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Fundamentals of
Signals and Systems
Using the Web and MATLAB[®]
(Third Edition)

信号与系统基础

——应用Web和MATLAB

(第三版)

Edward W. Kamen Bonnie S. Heck 著

(英文影印版)



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北 京

内 容 简 介

本书介绍了信号与系统的基本理论、基本分析方法及其应用,包括 11 章内容:第 1 章到第 7 章是信号与系统的基本内容,讨论了时域中信号与系统的各种特性以及连续和离散时间系统的各种模型;从频域的观点分析了信号与系统,讨论了离散时间傅里叶变换(DTFT)和离散傅里叶变换(DFT),系统的傅里叶分析,拉普拉斯变换, z 变换和线性时不变系统的传输函数表示法等内容;第 8 章以后为扩展内容,包括利用传输函数表示法分析线性时不变连续时间系统,将传输函数思想用于控制问题;将拉普拉斯和 z 变换的思想用于数字滤波器和控制器的设计;对线性时不变连续时间和离散时间系统的状态描述的基本理论进行了阐述。另外,本书还给出了大量的 MATLAB 软件仿真实例。

本书可作为电类各专业信号与系统课程的双语教材或参考书,也可供工程技术人员参考。

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Preface

With the presentation at an introductory level, this book contains a comprehensive treatment of continuous-time and discrete-time signals and systems, with demos on the textbook website, data downloaded from the Web, and illustrations of numerous MATLAB® commands for the solution of a wide range of problems arising in engineering and in other fields such as financial data analysis. The third edition is a major revision of the previous edition in that the degree of mathematical complexity has been reduced, practical applications involving downloaded data and other illustrations have been added, and the material has been reorganized in a significant way so that the flexibility in using the book in a one-quarter or one-semester course should be greatly enhanced. Highlights of the revised content of the third edition include the following:

1. The presentation has been simplified by deletion or rewrite of various mathematical parts of the previous edition, and by inclusion of new illustrations that should give additional insight into the meaning and significance of the mathematical formulations covered in the text. Summaries have been added at the end of the chapters to highlight the material covered in the chapters.
2. The core of the new edition consists of Chapters 1–7, most of which an instructor should be able to cover in a one-quarter course. For a one-semester course, an instructor should be able to cover the material in Chapters 1–7 and then select material on filtering, controls, and/or the state representation that can be found in Chapters 8–11.
3. The new edition contains practical applications that use actual data downloaded from the Web. It is shown how the data can be downloaded and then imported into MATLAB for analysis by techniques covered in the text. The focus is on the problem of data analysis in the presence of noise, which often arises in engineering, business and finance, and other fields. Details are given on the analysis of stock price data with the objective of determining if the trend in the stock price is up or down.
4. The new edition contains a major enhancement of the MATLAB component. In particular, the MATLAB Symbolic Math Toolbox that is available in the Student Version (7.0.1) of MATLAB is used throughout the text to complement and simplify various computational aspects of the theory and examples given in

The book includes a wide range of examples and problems on different areas in engineering, including electrical circuits, mechanical systems, and electromechanical devices (such as a dc motor). Examples are also given on data analysis, with part of the emphasis on the filtering or smoothing of noisy data (such as stock price data) for the purpose of revealing the trend of the data. It is also shown how the dominant cyclic components can be determined and extracted from time series data by use of the discrete Fourier transform (DFT). Other features of the book are a parallel treatment of continuous-time and discrete-time signals and systems, and three chapters on feedback control, digital filtering, and the state representation that prepare students for senior electives in these topics.

The book begins with the time-domain aspects of signals and systems in Chapters 1 and 2. These include the basic properties of signals and systems, the discrete-time convolution model, the input/output difference equation model, the input/output differential equation model, and the continuous-time convolution model. Chapter 3 begins the treatment of signals and systems from the frequency-domain standpoint. Starting with signals that are a sum of sinusoids, the presentation then goes into the Fourier series representation of periodic signals and on to the Fourier transform of nonperiodic signals. The use of the Fourier transform in the study of signal modulation is also considered in Chapter 3. Chapter 4 deals with the Fourier analysis of discrete-time signals with the focus on the discrete-time Fourier transform (DTFT) and the discrete Fourier transform (DFT). The DFT is used to determine the dominant sinusoidal components of a discrete-time signal in the presence of noise, with applications given in terms of data downloaded from the Web. Then in Chapter 5, the Fourier theory is applied to the study of both continuous-time and discrete-time systems. Applications to ideal analog filtering, sampling, signal reconstruction, and digital filtering are pursued in Chapter 5.

