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Mathematics Monograph Series 19

Economic Operation of Electricity Market and Its Mathematical Methods

Xiaojiao Tong Hongming Yang

（电力市场的经济运行及其数学方法）

SCIENCE PRESS
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Foreword

Modern operational research (OR) is generally regarded as being born during the second world war when scientists from different areas worked together to study how to make decisions for military operations using mathematical methods, though ideas of OR can be traced back to ancient China, an example in the warring state period (475BC-221BC) is the famous Tian Ji horse-racing, in which Sun Bin suggested a strategy to Tian Ji on how to utilize his horses to win two out of three rounds over the horses of the King Wei of Qi.

Operational Research has been playing an important role in social and economic development of modern China. Early highlights of OR application activities in China was conducted by late professor Loo-Keng Hua and his group during the Great Cultural Revolution in 1960-1970s. Hua and his group travelled to almost all the provinces in China to teach basic optimization techniques in factories and farm-fields to promote the applications of OR techniques and methods in production and daily operations. The famous "Chinese postman problem" proposed by professor Meigu Gwan was also initiated by studying real problems such as transportation and scheduling.

Traditionally, the OR community of China is very theoretical comparing with the international OR community. The reason for this is that the Operations Research Society of China was founded by mathematicians. However, internationally, the current main streams of OR researches are on interdisciplinary researches and newly emerging problems such as compressive sensing, wireless communication, image reconstruction, inverse problems, stochastic network, supply chains, computational biology and financial engineering. Moreover, due to the rapid economic development, China nowadays faces lots of complicated problems in energy, health care, environmental pollution, transportation, telecommunication, financial engineering, urban planning, transnational logistics, natural resource consumption, national defense and international relations. To obtain better solutions to these problems, OR methods and techniques will play an indispensible role.

I am very happy to see that in recent years more and more OR researchers in China turn their attentions to cross-disciplinary researches and study real-world problems, particularly those in engineering and management sciences. Prof Xiaojiao Tong is one of them. I have known her for a long time. In her earlier research career, she did nice researches on theories and methods for optimization, particularly for nonlinear equations. In recent years, founded by National Science Foundation of China's

"Problem-Drive Program", she has turned her researches interests into problems in electricity markets, and has achieved remarkable results. Currently she is one of the leading OR experts in Hunan province where I originally came from.

This book contains the research results of Prof Tong and her group in the past decade. It not only presents a very good summary of their scientific results over the years, but also provides a nice overview of OR methods for some important problems in electricity industry. The book discusses about the economic operation problems in power systems, and it gives an excellent study on how to mathematics can be used to model and solve the power system operation problems by combining new mathematical theories and algorithms with specific structured problems.

This book is a nice example of the integration of theory and practice, and it serves as a model for cross-disciplinary researches. It is an epitome of the rapid development of OR researches in China and indicates the expansion of OR into applications. Without doubt this book will make contributions to both scientific research and the development of power market operations. It will promote further cross-disciplinary OR related researches, trigger off more problem-driven projects and promote collaborations between OR researchers and engineers and scientists in other fields.

Ya-xiang Yuan
Beijing, Sept 27, 2011

Preface

This book mainly discusses about the economic operation problems in power systems which involve popular issues in system analysis since 1950s and optimization problems in the field of mathematics. Along with the working advance of market mechanism in power industry, some new problems and challenges of economic operations emerged in the complicated competitive environment. The key motivation of our research presented in this book is to study how to mathematically model and solve the power system operation problems by combining new mathematical theories and algorithms. This book covers our research results since 2002.

Our research aim is to apply new mathematical theories and methods to solve economic operation problems in power systems, especially in a market environment. We mainly focus on the modeling and solution algorithms for system operation based on its characteristics. The purpose of writing this book is twofold. Firstly, we want to introduce some scientific methods for solving the optimal operation of power system under the competitive mechanism and the new requirements of energy sources and environmental protection. Hopefully, the presented work can contribute to the scientific and rational development of power market operations. Secondly, by taking economic operation as an example, we hope that our work can motivate further research on the application of the mathematical modeling and solution methods discussed in this book for other practical scenarios.

