我也希望广大读者提出好的建议，以改进和完善丛书的出版工作。

总之，我对科学出版社引进外版书这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

A stylized, bold black ink signature of the character '王越' (Wang Yue).

中国科学院院士

中国工程院院士

2006年12月

## **Preface**

The purpose of this book is to introduce basic concepts of computer simulations and provide necessary knowledge on simulation developments. This book is primarily intended to be used in undergraduate courses in the universities, especially electrical engineering, electronics engineering, communication engineering, computer engineering, and mechanical engineering students who need the knowledge on computer simulations. In addition, given the comprehensive coverage and large number of detailed examples, this book should be useful as a primary reference to electrical, electronic and mechanical graduate students and professionals wishing to start research projects focusing on simulations. On the other hand, research scientist who would like to switch on to advanced simulations, starting from basic level simulations, would also reap benefits from this book.

Starting from introduction to system simulations in Chapter 1, it will cover the basic concepts on systems and system modeling. Chapter 2 will cover the details of software & hardware for your own simulations and some computing basics. You will get the background preparation on data visualization through Chapter 3. Chapter 4 and Chapter 5 will introduce different systems and system modeling principles respectively. System integration with different components will be discussed in Chapter 6. While advanced simulations and modeling of complex systems with few examples will cover in Chapter 7 and Chapter 8 respectively. Chapter 9 will bring one example focusing on an application. Chapter 10 presents some examples for getting experience on basic simulations.

**Lanka Udawatta**  
**Buddhika Jayasekara**

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## Chapter 1

# Introduction to System Simulations

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Computer Aided Simulation (CAS) is the discipline of designing a model of an actual or theoretical system and executing that model on a digital computer for analyzing its behavior.

### 1.1 INTRODUCTION

#### 1.1.1 Model Design

In today's highly competitive environment, different systems are being developed by engineers and scientists at a faster growth rate. These systems are catering various groups in the society at different levels. Technically, we can define the term system as "Pre-defined assembly of resources (A collection of personnel, equipment, or methods) in order to accomplish a set of specific functions or goals, via executing a clearly defined set of instructions". In certain situations, the system under consideration may consist of several sub-systems, which can be analyzed independently or as a single entity, presently referring as "system of systems". After the initial investigation, the first step is to specify a model of a given system mathematically, which will help us to understand and analyze the system behavior. To simulate something physical, we will first need to create this mathematical model, which represents that physical system. In fact, theoretically derived models or empirical models are also analyzed and simulated in certain situations. Model may consist of several variables which are going to decide the system behavior.

Models can take many forms including declarative, functional, constraint, spatial or multi-model. For an example, these mathematical models can be derived using Newton's laws of motion, if the system consists of moving masses and so on. If the system is an electrical one, you may apply Kirchhoff's circuit laws in obtaining a mathematical model. Most of the times, this will lead to a set of differential equations (DE) and these differential equations can be further deduced to different forms as required by the designer. Further more, these dynamic representations of systems attempt to capture and describe the behavior of the system over time under different operating conditions. Finally, the dynamic model consists of system variables and other parameters to be

## **2 Computer Aided Simulations**

investigated. However, there may be situations where it is difficult to derive a model, which represents the actual behavior of the system.

Theoretical models are derived from the first principles whereas empirical models are derived using observations of system variables. Mathematical models can be also synthesized for certain systems based on measured data. Fundamentally, you should have a scientific background to develop these models, may be in the fields of engineering, medicine, physics, biology, chemistry, finance, etc. The background information will help you to define the variables, identify the constraints, approximate the values, estimate the time frame of interest and make the necessary assumptions [1]-[3].

### **1.1.2 Model Execution**

Once the mathematical model has been developed, you are supposed to execute the model on a digital computer. Here, you need to create a computer program, which steps through the time while updating the state and event variables in your mathematical model. Furthermore, now-a-days it is a practice that you store the output on computer memory or visualize it on the computer screen. In any case, you need some insight measure in order to have a proper analysis of the system.

As the first step, you can develop a flow chart or an algorithm that gives you an idea of the data flow of the system through the time line. In large programs, you have to manage the computer memory where you may go for dynamic memory allocations. For an example, if you are going to use a genetic algorithm (GA) program for a large dynamic system, it is necessary to optimize the memory consumption through a dynamic memory allocation. Once you completed the coding; now you are ready to execute your program and get the results. The coding can be done by a conventional computer language like C or C++. For engineering and scientific research works, MATLAB (MATrix LABoratory) provides a better environment with powerful matrix manipulating capabilities [5],[6]. In fact present day scientific calculations need the capability of manipulating matrices when it comes to higher order system analysis. Nevertheless, FORTRAN, Visual Basic, Pascal, Pro-engineer, Autocad, PSCad or any scientific computer software may help you to execute your codes of the algorithm of that particular model. In this book, Matlab codes are used in order to illustrate most of the examples. However, you are free to use any computer language or software in executing your algorithm.