After the Fourier theory, the study of the Laplace transform begins in Chapter 6 with the definition and properties of the Laplace transform and the transfer function representation of linear time-invariant continuous-time systems. In Chapter 7, the z -transform is introduced, and the transfer function representation of linear time-invariant discrete-time systems is pursued. This leads to the notion of the frequency response function, which is first considered in Chapter 5. In Chapter 8, the analysis of linear time-invariant continuous-time systems is carried out by the use of the transfer function representation. The transfer function framework is then applied to the problem of control in Chapter 9; and in Chapter 10, the Laplace and z -transform frameworks are applied to the design of digital filters and controllers. In Chapter 11, the fundamentals of the state description of linear time-invariant continuous-time and discrete-time systems are presented.

As noted, the book can be used in a one-quarter or one-semester course in signals and systems, with Chapters 1–7 (or parts of these chapters) covered in a one-quarter course and parts of Chapters 1–11 covered in a one-semester course. By selecting appropriate sections and chapters from the book, an instructor could cover the continuous-time case in one course and the discrete-time case in a second course.

The authors wish to thank the faculty who have used the previous editions of the book in a course, and the students who have taken a course with the book designated as the course text, for their numerous helpful comments. We also appreciate the written comments made by the following reviewers: Professor Charles W. Brice, University of South

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EDWARD W. KAMEN
BONNIE S. HECK

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Fundamental Concepts

The concepts of signals and systems arise in virtually all areas of technology, including electrical circuits, communication devices, signal processing devices, robotics and automation, automobiles, aircraft and spacecraft, biomedical devices, chemical processes, and heating and cooling devices. The concepts of signals and systems are also of great importance in other areas of human endeavor, such as in science and economics. In this chapter various fundamental aspects of signals and systems are considered. The chapter begins with a brief introduction to continuous-time and discrete-time signals given in Sections 1.1 and 1.2. In Section 1.2 it is shown how discrete-time data can be acquired for analysis by downloading data from the Web. Then in Section 1.3 the concept of a system is introduced, and in Section 1.4 three specific examples of a system are given. In Section 1.5 of the chapter, the basic system properties of causality, linearity, and time invariance are defined. A summary of the chapter is given in Section 1.6.

1.1 CONTINUOUS-TIME SIGNALS

A signal $x(t)$ is a *real-valued*, or *scalar-valued*, function of the time variable t . The term *real valued* means that for any fixed value of the time variable t , the value of the signal at time t is a real number. When the time variable t takes its values from the set of real numbers, t is said to be a *continuous-time variable* and the signal $x(t)$ is said to be a *continuous-time signal* or an *analog signal*. Common examples of continuous-time signals are voltage or current waveforms in an electrical circuit, audio signals such as speech or music waveforms, positions or velocities of moving objects, forces or torques in a mechanical system, bioelectric signals such as an electrocardiogram (ECG) or an electroencephalogram (EEG), flow rates of liquids or gases in a chemical process, and so on.

Given a signal $x(t)$ that is very complicated, it is often not possible to determine a mathematical function that is exactly equal to $x(t)$. An example is a speech signal, such as the 50-millisecond (ms) segment of speech shown in Figure 1.1. The segment of speech shown in Figure 1.1 is the “sh”-to-“u” transition in the utterance of the word “should.” Due to their complexity, signals such as speech waveforms are usually not specified in mathematical form. Instead, they may be given by a set of sample values. For example, if $x(t)$ denotes the speech signal in Figure 1.1, the signal can be represented by the set of sample values