The rest of the book is organized as followings. Chapter 1 discusses the basis of our research. An introduction of electricity market operation is given. New mathematical theories and algorithms for optimization problems focusing on solution algorithms with satisfactory convergence, such as semismooth function, semismooth and smoothing Newton method and the KKT system of semi-infinite programming, are also studied. Some of these topics are included in our previous works in mathematical field. Chapter 2 focuses on the available transfer capability (ATC) of power systems. The mathematical modeling and calculating method of ATC, considering the security of systems, are studied. Determination of ATC is formulated as a problem of finding the solutions to a system of equations using the pointwise maximum function (PMF) which collapses all system operating constraints into one equation. A new semismooth ATC model is introduced with two solution algorithms, smoothing Newton method and decoupled Newton method. Smoothing Newton method treats smoothing parameter as an independent variable, whereas, the decoupled Newton method takes account of the inherent weak-coupling characteristics of power system and is suitable for solving large scale problems. Furthermore, the ATC region (ATCR) is proposed to obtain the global message for different trades, which is an extension of ATC issue. In Chapter 3, we study the optimal power flow (OPF) approach in power system from the point of view of models and algorithms. Two classes of OPF problems are discussed. One is the traditional OPF in which the optimal operation of system with security constraints is considered. The other one is OPF with transient stability (OTS) constraints which include both the security and stability limits of systems. The convergence of traditional decoupled OPF approach is studied as well. All aforementioned problems are based on the nonsmooth analysis and the semismooth Newton method. Chapter 4 considers some economic operation issues in power systems under the conditional value-at-risk (CVaR) management. A new calculating method having high computation efficiency is proposed for solving the portfolio optimization with CVaR. Then the approach is used to solve the allocation of generation asset in electricity market. The worst-case CVaR (WCVaR) is also presented for cases that the message of random variable is partially known and the methods based on WCVaR is tested for scenario planning in electricity market. Furthermore, by using the CVaR management, a security-constrained economic dispatch model is presented and studied from the modeling and approach issues. In Chapter 5, we study the dynamic equilibrium of power market that considers the constraints of realistic power networks with emphasis on dynamic model, Nash equilibrium and the stability of power markets. The dynamic model is formulated as a set of discrete difference equations embedded within an optimization problem of market clearing. Using a nonlinear complementary function, the complex discrete difference dynamic model is transformed into a set of familiar discrete difference algebraic equations based on which Nash equilibrium of power market and its stability are calculated. In Chapter 6, we investigate the chaos control of power market when the market is beyond the stability region of Nash equilibrium. Given the poor economic performance in the chaotic state, the time-delayed feedback chaos control is proposed to improve the economic benefit of power market. In the writing of this book, the last section of each chapter contains the comments and notes that cover the status of the studied problems, the characteristics of our research and potential research topics. We hope this writing can provide readers with more messages and interesting research issues.

Chapter 1 to 4 are written by Xiaojiao Tong, except Section 1.1 written by Hongming Yang. Chapter 5 and Chapter 6 are written by Hongming Yang. Dr. Mei Li took part in the writing of Section 1.1. The overall plan of this book is done by Xiaojiao Tong. We would like to thank Professor Liqun Qi from the Hong Kong Polytechnic University and Professor Felix Wu from Hong Kong University who contributed and supervised most of the works introduced in this book. We would like to show our appreciation to the National Natural Science Foundation of China (11171095, 10871031, 10926189, 10826099, 60474070, 71071025, 70601003), the Natural Science United Foundation of Hunan-Hengyang (10JJ8008), and the Outstanding Youth Science Foundation of Hunan Province (10JJ1010) that supported our research works as well as this book (10871031, 10926189, 10826099, 60474070, 10JJ8008). We also want to thank Changsha University of Science and Technology and Hengyang Normal University for supporting our research.

We enjoyed our research and made great effort to two research issues. One is the application of new mathematical methods in power systems, and the other is the study of some new mathematical problems arising from power systems. Operation problems in uncertain market environment are interesting topics for our future research. We welcome the readers to share their valuable comments on our work and help improve its theoretical and practical research value.

