Self-thinning and Growth Modelling for Even-aged Chinese Fir (*Cunninghamia Lanceolata* (Lamb.) Hook.) Stands

杉木自然稀疏与生长模拟

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Brief introduction of content

Based on long-term re-measurement data obtained from permanent plots representing different planting densities of chinese fir plantations, dominant height growth model, basal area growth model, diameter distribution model and self-thinning rule were systematically built and discussed, and some new modelling technologies were put forward and used. This book includes nine chapters, and can be used and referred for those working toward the healthy management of fast-growing and high-yielding plantation.

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Preface

Forest managers, planners and policy makers forecast the outcomes of different forest management measurement in order to make wise decisions for the biggest benefit. Efficient and readily understandable models of growth and yield can become invaluable tools. With suitable inventory and other resource data, growth models provide a reliable way to examine silvicultural and harvesting options, to determine the sustainable timber yield, and examine the impacts of forest management and harvesting on other values of the forest.

In stand growth model system, dominant height growth model, basal area growth model, diameter distribution model are the three nuclear components. The self-thinning rule describes a density-dependent upper boundary of stand biomass for even-aged pure plant stands in a given environment, so far, the debate about it has not been adequately resolved. Therefore, the building of dominant height, basal area and diameter distribution models and self-thinning rule were considered as main research fields in this book.

However, growth models are of limited use on their own, and need long-term experimental data to testify, also require ancillary data to provide useful information. chinese fir is one of the most important coniferous species for timber production in southern China, with 9.21 million ha of single species or mixed stands occurring in both artificial plantations and natural forests. Long-term re-measurement data obtained from permanent plots representing different planting densities of chinese fir plantations were used to fit different models with different modelling technologies.

In our studies, algebraic difference method and generalized algebraic difference method were provided to build polymorphic dominant height model and polymorphic site index equations. A new high-performance diameter distribution function and Fuzzy functions were innovatively introduced and applied to model stand diameter distributions, and reason for differing simulation accuracies among growth equations was revealed. Self-thinning rule was testified and discussed for chinese fir plantations, the selection methods of data points and regression methods were compared and analyzed to estimate the self-thinning boundary line.

This book was researched and written over a period of several years, and many people and several institutions have supported this work in various ways. We are grateful for the long-term support of Subtropical Forest Experiment Centre of Chinese Academy of Forestry, Weimin State-owned Forest Farm in Shaowu City of Fujian province. Very thanks to Professor Tong Shuzhen for experiment design and selfless support.

Zhang Jianguo, Duan Aiguo, Sun Honggang, Fu Lihua

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Contents

CHAPTER ONE:
Modelling of dominant height growth and building of polymorphic site index equations of chinese fir plantations

Abstract Difference methods based on six growth equations such as Richards, Weibull, Korf, Logistic, Schumacher and Sloboda were adopted to build polymorphic dominant height and site index equations for chinese fir plantations in southern China. Data from stem analysis of 157 trees were used for model construction. The performance of fifteen equations including ten kinds of difference equations was compared under different conditions. Effects on modeling precision caused by the variation of fitting data sets, site index, stands age and freedom parameter were analyzed and discussed. Results showed that the attributes of inflection points of the biological growth equations had very important effects on their precision while modeling dominant height. Difference equations had a better modeling precision for regional data sets than the prototypes of equations. The polymorphic dominant height equations, such as the two-parameter polymorphic forms of Korf, Richards, Weibull and three-parameter polymorphic form of Sloboda, showed higher precision. The two-parameter polymorphic form of Korf equation was selected to build polymorphic site index equation for chinese fir plantations.
Key words Dominant height modeling; polymorphic site index model; Difference method

1
Introduction

Dominant height model and site index curve plays an important role in the stand growth and management model system (Clutter *et al.*, 1983; Avery and Burkhart, 1994). Currently, two kinds of methods are often adopted for building dominant height model. One method is to directly apply the prototype of a theoretical equation, many single-variable functions, having the asymptotic value and inflection point, can be used for modelling stand dominant height growth(Zeide, 1993; Garçia, 1997); the other is using the differential form of theoretical equation(Border *et al.*, 1988; Amaro *et al.*, 1998). The latter approach is more flexible, and has increasingly become main research methods, but there is not a few specific applications of such method. For the development of site index curve, 3 ways usually are selected as follows: ① parameter estimation method (Mark and Nick, 1998); ② guide curve method (Newberry and Pienaar, 1978; Lee and Hong, 1999); ③ difference equation (Border *et al.*, 1984; Lee, 1999; Kalle, 2002). Generally, polymorphic dominant height model can explain and describe the phenomenon that a site index curve decides a dominant height curve, better simulate dominant height growth than the simplex model, and has good theoretical explain (Devan and Burkhart, 1982 ; Mark and Nick, 1998).