During the development time of the computer simulations, researcher should plan for better approaches where it brings several outcomes simultaneously that make your computer simulation a great success. For an example, program might provide data storing, data visualizing, performance analyzing, mathematical computing, and other aspects of the simulation. This will save your time in many ways. When you need graphs or numerical results of the computer simulation for writing your reports, just running your program can bring all these. In fact, you may write several programs and get the same thing done consuming more time.

In general, dynamic systems are complex and nonlinear. Systems are simplified under certain assumptions in order to have flexible computer models. In fact, we have to make sure that the desired features of the original system remains unchanged in the synthesized computer model. For an example, the sampling time for a computing model can be

selected at different rates; data handling capability of the program can be greatly affected according to the selection. On the other hand, in an electrical or electronic circuit simulation, you might change the values of the devices, which are under the consideration.

### **1.1.3 Model Analysis through Outputs**

After executing your program, you will obtain certain results under different conditions or under different assumptions. As an example, if you can graphically visualize the state variables of a dynamic system through the time line, you are in a position to comment on the stability of it. Sometimes, it is necessary that the model output is to be visualized in a 3D computer animation. Once the simulation is completed under a selected software environment, in order to analyze the output further, you might switch on to a completely different software environment. The output data can be stored in data files, which can be used later.

The system response describes how the output changes over time. For an example, when it comes to systems control, by looking at the output, we might be able to infer additional properties of the system. Typically, we will observe the settling time, transient stability, maximum overshoot, steady-state error and so on. Once you created the simulation model you are in a position to do all these sensitivity analysis or changes without putting much effort compared to the original work. A good computer simulation can bring all these aspects through an innovative thinking. However, there are several discussions in the incoming chapters on developing background knowledge for a better simulation in order to obtain the desired outputs.

Assume that you want to develop a forecasting model using some past data. There are various techniques and methodologies that can be employed in order to derive a forecasting model. As examples, linear, quadratic or double exponential models can be used in order to model your past data set. As the first step, you may select a simple linear forecasting model which might fit to the given data set and desired coefficients are computed. In fact, it will be convenient if you visualize the model graphically. Furthermore, you can visualize the predicted outputs for different time frames which will give you an overall idea of the future behavior. In the same time, programmer can get the exact numerical values of the predictions through that model as outputs. In this particular example, you can compute the modeling error or fitting accuracy that will give you an idea about the model. These three outputs will give you an insight to the model and you may then decide whether it is necessary to develop another model or not for that data set. Once you develop your program in a systematic manner as explained in the above subsection, your program has the flexibility of including all other possible models. In the same program, you can visualize different models and get the predicted values. Moreover, you can compare accuracy of fitting for these different models and select the best one. When you design the program for these outputs for different models it leads to a better computer simulation.

In certain situations, it may be difficult to obtain all these aspects through a single program. You might need to run software and switch on to another completely different program in order to complete your simulation. Finally, you should be able to analyze your model through this computer simulation and obtain the desired results of the model. For different systems, you have to find the desired software or programming environment in order to obtain the outputs. For an example, if you analyze a biological system that

## 4 Computer Aided Simulations

may require a completely different simulation environment compared to an engineering simulation. In these situations, it may require different interfacing or linking to other devices and programs where you run your main simulation program for getting the final outputs of the model.

### 1.2 DYNAMIC SYSTEM MODELING

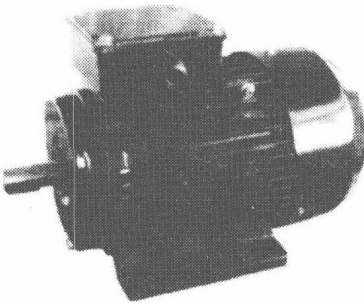
Physical systems that we find in our day-to-day life have different dynamic features and dynamic models according to the application. These systems are designed and optimized after analyzing their dynamic models. Figure 1.1 shows four different physical systems and they have different dynamic properties or dynamic models. Most physical systems available are continuous and they can be described using differential equations (DE). On the other hand, discrete systems can be described using difference equations dealing with discrete time signals. In fact, some continuous systems are transformed into discrete systems according to the requirement. As an example, if we consider the electric motor in Figure 1.1(A), for a given input power amount there will be a corresponding output, i.e., the torque generated on the motor shaft.

How to control the angular speed and the desired torque can be investigated using a dynamic model. Once you develop the dynamic model, modifications and different analysis can be easily done through a computer simulation.

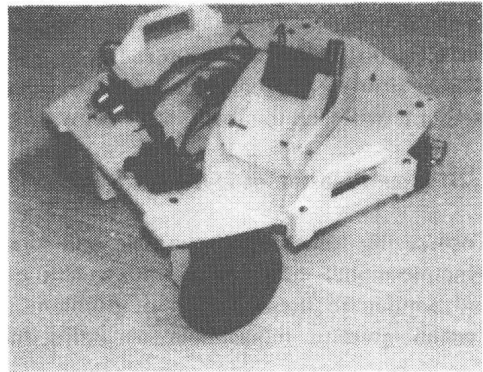
The mobile robot in Figure 1.1(B) is a test bed for experimenting different research and development works on mobile platforms. Two independently controllable wheels are located on right-hand and left-hand sides and it is further supported by another two small wheels. To control a mobile robot Position, a dynamic model is necessary and it can be used to employ different control algorithms. This mobile robot needs different navigation algorithms for different situations [4]. These algorithms can be tested in simulation environments before going into the actual practice. In some cases, artificial intelligence concepts are employed in order to have the desired behaviors of the robot whereas in some cases classical control theory based mathematical models are used.