Xiaojiao Tong, Hongmin Yang
April, 2011 in Changsha

List of Notation

A^{T}:	transpose of matrix (vector) A, 4
VaR (CVaR):	value-at-risk (conditional VaR), 6
$[x]^{+}$:	$= \max\{x, 0\}$, 7
$p(y)$:	density function of y, 6
∇F:	derivative of differentiable function F, 8
∂F,	Clarke generalized Jacobian of F, 9
$\mathrm{conv}\{C\}$:	convex hell of set C, 18
$o(c)$ $(O(c))$:	high (same) order small quantity than scalar C, 9
$a \perp b$:	vectors a and b orthogonal, 10
$a \circ b$:	Hadamard product of a and b, 10
$\mathbb{R}^n$:	n dimensional space,
NCP:	nonlinear complementarity problem, 10
x^k:	kth iteration of x, 16
$\{x^k\}$:	iterative sequence, 16
$\|A\|$:	norm of matrix (vector) of A , 17
$\mathrm{dist}(x, C)$:	distance from point x to set C, 18
$\|I\|$:	number of index set I, 18
$\mathbb{R}_{+}$:	nonnegative real number set, 20
$\mathbb{R}_{++}$:	strictly positive real number set, 20
$F'(x)$:	$= \nabla F(x)^{\mathrm{T}}$, derivative of differentiable function, 21
Ω_{SR}:	security region, 25
Δx:	updated value of x, 28
$\partial_i F(x_1, x_2)$:	partial generalized derivative for x_i, 31
Ω_{ATCR}:	available transfer capability region, 42
$\partial\Omega$:	boundary of set Ω, 44
$\dfrac{\partial^2 f}{\partial x_i \partial x_j}$, $\partial^2_{x_i x_j} f$:	second derivative of f for x_i, x_j, 45
$P_{\mathrm{E}}, P_{\mathrm{I}}$:	functions for real power, 58
$Q_{\mathrm{E}}, Q_{\mathrm{I}}$:	functions for reactive power, 58
$L(\cdot,\cdot)$:	Lagrangian function, 60
$\pi_x \partial H, \pi_y \partial H$:	matrix sets with $(\pi_x \partial H, \pi_y \partial H) = \partial H$, 63, 64
$\dot{x}$:	differential of x for time t, 75
$\det(A)$:	determinant of matrix A, 75
$E[\cdot]$:	expectation of random, 105
$\arg\min F$:	optimal solution of $\min F$, 110
$\sup X$:	minimal upper bound in X, 116
WCVaR:	worst-case CVaR, 116
$\mathcal{P}_M$:	mixture distribution for random variable, 117
$\mathcal{P}_D$:	discrete distribution, 117
$\mathrm{mid}\{a, b, c\}$:	midpoint value function for a, b, c, 156
ISO:	independent system operator, 157
LSE:	load serving entity, 158
q^*_{-i}:	dropped the ith component of vector q^*, 165
MWh:	megawatt -hour, 167
$\mathcal{N}(q)$:	open neighbourhood of point q, 170

Contents

Chapter 1

Electricity Markets and Preliminaries of Mathematics

This chapter establishes the basis for the following chapters. We first introduce the changes in electric power industry, including its deregulation and reformation, competitive structure of electricity market and other concerns emerged with new market mechanism. A brief introduction of new mathematical concepts and methods is also given for related optimization issues, such as conditional value-at-risk (CVaR) management, nonlinear complementarity problem (NCP), semismooth and smoothing Newton method.

1.1 Electricity Market

1.1.1 Deregulation and reformation of power industry

Traditional electric power industry is vertically integrated monopoly utility. In 1980s, some economists started to argue this operation mode due to its low efficiency and the high price. A competition mechanism is urged to be implemented in the electric power industry. In late 1970s, Chile became the first country to introduce privatization and market concepts to electric power systems. However, the reform failed to achieve revolutionary progress at that time. The key event of electricity market reform occurred in 1990 when the UK Government privatized the UK electricity supply utilities and deregulated the electric power industry. The revolution of electric power industry in UK was later followed by other Commonwealth countries, notably Australia, New Zealand, and some regional markets such as Alberta. The enactment of Energy Policy Act in 1992 and the subsequent FERC's orders 888 and 889 in 1996 marked the start of deregulation of electricity supply in the United States. The PJM and New York Independent System Operator (ISO) are now one of the most successful electricity markets in the world. Currently, the worldwide reform of electric power industry is progressing at an unprecedented pace in the history. Electricity market is a system which organizes, manages and coordinates the power system by means of laws and economic tools under the principle of openness, competition and fairness.