For the building of polymorphic site index equation of chinese fir plantation, the methods ①, ② are used to be selected. The method ① expresses all or some equation parameters as a function of site index (Scientific research coordination group for chinese fir cultivation in southern China, 1982; Luo *et al*., 1989; Liu and Tong, 1996), the advantages of this method is clearly expressing the polymorphic meaning of site index equation, but often having the problems that the dominant height of standard age is inconsistent with the value of site index and the site index is not easily given when the dominant height and stand age are known; the method ② directly applies the theoretical growth equation with polymorphic meaning, such as sloboda equation that is applied many times at present (Gadow and Hui, 1998), but still lacks of the studies of polymorphic expression form of other common theoretical growth equation. When differential equations are developed to build site index equations, since the freedom of choice or operation 'parameters are different in different ways, and the resulting site index equations may produce two forms including single form and polymorphic form. It is worth to make clear that the application of guide curve method and differential equation method both can get site index curve with the characteristics of single or polymorphic form while building site index curve.

In summary, although the dominant high model and polymorphic site index equation have been studied widely in the world, a large number of single and polymorphic dominant hight equations are still lacking of systematic comparison studies, and failed to specify the inherent reasons that cause high or low precision for different dominant height model. The lack of deeply understanding and exploration for the polymorphic forms of many theoretical equations virtually restricts application of theoretical equations in this aspect, and thus has affected the development of dominant high model and polymorphic site index equation for many tree species.

Based on several common dominant hight equations, the polymorphic expressions were built by adopting differential equation method, and the polymorphic expression mechanism and the advantages and disadvantages of polymorphic dominant hight equations was comprehensively discussed and analyzed. In order to provide good theoretical and practical basis for the establishment of advantages of dominant high models and site index equations of chinese fir and other tree species plantations.

2
Material and methods

2.1
Material

Data used in this article were collected from 157 analytical stems of chinese fir dominant trees in southern China. The standard sites located in Huitong county of Hunan province, Damiao Mountain of Guangxi province, Liping county of Guizhou province and Nanping and Sanyuan county of Fujian province, these regions all belonged to the central districts for chinese fir. The survey years was 1981, 1954, 1955 and 1956 respectively, the number of analytical stems were 31, 35, 43, 48 respectively, the total stems were 157.

The area of standard site of Huitong county of Hunan province was 300~500 m^2, other provinces were 1000 m^2. The stands in Fujian province originated from cutting seedlings, other provinces from seedlings.

The age of dominant trees all arrived at or near the index age (20a) for chinese fir. For trees whose age exceeded the index age, we directly used tree height at 20a to determine site index classes, for trees whose age were lower than 20a, the national site index table was applied to get respective site index class. The statistical data of standard sites in different provinces were shown as Table 1.

Table 1 Statistical table of stem analysis data of chinese fir

Standard sites	Age/a			Height/m		
	Min.	Max.	Mean	Min.	Max.	Mean
Huitong county of Hunan province	5	63	19	2.80	28.21	14.37
Damiao mountain of Guangxi province	5	43	21	2.65	27.00	17.32
Liping county of Guizhou province	5	46	23	2.33	27.65	16.18
Nanping and Sanyuan county of Fujian province	5	36	21	3.30	30.70	15.95
Total	5	63	21	2.33	30.70	15.79

2.2
Methods

2.2.1
Dominant height growth equation

Five theoretical growth equations, Richards equation, Weibull equation, Korf equation, Logistic equation and Schumacher equation, were selected as candidate equations for modelling dominant height growth process. these equations were widely used for the simulation of tree growth, especially for the Richards Equation, (Rennolls, 1995; Li, 1996; Amaro et al., 1998; Gadow and Hui, 1998; Li *et al.*, 1999). Mathematical expression of the equations were shown as Table 2.