$$\{x(t_0), x(t_1), x(t_2), x(t_3), \dots, x(t_N)\}$$

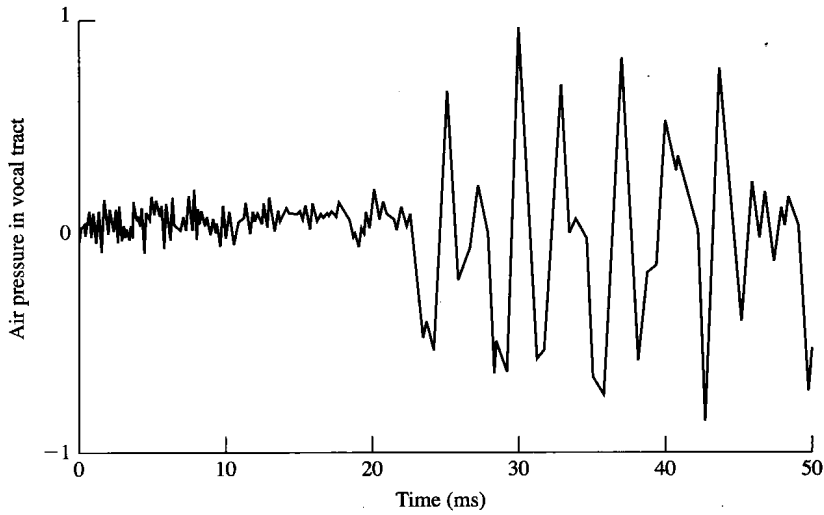


FIGURE 1.1
Segment of speech.

where $x(t_i)$ is the value of the signal at time t_i , $i = 0, 1, 2, \dots, N$, and $N + 1$ is the number of sample points. This type of signal representation can be generated by sampling the speech signal. Sampling is discussed briefly in Section 1.2 and then is studied in depth in later chapters.

In addition to the representation of a signal in mathematical form or by a set of sample values, signals can also be characterized in terms of their “frequency content” or “frequency spectrum.” The representation of signals in terms of the frequency spectrum is accomplished by using the Fourier transform, which is studied in Chapters 3 to 5.

Some simple examples of continuous-time signals that can be expressed in mathematical form are given next.

1.1.1 Step and Ramp Functions

Two simple examples of continuous-time signals are the unit-step function $u(t)$ and the unit-ramp function $r(t)$. These functions are plotted in Figure 1.2.

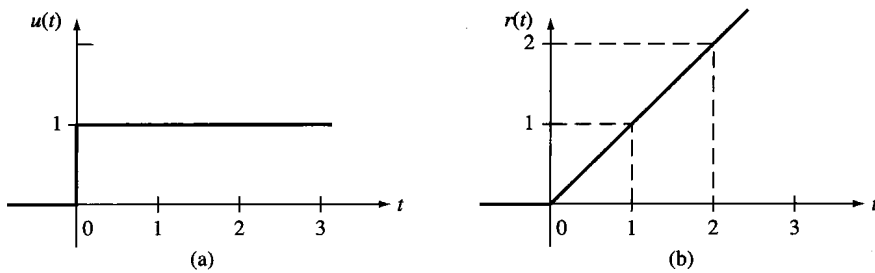


FIGURE 1.2
(a) Unit-step and (b) unit-ramp functions.

The *unit-step function* $u(t)$ is defined mathematically by

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Here *unit step* means that the amplitude of $u(t)$ is equal to 1 for all $t \geq 0$. [Note that $u(0) = 1$; in some textbooks, $u(0)$ is defined to be zero.] If K is an arbitrary nonzero real number, $Ku(t)$ is the step function with amplitude K for $t \geq 0$.

For any continuous-time signal $x(t)$, the product $x(t)u(t)$ is equal to $x(t)$ for $t \geq 0$ and is equal to zero for $t < 0$. Thus multiplication of a signal $x(t)$ with $u(t)$ eliminates any nonzero values of $x(t)$ for $t < 0$.

The *unit-ramp function* $r(t)$ is defined mathematically by

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Note that for $t \geq 0$, the slope of $r(t)$ is 1. Thus $r(t)$ has “unit slope,” which is the reason $r(t)$ is called the unit-ramp function. If K is an arbitrary nonzero scalar (real number), the ramp function $Kr(t)$ has slope K for $t \geq 0$.