The aim of electricity market is to improve the efficiency of electric power industry, reduce the electricity price, and at the same time, ensure the security of power system. Meanwhile, the introduction of competition mechanism in electric power system also generated some new issues to be concerned.

1.1.2 Competitive structure of electricity market

Fig. 1.1 illustrates the evolution of the electricity industry from regulated monopoly

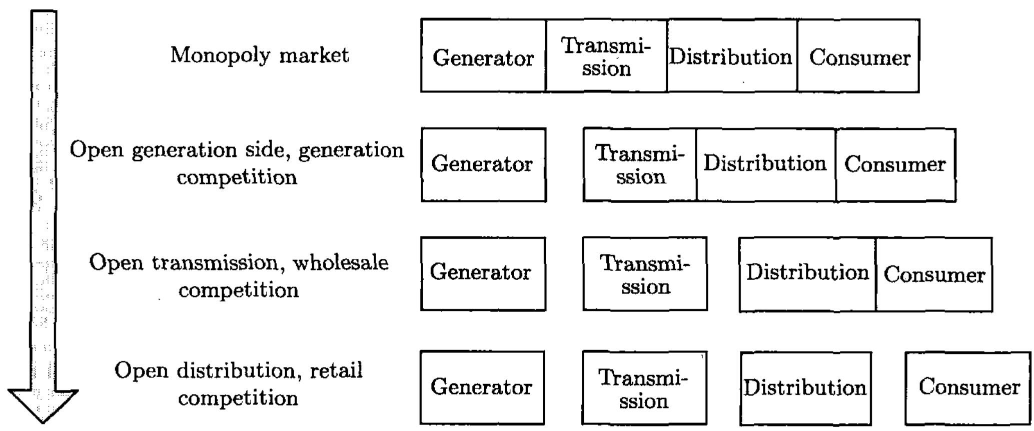

Fig. 1.1 Evolution of electricity market

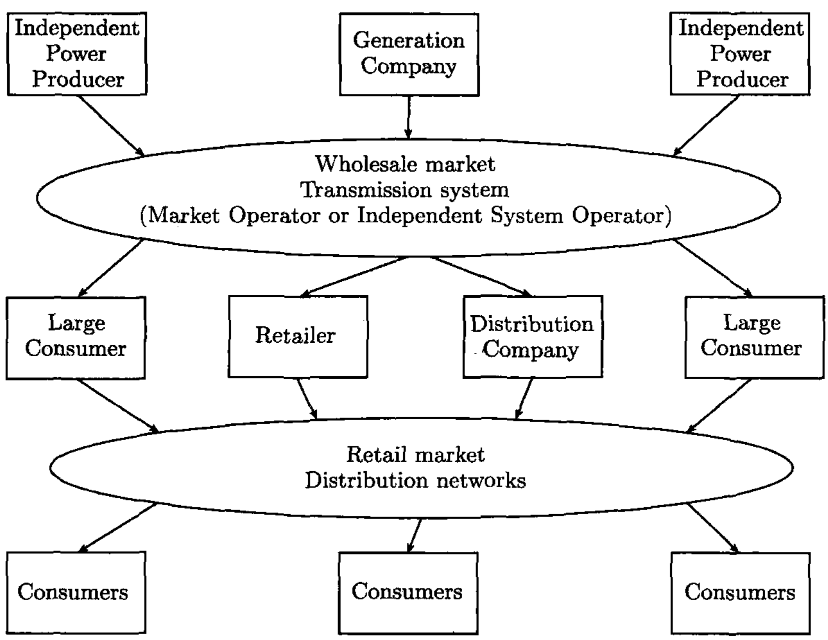

Fig. 1.2 Mode of competitive electricity market

to retail competition. The traditional power industry used to be managed under the integrated utility that owns the generators, transmission and distribution networks to supply electric power to the consumers, as shown in the first line of the figure. The first step of deregulation is to introduce competition on the electricity supply side. The subsequent openness of transmission network realize the wholesale market. sequentially the openness of distribution network leads to the establishment of retail market as shown in Fig. 1.2.

