国外数学名著系列
(影印版)68

Dmitry Kozlov

Combinatorial Algebraic Topology

组合代数拓扑

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。 这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王 元

2005 年 12 月 3 日

Dedicated to my family

Preface

The intent of this book is to introduce the reader to the beautiful world of Combinatorial Algebraic Topology. While the main purpose is to describe the modern research tools and latest applications of this field, an attempt has been made to keep the presentation as self-contained as possible.

A book to teach from

The text is divided into three major parts, which provide several options for adoption for course purposes, depending on the time available to the instructor.

The first part furnishes a brisk walk through some of the basic concepts of algebraic topology. While it is in no way meant to replace a standard course in that field, it could prove helpful at the beginning of the lectures, in case the audience does not have much prior knowledge of algebraic topology or would like to focus on refreshing those notions that will be needed in subsequent chapters. The first part can be read by itself, or used as a blueprint with a standard textbook in algebraic topology such as [Mun84] or [Hat02] as additional reading. Alternatively, it could also be used for an independent course or for a student seminar.

If the audience is sufficiently familiar with algebraic topology, then one could start directly with the second part. This is suitable for a graduate or advanced undergraduate course whose purpose would be to learn contemporary tools of Combinatorial Algebraic Topology and to see them in use on some examples. At the end of the course, a successful student should be able to conduct independent research on this topic.

The third and last part of the book is a foray into one specific realm of a present-day application: the topology of complexes of graph homomorphisms. It fits well at the end of the envisioned graduate course, and is meant as a source of illustrations of various techniques developed in the second part. Another possibility would be to use it as material for a reading seminar.

What is different in our presentation

In the second part we lay the foundations of Combinatorial Algebraic Topology. In particular, we survey many of the tools that have been used in research in topological combinatorics over the last 20 years. However, our approach is at times quite different from the one prevailing in some of the literature.

Perhaps the major novelty is the general shift of focus from the category of posets to the category of acyclic categories. Correspondingly, the entire Chapter 10 is devoted to the development of the fundamental theory of acyclic categories and of the topology of their nerves, which in turn are no longer abstract simplicial complexes, but rather regular triangulated spaces.

Also, Chapter 11 is designed to give quite a different take on discrete Morse theory. The theory is broken into three major branches: combinatorial, topological, and algebraic; each one with its own specifics. A very new feature here is the recasting of discrete Morse theory for posets in terms of *poset maps with small fibers*. This, together with the existence of a universal object associated to every acyclic matching and the Patchwork Theorem allows for a structural understanding of the techniques that have been used until now.

There are further novelties scattered in the remaining four chapters of the second part. In Chapter 13 we connect the notion of evasiveness with monotone poset maps, and introduce the notion of NE-reduction. After that, the importance of colimits in Combinatorial Algebraic Topology is emphasized. We look at regular colimits and their relation with group actions in Chapter 14, and at homotopy colimits in Chapter 15. We provide complete proofs for all the statements in Chapter 15, based on the previous groundwork pertaining to cofibrations in Chapter 7. Finally, in Chapter 16, we take a daring step of counting the machinery of spectral sequences to the core methods of Combinatorial Algebraic Topology.

Let us also comment briefly on our citation policy. As far as possible we have tried to avoid citations directly in the text, choosing to present material in the way that appeared to us to be most coherent from the contemporary point of view. Instead, each chapter in the second and third parts ends with a detailed bibliographic account of the contents of that chapter. Since the mathematics of the first part is much more classical, we skip bibliographic information there almost completely, giving only general references to the existing textbooks. An exception is provided by Chapter 8, where the material is slightly less standard, thus justifying making some reading suggestions.

Acknowledgments

Many organizations as well as individuals have made it possible for this book project to be completed. To start with, it certainly would not have materialized without the generous financial support of the Swiss National Science Foundation and of the Institute of Theoretical Computer Science at the Swiss Federal Institute of Technology in Zürich. Furthermore, a major part of this work has been done while the author was in residence as a research professor at the Mathematical Science Research Institute in Berkeley, whose hospitality, as well as the collegiality of the organizers of the special program during the fall term 2006, is warmly appreciated. The last academic institution that the author would like to thank is the University of Bremen, which has generously granted him a research leave, so that in particular this project could be completed.

The staff at Springer has been most encouraging and helpful indeed. Many thanks go to Martin Peters and Ruth Allewelt, who have managed to circumvent all the clever excuses that I kept fabricating for not being able to finish the writing.

This text has grown from a one-year graduate course that I have given at ETH Zürich to an enthusiastic group of students. Their comments have been most welcome and have led to substantial improvements. Special thanks go to Peter Csorba, whose additional careful proofreading of the text has revealed many inconsistencies both notational and mathematical.

The head of the Institute of Theoretical Computer Science at ETH Zürich, Emo Welzl, has been a congenial host and a thoughtful mentor during my sojourn as an assistant professor here. For this he has my deepest gratitude.

Much of the material in this book is based on joint research with my longtime collaborator Eric Babson, from UC Davis. Without him there would be no book. He is the spiritual coauthor and I thank him for this.

Writing a text of this length can be a daunting task, and it is invaluable when someone's support is guaranteed come rain or come shine. During this work, I was in a singularly fortunate situation of having my mathematical collaborator and my wife, Eva-Maria Feichtner, by my side, to help me to persevere when it seemed all but futile. There is no way I can thank her enough for all the advice, comfort, and reassurance that she lent me.

Finally, I would like to thank my daughter, Esther Yael Feichtner, who was born in the middle of this project and immediately introduced an element of randomness into the timetable. The future looks bright for her, as the opportunities for this welcome sabotage abound.

ETH Zürich, Switzerland

Dmitry N. Kozlov
March 2007

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