Table 2 The mathematical expression of five theoretical growth equations

Equation	Expression	Inflection point		Parameter
		Abscissa	Ordinate	
Richards	$y=a(1-\exp(-bx))^c$	$1/(b\ln c)$	$a(1-1/c)^c$	*a, b*>0
Weibull	$y=a(1-\exp(-bx^c))$	$((c-1)/bc)^{1/c}$	$a(1-\exp(1-c)/c)$	*a, b, c*>0
Korf	$y=a\exp(-b/x^c)$	$((c+1)/bc)^{-1/c}$	$a\exp((c-1)/c)$	*a, b, c*>0
Logistic	$y=a/(1+\exp(b-cx))$	b/c	$a/2$	*a, c*>0
Schumacher	$y=a\exp(-b/x)$	$b/2$	ae^{-2}	*a, b*>0

The five above-mentioned equations were all S-shaped growth equations with inflection points and asymptotic lines. In which, equations, such as Richards, Weibull and Korf equation, had the charateristics that the coordinates of inflection points were variable multiples of asymptotic values, while Logistic equation and Schumacher equation presented a fixed multiple. The meanings of parameters and their complex relationship of these equations were explained by Duan *et al.* (2003).

2.2.2
Difference equation

For any equation that can reflect the relationship between tree height and age, the differential form always can be gotten by using the differential method, and the differential equations can simulate the height growth process of stand dominant trees. The data for fitting differential equation can be derived from the permanent plots, interval plots and temporary plots. When the data comes from long-term observation data or analytical stem materials, the use of differential equations is more appropriate (Amaro et al., 1998).

Differential method was used for five theoretical growth equations, and their differential forms were gotten. Through viewing parameter *b* as freedom parameter, retaining asymptotic parameter *a* and shape parameter *c*, four two-parameter differential equations and one one-parameter differential equation were gotten after differential elimination method.

Through differential but no elimination method, three-parameter Richards and Weibull differential equations were obtained. In order to compare the simulation effects of differential equations with different freedom parameters, two differential equations, respectively with parameter *a* or *c* as freedom parameter, were gotten from Richards function. Korf equation was taken as an example to elaborate the basic form process of every differential equation.

Through selecting any two pairs of stem analysis data of dominants and heights (t_1, H_1) and (t_2, H_2), and substituting them into Korf equation, formula (1) and (2) could be obtained as follows:

$$H_1 = a\exp(-b/t_1^c) \quad (1)$$

$$H_2 = a\exp(-b/t_2^c) \quad (2)$$

and convert to:

$$\ln H_1 - \ln a = -b/t_1^c \quad (3)$$

$$\ln H_2 - \ln a = -b/t_2^c \quad (4)$$

After divided formula (3) by (4), the difference equation of Korf could be gotten as follows.

$$H_2 = H_1^{t_1^c/t_2^c} \cdot a^{1-t_1^c/t_2^c} \quad (5)$$

The difference forms of other equations could be obtained like Korf equation (Table 3). In order to comprehensively introduce good dominant height growth equations and compare the fitting characteristics of three-parameter difference equations, the difference form of Sloboda equation was also listed (Gadow and Hui, 1998).

If letting $H = H_2$, $t = t_2$; $SI = H_1$, $T = t_1$, where SI and T respectively stand for site index and index age, and substituting into all difference equations in Table 3, the site index equations originated from corresponding difference equations could be gotten (Table 4). The site index equation of Korf was built as follows.

$$H = SI^{T^c/t^c} \cdot a^{1-T^c/t^c}$$

The dominant growth curves of different site indices (e.g. 16, 18, 20) could be obtained through above-mentioned formula. When dominant height H and stem age t were known, the stand site index could be calculated through this formula.