The unit-ramp function $r(t)$ is equal to the integral of the unit-step function $u(t)$; that is,

$$r(t) = \int_{-\infty}^t u(\lambda) d\lambda$$

Conversely, the first derivative of $r(t)$ with respect to t is equal to $u(t)$, except at $t = 0$, where the derivative of $r(t)$ is not defined.

1.1.2 The Impulse

The *unit impulse* $\delta(t)$, also called the *delta function* or the *Dirac distribution*, is defined by

$$\delta(t) = 0, \quad t \neq 0$$

$$\int_{-\varepsilon}^{\varepsilon} \delta(\lambda) d\lambda = 1, \quad \text{for any real number } \varepsilon > 0$$

The first condition states that $\delta(t)$ is zero for all nonzero values of t , while the second condition states that the area under the impulse is 1, so $\delta(t)$ has unit area.

It is important to point out that the value $\delta(0)$ of $\delta(t)$ at $t = 0$ is not defined; in particular, $\delta(0)$ is not equal to infinity. The impulse $\delta(t)$ can be approximated by a pulse centered at the origin with amplitude A and time duration $1/A$, where A is a very large positive number. The pulse interpretation of $\delta(t)$ is displayed in Figure 1.3.

For any real number K , $K\delta(t)$ is the impulse with area K . It is defined by

$$K\delta(t) = 0, \quad t \neq 0$$

$$\int_{-\varepsilon}^{\varepsilon} K\delta(\lambda) d\lambda = K, \quad \text{for any real number } \varepsilon > 0$$

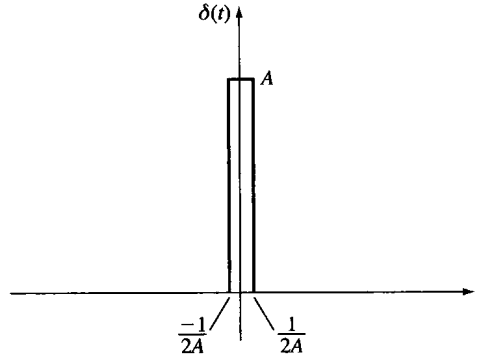


FIGURE 1.3
Pulse interpretation of $\delta(t)$.

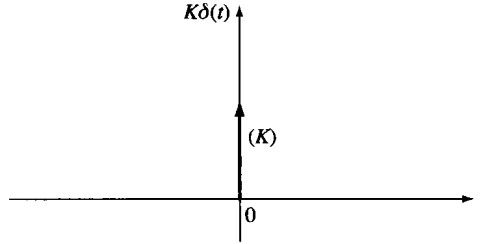


FIGURE 1.4
Graphical representation of the impulse $K\delta(t)$.

The graphical representation of $K\delta(t)$ is shown in Figure 1.4. The notation “(K)” in the figure refers to the area of the impulse $K\delta(t)$.

The unit-step function $u(t)$ is equal to the integral of the unit impulse $\delta(t)$; more precisely,

$$u(t) = \int_{-\infty}^t \delta(\lambda) d\lambda, \text{ all } t \text{ except } t = 0$$

To verify this relationship, first note that for $t < 0$,

$$\int_{-\infty}^t \delta(\lambda) d\lambda = 0, \text{ since } \delta(t) = 0 \text{ for all } t < 0$$

For $t > 0$,

$$\int_{-\infty}^t \delta(\lambda) d\lambda = \int_{-t}^t \delta(\lambda) d\lambda = 1, \text{ since } \int_{-\varepsilon}^{\varepsilon} \delta(\lambda) d\lambda = 1 \text{ for any } \varepsilon > 0$$

1.1.3 Periodic Signals

Let T be a fixed positive real number. A continuous-time signal $x(t)$ is said to be periodic with period T if

$$x(t + T) = x(t) \text{ for all } t, -\infty < t < \infty \quad (1.1)$$



Signals
and
Sounds

Note that if $x(t)$ is periodic with period T , it is also periodic with period qT , where q is any positive integer. The *fundamental period* is the smallest positive number T for which (1.1) holds.

An example of a periodic signal is the sinusoid

$$x(t) = A \cos(\omega t + \theta), -\infty < t < \infty \quad (1.2)$$

Here A is the amplitude, ω the frequency in radians per second (rad/sec), and θ the phase in radians. The frequency f in hertz (Hz) (or cycles per second) is $f = \omega/2\pi$.

