STOCHASTIC
MODELLING
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Marek Musiela
Marek Rutkowski

Martingale Methods in Financial Modelling

Second Edition

金融建模中的鞅方法

第 2 版

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Stochastic Optimization
Stochastic Control
Stochastic Models in Life Sciences

Stochastic Modelling and Applied Probability

(Formerly: Applications of Mathematics)

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Stochastic Modelling and Applied Probability

formerly: Applications of Mathematics

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Preface to the First Edition

The origin of this book can be traced to courses on financial mathematics taught by us at the University of New South Wales in Sydney, Warsaw University of Technology (Politechnika Warszawska) and Institut National Polytechnique de Grenoble. Our initial aim was to write a short text around the material used in two one-semester graduate courses attended by students with diverse disciplinary backgrounds (mathematics, physics, computer science, engineering, economics and commerce). The anticipated diversity of potential readers explains the somewhat unusual way in which the book is written. It starts at a very elementary mathematical level and does not assume any prior knowledge of financial markets. Later, it develops into a text which requires some familiarity with concepts of stochastic calculus (the basic relevant notions and results are collected in the appendix). Over time, what was meant to be a short text acquired a life of its own and started to grow The final version can be used as a textbook for three one-semester courses – one at undergraduate level, the other two as graduate courses.

The first part of the book deals with the more classical concepts and results of arbitrage pricing theory, developed over the last thirty years and currently widely applied in financial markets. The second part, devoted to interest rate modelling is more subjective and thus less standard. A concise survey of short-term interest rate models is presented. However, the special emphasis is put on recently developed models built upon market interest rates.

We are grateful to the Australian Research Council for providing partial financial support throughout the development of this book. We would like to thank Alan Brace, Ben Goldys, Dieter Sondermann, Erik Schlögl, Lutz Schlögl, Alexander Mürmann, and Alexander Zilberman, who offered useful comments on the first draft, and Barry Gordon, who helped with editing.

Our hope is that this book will help to bring the mathematical and financial communities closer together, by introducing mathematicians to some important problems arising in the theory and practice of financial markets, and by providing finance professionals with a set of useful mathematical tools in a comprehensive and self-contained manner.

Sydney
March 1997

Marek Musiela
Marek Rutkowski

Preface to the Second Edition

During the seven years that elapsed between the first and second editions of the present book, considerable progress was achieved in the area of financial modelling and pricing of derivatives. Needless to say, it was our intention to incorporate into the second edition at least the most relevant and commonly accepted of these developments. Since at the same time we had the strong intention not to expand the book to an unbearable size, we decided to leave out from the first edition of this book some portions of material of lesser practical importance.

Let us stress that we have only taken out few sections that, in our opinion, were of marginal importance for the understanding of the fundamental principles of financial modelling of arbitrage valuation of derivatives. In view of the abundance of new results in the area, it would be in any case unimaginable to cover all existing approaches to pricing and hedging financial derivatives (not to mention all important results) in a single book, no matter how voluminous it were. Hence, several intensively studied areas, such as: mean-variance hedging, utility-based pricing, entropy-based approach, financial models with frictions (e.g., short-selling constraints, bid-ask spreads, transaction costs, etc.) either remain unmentioned in this text, or are presented very succinctly. Although the issue of market incompleteness is not totally neglected, it is examined primarily in the framework of models of stochastic (or uncertain) volatility. Luckily enough, the afore-mentioned approaches and results are covered exhaustively in several excellent monographs written in recent years by our distinguished colleagues, and thus it is our pleasure to be able to refer the interested reader to these texts.

Let us comment briefly on the content of the second edition and the differences with respect to the first edition.

Part I was modified to a lesser extent and thus is not very dissimilar to Part I in the first edition. However, since, as was mentioned already, some sections from the first edition were deliberately taken out, we decided for the sake of better readability to merge some chapters. Also, we included in Part I a new chapter entirely devoted to volatility risk and related modelling issues. As a consequence, the issues of hedging of plain-vanilla options and valuation of exotic options are no longer limited to the classical Black-Scholes framework with constant volatility. The theme of stochastic volatility also reappears systematically in the second part of the book.

Part II has been substantially revised and thus its new version constitutes a major improvement of the present edition with respect to the first one. We present there alternative interest rate models, and we provide the reader with an analysis of each of them, which is very much more detailed than in the first edition. Although we did not even try to appraise the efficiency of real-life implementations of each approach, we have stressed on each occasion that, when dealing with derivatives pricing models, one should always have in mind a specific practical perspective. Put another way, we advocate the opinion, put forward by many researchers, that the choice of model should be tied to observed real features of a particular sector of the financial market or even a product class. Consequently, a necessary first step in modelling is a detailed study of functioning of a given market we wish to model. The goal of this preliminary stage is to become familiar with existing liquid primary and derivative assets (together with their sometimes complex specifications), and to identify sources of risks associated with trading in these instruments.

It was our hope that by concentrating on the most pertinent and widely accepted modelling approaches, we will be able to provide the reader with a text focused on practical aspects of financial modelling, rather than theoretical ones. We leave it, of course, to the reader to assess whether we have succeeded achieving this goal to a satisfactory level.

Marek Rutkowski expresses his gratitude to Marek Musiela and the members of the Fixed Income Research and Strategies Team at BNP Paribas for their hospitality during his numerous visits to London.

Marek Rutkowski gratefully acknowledges partial support received from the Polish State Committee for Scientific Research under grant PBZ-KBN-016/P03/1999.

We would like to express our gratitude to the staff of Springer-Verlag. We thank Catriona Byrne for her encouragement and invaluable editorial supervision, as well as Susanne Denskus for her invaluable technical assistance.

London and Sydney
September 2004

Marek Musiela
Marek Rutkowski

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Part I

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