Up thrust generated through gas firing of a hot air balloon is the main controlling force in the dynamic system in Figure 1.1(C). The amount of gas required to fire in a given time depends the factors like number of people carried, wind speed, surface of the balloon, and outside air temperature. Including all these parameters, you can have a dynamic model for the hot air balloon.

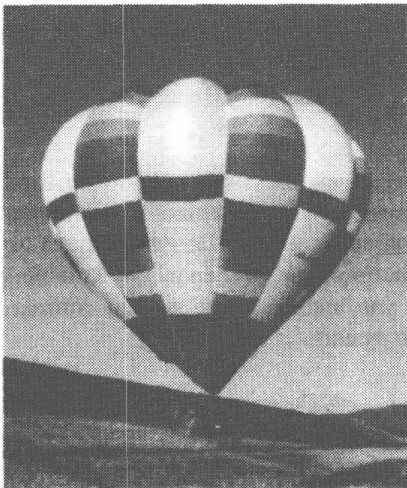
In recent years, enormous progress has been made in the development of wind turbines for electricity generation ranging from small scale wind turbines to large scale turbines. Wind power is the conversion of wind energy into more useful forms, usually electricity using wind turbines. Wind wane measures wind direction and communicates with the yaw drive to orient the wind turbine with respect to the wind direction. Wind turbine in Figure 1.1(D) converts energy available in the wind into electrical energy. The power output depends on the wind velocity and rotor swept area. These parameters can be modeled to a dynamic system and analyzed. The power in wind is proportional to the third power of the momentary wind speed and the dynamic model consists of several other parameters. However, once you developed the model you can simulate various scenarios on wind energy, varying from economic analysis to technical analysis.



A. Electric motor



B. Mobile robot



C. Hot air balloon



D. Wind turbine

Figure 1.1: Different Physical Systems

Above explained four systems are examples for different dynamic systems and let us try to understand the basic modeling concepts behind them in the next few chapters. You can use your engineering, scientific, and general knowledge in developing these models. In particular, theoretical background on Newton's laws of motion, Kirchhoff's circuit laws, Maxwell equations, laws of thermodynamics, differential equations, numerical methods, Euler and Lagrangian equations, kinematics and dynamics, nonlinear systems, and transformation techniques might be useful in modeling physical systems. In certain situations, it is necessary to start with a very basic model and develop an advanced version when you progress. This is basically due to the lack of information or knowledge about the system under investigation at the initial stage. After several experiments or simulations, you are in a position to validate the developed model and test the accuracy of it. In fact one can make a case that greater dependence on pure mathematical approach



## **6 Computer Aided Simulations**

has sometimes led to neglect of other approaches which might give you a better solution. There is little communication between people in quantitative fields and those in other fields. However, greater cooperation between different fields like engineering and medicine will lead to a better know-how in system modeling [7]-[10].

### **1.2.1 Classification of Dynamic Systems**

Considering inherent properties and characteristics, different systems or different dynamic models are grouped into several classes. Mathematical techniques are identified and applied to these classes of problems [2]. There are various ways of categorizing dynamic systems. In particular, we define them according to five characteristics as below:

1. Linear or Nonlinear
2. Constant or Variable Coefficients
3. Forced or Unforced
4. Lumped or Distributed Parameter
5. Deterministic or Stochastic

#### ***1. Linear or Nonlinear Systems***

In linear systems, dependent variables and their derivatives are risen only to the first power (products of dependent variables are not allowed). When it comes to nonlinear systems, the system must be described by dependent variables or their derivatives risen to powers other than the first. Systems, which contain products of dependent variables and/or derivatives, are also nonlinear. Moreover, if you analyze these systems further, you will observe the inherent features of that particular system.

#### ***2. Constant or Variable Coefficients***

In constant coefficient systems, coefficients are not functions of the independent variables whereas in variable coefficient systems coefficients are functions of the independent variables.

#### ***3. Forced or Unforced***

Forced systems that contain non-homogeneous terms (*i.e.*, terms that do not contain the dependent variable) are coming under this category. In an unforced system, it is described by homogeneous equations (*i.e.*, all terms contain the dependent variable). In practice, to control a physical system, it is needed to have a controlled force or an input in order to keep the system states at desired conditions.

#### ***4. Lumped or Distributed Parameter***

Lumped parameter system can be described with ordinary difference or differential equations with time as the independent variable. In distributed parameter systems, it is described by partial differential equations with space and time dependence.