Table 3 The expression of every difference equation

Prototype of equation	Difference equation	Freedom parameter	Designation
Korf	$H_2 = H_1^{t_1^c/t_2^c} \cdot a^{1-t_1^c/t_2^c}$	b	(5)
Richards	$H_2 = a(1-(1-(H_1/a)^{1/c})^{t_2/t_1})^c$	b	(6a)
	$H_2 = a \cdot \exp(\ln(H_1/a) \cdot \ln(1-e^{-bt_2})/\ln(1-e^{-bt_1}))$	c	(6b)
	$H_2 = H_1((1-\exp(-bt_2))/(1-\exp(-bt_1)))^c$	a	(6c)
	$H_2 = a(1-(1-a^{-1/c}H_1^{1/c}) \cdot \exp(-b(t_2-t_1)))^c$		(6d)
Weibull	$H_2 = a - a((a-H_1)/a)^{t_2^c/t_1^c}$	b	(7a)
	$H_2 = a + (H_1 - a) \cdot \exp(-bt_2^c + bt_1^c)$		(7b)
Logistic	$H_2 = a/((a/H_1 - 1) \cdot \exp(ct_1 - ct_2) + 1)$	b	(8)
Schumacher	$H_2 = a \cdot \exp(t_1/t_2 \cdot \ln(H_1/a))$	b	(9)
Sloboda	$H_2 = a(H_1/a)^{\exp(-b/((c-1) \cdot t_1^{c-1}) + b/((c-1) \cdot t_2^{c-1}))}$	d	(10)

Table 4 Ten kinds of site index equations and their expressions of inflection point①

Prototype of equation	Site index equation	Expression of inflection point	Number
Korf	$H = SI^{T^c/t^c} \cdot a^{1-T^c/t^c}$	$t = T(\ln(a/SI)/(1+1/c))^{1/c}$	(5)
Richards	$H = a(1-(1-(SI/a)^{1/c})^{t/T})^c$	$t = -T\ln(c+1)/\ln(1-(SI/a)^{1/c})$	(6a)
	$H = a \cdot \exp(\ln(SI/a) \cdot \ln(1-e^{-bt})/\ln(1-e^{-bT}))$	$t = \frac{1}{b} \cdot \ln \frac{\ln(SI/a)}{\ln(1-\exp(-bT))}$	(6b)
	$H = SI((1-\exp(-bt))/(1-\exp(-bT)))^c$	$t = \ln c/b$	(6c)
	$H = a(1-(1-a^{-1/c}SI^{1/c}) \cdot \exp(-b(t-T)))^c$	$t = T + 1/b \cdot \ln(c \cdot (1-a^{-1/c}SI^{1/c}))$	(6d)
Weibull	$H = a - a((a-SI)/a)^{t^c/T^c}$	$t = T((1/c-1)/\ln(1-SI/a))^{1/c}$	(7a)
	$H = a + (SI - a) \cdot \exp(-bt^c + bT^c)$	$t = ((c-1)/(bc))^{1/c}$	(7b)
Logistic	$H = a/((a/SI - 1) \cdot \exp(cT - ct) + 1)$	$t = T + 1/c \cdot \ln(a/SI - 1)$	(8)
Schumacher	$H = a \cdot \exp(T/t \cdot \ln(SI/a))$	$t = -T\ln(SI/a)/2$	(9)
Sloboda	$H = a(SI/a)^{\exp(-c/((d-1) \cdot T^{D-1}) + c/((d-1) \cdot t^{d-1}))}$	$c\ln\frac{SI}{a} \cdot \exp(m + \frac{c}{(d-1)t^{d-1}}) + dt^{d-1} + c = 0$	(10)

① The expression of inflection point is the abscissa.

2.2.3 Polymorphic site index equation

For the purpose of obtaining the polymorphic expression forms of many theoretical equations, difference method was used and ten difference equations were produced (Table 3). Based on analysis for inflection points of difference equations, the characteristics of single or polymorphic form of the ten equations was discussed, and ten corresponding site index equations were conducted (Table 4).

For all site index equations in Table 4, if $t = T$, then there is $y = SI$. This means that these site index equations, obtained by the difference method, will not produce the contradiction that the value of tree height at index age is inconsistent with the site index value. From the variation of inflection points of every equation in Table 4, the abscissa of inflection points of those equations numbered 5, 6a, 6b, 6d, 7a, 8, 9, 10 are correlative to site index, which indicates that the obtained site index equations can ensure that different site index has a different height growth curve, that is, the eight site index equations are all polymorphic. Then there is another question to be answerd that if these polymorphic site index equations can guarantee the biological significance of inflection points? Which needs a further exploration. For the inflection points of equation 5, 6a, 6d, 8, 9, when *SI* increases, *t* decreases; for equation 6b, due to $\ln(1-\exp(-bT))<0$, the age *t* at inflection point decreases with the increasing of *SI*; for equation 7a, the relationship between *t* and *SI* is correlative to parameter *c*; for equation 10, originated from Sloboda, the variation of *t* at inflection point is not obvious with *SI*.