To see that the sinusoid given by (1.2) is periodic, note that for any value of the time variable t ,

$$A \cos\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \theta\right] = A \cos(\omega t + 2\pi + \theta) = A \cos(\omega t + \theta)$$

Thus the sinusoid is periodic with period $T = 2\pi/\omega$, and in fact, $2\pi/\omega$ is the fundamental period. The sinusoid $x(t) = A \cos(\omega t + \theta)$ is plotted in Figure 1.5 for the case when $-\pi/2 < \theta < 0$. Note that if $\theta = -\pi/2$, then

$$x(t) = A \cos(\omega t + \theta) = A \sin \omega t$$

An important question for signal analysis is whether or not the sum of two periodic signals is periodic. Suppose that $x_1(t)$ and $x_2(t)$ are periodic signals with fundamental

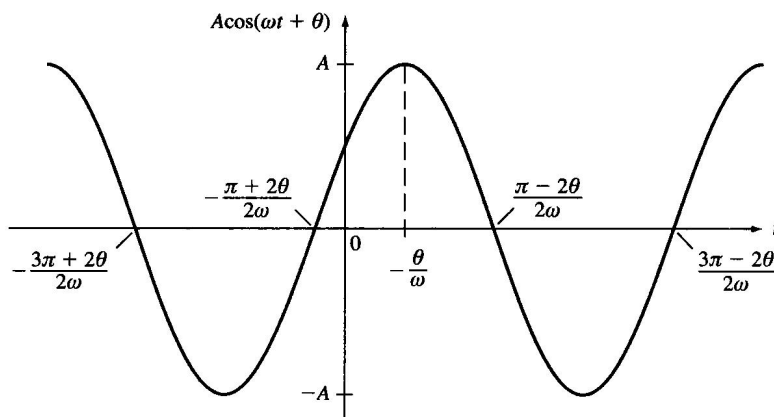


FIGURE 1.5

Sinusoid $x(t) = A \cos(\omega t + \theta)$ with $-\pi/2 < \theta < 0$.

periods T_1 and T_2 , respectively. Then is the sum $x_1(t) + x_2(t)$ periodic; that is, is there a positive number T such that

$$x_1(t + T) + x_2(t + T) = x_1(t) + x_2(t) \quad \text{for all } t? \quad (1.3)$$

It turns out that (1.3) is satisfied if and only if the ratio T_1/T_2 can be written as the ratio q/r of two integers q and r . This can be shown by noting that if $T_1/T_2 = q/r$, then $rT_1 = qT_2$, and since r and q are integers, $x_1(t)$ and $x_2(t)$ are periodic with period rT_1 . Thus the expression (1.3) follows with $T = rT_1$. In addition, if r and q are coprime (i.e., r and q have no common integer factors other than 1), then $T = rT_1$ is the fundamental period of the sum $x_1(t) + x_2(t)$.



Periodicity of Sums of Sinusoids

Example 1.1 Sum of Periodic Signals

Let $x_1(t) = \cos(\pi t/2)$ and $x_2(t) = \cos(\pi t/3)$. Then $x_1(t)$ and $x_2(t)$ are periodic with fundamental periods $T_1 = 4$ and $T_2 = 6$, respectively. Now

$$\frac{T_1}{T_2} = \frac{4}{6} = \frac{2}{3}$$

Then with $q = 2$ and $r = 3$, it follows that the sum $x_1(t) + x_2(t)$ is periodic with fundamental period $rT_1 = (3)(4) = 12$ seconds.

1.1.4 Time-Shifted Signals

Given a continuous-time signal $x(t)$, it is often necessary to consider a *time-shifted* version of $x(t)$: If t_1 is a positive real number, the signal $x(t - t_1)$ is $x(t)$ shifted to the right by t_1 seconds and $x(t + t_1)$ is $x(t)$ shifted to the left by t_1 seconds. For instance, if $x(t)$ is the unit-step function $u(t)$ and $t_1 = 2$, then $u(t - 2)$ is the 2-second right shift of $u(t)$ and $u(t + 2)$ is the 2-second left shift of $u(t)$. These shifted signals are plotted in Figure 1.6. To verify that $u(t - 2)$ is given by the plot in Figure 1.6a, evaluate $u(t - 2)$ for various values of t . For example, $u(t - 2) = u(-2) = 0$ when $t = 0$, $u(t - 2) = u(-1) = 0$ when $t = 1$, $u(t - 2) = u(0) = 1$ when $t = 2$, and so on.