In the electricity market, the generation and retail sectors are separated from natural monopoly of transmission and distribution. Electricity market operates through submitting bids to buy and sell, and using supply and demand principles to set the price with the management of market operator (MO). Large consumers can purchase energy directly on the wholesale market. Some long-term trades are generally considered as private bilateral contracts between participants. In the retail market, the consumers could choose their suppliers freely. Such form of electricity market is beneficial to the openness and fairness of the operation of electric power industry.

1.1.3 Concerned issues in electricity market

Different from other markets, electricity market has not only economic problem but also physical problems. The results obtained by using pure economical principle will affect the operation of power system, and vice versa. For example, the inherent characteristics of power system determines how electric power flows through a transmission network. Electricity is by its nature difficult to be stored and has to be available on demand. Due to instantaneous balance between demand and supply, the operation and control of electricity market are quite different from other markets. On the other hand, in a competitive electricity market, the participants always try to avoid high risks resulted from market uncertainties like electricity price and maximize their own profits at the same time. Hence, the operation of electricity market aims to ensure the security of power system and coordinates the benefits of various market participants. As mentioned above, during the operation of a competitive electricity market, there are many problems needs to be resolved.

Problem 1 Economic operation of electricity market.

Prior to restructuring the electric industry, the economic operation essentially is economic dispatch. Economic dispatch with operation limits can be regarded as security constrained optimal power flow (OPF). OPF determines the optimal operation state of power system under specific criteria and operating constraints, by means of adjusting relative control variables. It is a fundamental tool for power system planning, operation, economic dispatch, security re-dispatch, reactive power management and etc.

OPF plays a more important role in competitive electricity market. It transfers the most concerned problem in electricity market to the optimization of the interests of market participants. Moreover, the coordination of economic operation and system security, real power and reactive power makes OPF become more indispensable than ever.

Mathematically, an OPF problem is a nonlinear optimization problem as follows:

$$\begin{aligned} \min \quad & C(x_P, x_Q) \\ \text{s.t.} \quad & P_{\mathrm{E}}(x_P, x_Q) = 0, \quad P_{\mathrm{I}}(x_P, x_Q) \leqslant 0, \\ & Q_{\mathrm{E}}(x_P, x_Q) = 0, \quad Q_{\mathrm{I}}(x_P, x_Q) \leqslant 0, \end{aligned} \tag{1.1}$$

where x_P is a vector called *active power variables* including vectors of real power generations P_{G} and bus voltage angles θ; x_Q is a vector called *reactive power variables* including vectors of reactive power generations Q_{G}, bus voltage magnitudes V and transformer taps t; $P_{\mathrm{E}}(\cdot)$ and $P_{\mathrm{I}}(\cdot)$ represent equation and inequality constraints with respect to active power $Q_{\mathrm{E}}(\cdot)$ and $Q_{\mathrm{I}}(\cdot)$ are vectors of equation and inequality coustraints with respect to reactive power.

In a competitive electricity market, generation companies prefer strategic bidding for more profits. But it will bring generation companies high risks. One solution to risk aversion is called portfolio optimization. As we know, besides energy market, there are financial derivatives, such as futures and option contracts, in the electricity market. Therefore, generation companies choose different optimal portfolio in the market to avoid risks. This problem can be formulated as an OPF problem.

Problem 2 Equilibrium analysis of electricity market.

As mentioned above in a competitive electricity market, many rational generation companies dynamically adjust their bid parameters to maximize their profits by analyzing the ISO published clearing price and power quantity. If no generation company has any incentives to unilaterally deviate from its bidding strategy, the market reaches Nash equilibrium which is defined as follows.

Definition 1.1.1 *A Nash equilibrium is a vector* $a^* = (a_0^*, a_1^*, \cdots, a_i^*, \cdots, a_N^*)^{\mathrm{T}}$ *such that for each participant* i*, given all other participants' bid* $a_{-i}^* = (a_0^*, a_1^*, \cdots, a_{i-1}^*, a_{i+1}^*, \cdots, a_N^*)^{\mathrm{T}}$, a_i^* *maximizes the* i*-th participant's profit, that is*

$$a_i^* \in \underset{a_{li} \leqslant a_i \leqslant a_{ui}}{\operatorname{argmax}} \ \pi_i(a_i, a_{-i}^*), \tag{1.2}$$

where π_i *is the profit objective of the* i*-th participant.*

Problem 3 System security under electricity market environment. Security is one of the most important issues in power systems. However, since inde-