Thus, at least the six equations numbered 5, 6a, 6b, 6d, 8, 9 have good biological sense, that is, the better the site condition (*SI*) is, the earlier the inflection point occurs, on the contrary, the worse the site condition is, the later the inflection point occurs, which fully reflects the biological law that trees arrive fast-growing year earlier in the better site.

2.2.4 Parameter estimation

The data collected was organized into two forms. One was the pair data of dominant heights and ages, which was used to fit the prototype of five theoretical growth equations. The other is double pairs data of dominant heights and ages for fitting ten difference equations. While fitting, all the data were divided into three levels including site indices, provinces and districts, to compare simulation accuracy of the candidate equations at three levels. As the candidate equations all are nonlinear, so the nonlinear regression method of SAS software was adopted for parameter estimation.

2.2.5 Test statistics

Generally, the methods for model test often include two points, one is the biological meaning of models and its parameters, and the other is characterized by the statistical indices that describe the actual fitting effect of models, but often a compromise between the two is considered (Amaro et al., 1998). The statistical variables used here are average residual (*MR*), absolute mean residual (*AMR*), relative absolute residual (*RAR*), residual sum of squares (*RSS*), standard residual (*SR*) and coefficient of determination (R^2), in which, *AMR*, *RAR* , *RSS*, *SE* are the index for the accuracy of the model, *AMR* and R^2, respectively, stand for the model bias and efficiency. The calculation formula of these statistics were listed in Table 5.

Table 5 The statistics used for test of models①

Statistics index	Symbol	Formula	Ideal value
Mean residual	*MR*	$\sum_{i=1}^{n}\frac{(obs_i - est_i)}{n}$	0
Absolute mean residual	*AMR*	$\sum_{i=1}^{n}\frac{\lvert obs_i - est_i\rvert}{n}$	0
Relative absolute residual	*RAR*	$\frac{1}{n}\sum_{i=1}^{n}\frac{\lvert obs_i - est_i\rvert}{obs_i}$	0
Residual sum of square	*RSS*	$\sum_{i=1}^{n}(obs_i - est_i)^2$	0
Standard residual	*SR*	$\sqrt{\frac{\sum_{i=1}^{n}(obs_i - est_i)^2}{n}}$	0
Coefficient of determination	R^2	$1-\frac{\sum_{i=1}^{n}(obs_i - est_i)^2}{\sum_{i=1}^{n}(obs_i - \overline{obs_i})^2}$	1

① obs_i, est_i, n respectively stand for the *i*th observed value, the *i*th estimated value, number of observations.

3 Results and analysis

3.1 Comparison of modelling precision of dominant growth models

Table 6 showed the values of parameters and modelling precision indices of five theoretical growth equations and ten difference equations while modelling dominant height growth.

3.1.1
Factors for difference of modelling precision

When the data originated from the stands with same site index of same district, the selected statistics all showed that the size sequence of modelling precision for five theoretical growth equations was Korf> Richards> Weibull> Schumacher> Logistic. Through substituting evaluated parameters into formula of inflection points of five equations, the values of inflection points of five equations were obtained while modelling stands dominant height growth. Then it could be found that the relative location of inflection points, namely the rates of coordinates of inflection points to asymptote values of five equations, were 0.0001~0.1786, 0.0498~0.3103, 0.0303~0.3940, 0.1353 and 0.5 respectively.

In the view of modelling precision of equations, it could be found that the relative position of inflection point of equations had close correlation with modelling precision of equations for stand dominant height. For the equations with fixed inflection points, the equation that had a smaller fixed inflection point had a higher simulation accuracy, and this phenomenon had nothing to do with the number of parameters of equation (such as the two-parameter Schumacher equation and three-parameter Logistic equation). Which showed that the inflection point of stand dominant height growth curve occurred early, and meant that the fast-growth period of stand dominant height already appeared at the young years.