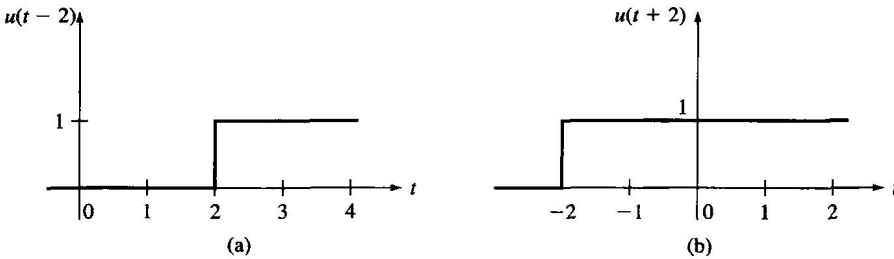


FIGURE 1.6 Two-second shifts of $u(t)$: (a) right shift; (b) left shift.

For any fixed positive or negative real number t_1 , the time shift $K\delta(t - t_1)$ of the impulse $K\delta(t)$ is equal to the impulse with area K located at the point $t = t_1$; in other words,

$$K\delta(t - t_1) = 0, \quad t \neq t_1$$

$$\int_{t_1-\varepsilon}^{t_1+\varepsilon} K\delta(\lambda - t_1) d\lambda = K, \quad \text{any } \varepsilon > 0$$

The time-shifted unit impulse $\delta(t - t_1)$ is useful in defining the *sifting property* of the impulse given by

$$\int_{t_1-\varepsilon}^{t_1+\varepsilon} f(\lambda)\delta(\lambda - t_1) d\lambda = f(t_1), \quad \text{for any } \varepsilon > 0$$

where $f(t)$ is any real-valued function that is continuous at $t = t_1$. (Continuity of a function is defined subsequently.) To prove the sifting property, first note that since $\delta(\lambda - t_1) = 0$ for all $\lambda \neq t_1$, it follows that

$$f(\lambda)\delta(\lambda - t_1) = f(t_1)\delta(\lambda - t_1)$$

Thus

$$\begin{aligned} \int_{t_1-\varepsilon}^{t_1+\varepsilon} f(\lambda)\delta(\lambda - t_1) d\lambda &= f(t_1) \int_{t_1-\varepsilon}^{t_1+\varepsilon} \delta(\lambda - t_1) d\lambda \\ &= f(t_1) \end{aligned}$$

which proves the sifting property.

1.1.5 Continuous and Piecewise-Continuous Signals

A continuous-time signal $x(t)$ is said to be *discontinuous* at a fixed point t_1 if $x(t_1^-) \neq x(t_1^+)$, where $t_1 - t_1^-$ and $t_1^+ - t_1$ are infinitesimal positive numbers. Roughly speaking, a signal $x(t)$ is discontinuous at a point t_1 if the value of $x(t)$ “jumps in value” as t goes through the point t_1 .

A signal $x(t)$ is *continuous* at the point t_1 if $x(t_1^-) = x(t_1) = x(t_1^+)$. If a signal $x(t)$ is continuous at all points t , $x(t)$ is said to be a *continuous signal*. The reader should note that the term *continuous* is used in two different ways; that is, there is the notion of a continuous-time signal and there is the notion of a continuous-time signal that is continuous (as a function of t). This dual use of *continuous* should be clear from the context.

Many continuous-time signals of interest in engineering are continuous. Examples are the ramp function $Kr(t)$ and the sinusoid $x(t) = A \cos(\omega t + \theta)$. Another example of a continuous signal is the triangular pulse function displayed in Figure 1.7. As indicated in the figure, the triangular pulse is equal to $(2t/\tau) + 1$ for $-\tau/2 \leq t \leq 0$ and is equal to $(-2t/\tau) + 1$ for $0 \leq t \leq \tau/2$.