In fact, for many tree species, due to the rapid growth in early time, the inflection point of stand dominant height growth curve does not exist, the growth pattern is more accordant with a convex-shape curve, which may be the factor that Logistic equation is not suitable for modelling dominant height growth. From the occurrence time of age of inflection point of Korf equation, the fast-growth period of stand dominant height of chinese fir plantation occurred at 2 or 8 years old for collected data.

3.1.2
Modelling precision of difference equations

3.1.2.1
Effect of fitting data

From Table 6, it could be found that *AMR* of equations was less than 0.5 m or a little higher, and *RAR* is less than 0.05 when the fitting data based on the site level. Which indicated that the selected equations all can well simulate the dominant height growth process. Statistical variables *AMR*, *RAR*, *SR* showed that the size sequence of modelling precision of selected equations was Korf, 7b, Richards, 6d, 10, Weibull, 5, 6a, 7a, Schumacher, Logistic, 9 and 8. The results showed that the equations with fixed inflection points, such as Schumacher and Logistic, had relative low modelling precision than other equations with floating inflection points regardless of whether the equations were difference equations or not, the equations with three parameters (excepting Logistic equation) had more higher modelling precision than equations with two parameters, the prototype of every theoretical growth equation had more higher modelling precision than its difference form that only had one parameter. For two-parameter difference form of Weibull equation (7a), the prediction values of parameter *c* were all greater than 1, which indicated that equation 7a was a polymorphic equation. Obviously, when the fitting data based on the site level, the modelling precision of polymorphic equations were not more higher than the single-form equations. The *MR* of 6a, Richards, Korf were relatively small, indicating that the errors distribution of these three equations were more symmetrical nearby *x* axis.

Table 6 Dominant height fitting results for site index-level data set

Equation	Parameters			*MR*	*AMR*	*RAR*	*RSS*	*SR*	R^2
	a	*b*	*c*						
Korf	15.6990~87.4769	5.0164~16.4066	0.1200~1.3844	0.0022	0.2389	0.0199	27.6038	0.2662	0.9930~0.9997
Richards	12.5186~59.5427	0.0130~0.2132	1.0633~3.6214	−0.0035	0.2697	0.0209	35.2862	0.2900	0.9879~0.9994
Weibull	11.8894~51.3291	0.0063~0.0533	1.0318~2.0038	−0.0087	0.2935	0.0236	42.2420	0.3104	0.9863~0.9994
Schumacher	16.4651~43.9068	8.2336~24.7520		0.0438	0.3902	0.0433	77.4831	0.4807	0.9739~0.9989
Logistic	11.6704~31.3989	1.5846~3.0959	0.0980~0.3840	−0.0258	0.5034	0.0455	117.7375	0.5495	0.96[illegible]9~0.9993
(5)	15.1023~70.5481		0.0760~1.4662	0.0077	0.3069	0.0206	41.5303	0.3422	0.9606~0.9979
(6a)	12.2659~34.7447		1.1344~4.0557	−0.0005	0.3224	0.0205	45.1675	0.3554	0.9338~0.9981
(6d)	14.9681~39.4285	0.0009~0.1352	0.3151~3.8750	−0.0141	0.2729	0.0193	31.1026	0.2926	0.9724~0.9999
(7a)	11.5729~32.9764		1.1284~2.1800	−0.0077	0.3455	0.0218	53.7170	0.3839	0.9041~0.9979
(7b)	14.0592~40.1788	0.0025~1.7257	0.0873~2.1717	0.0120	0.2549	0.0158	31.3702	0.2849	0.9721~0.9991
(8)	11.5692~30.2524		0.1003~0.4059	0.0391	0.5548	0.0373	130.7375	0.6200	0.9040~0.9990
(9)	15.3130~41.0334			0.0726	0.4532	0.0354	98.1112	0.5499	0.9340~0.9969
(10)	14.6754~58.7902	0.1059~2.7311	0~1.6081	0.0164	0.2559	0.0161	32.6364	0.2948	0.9642~0.9986

When fitting data based on district level, the maximum *SR* and *AMR* of eight selected difference equations were respectively 1.7032, 1.0087, far less than the