You-Quan Li

# Lectures on Quantum Mechanics

量子力学简明教程

ZHEJIANG UNIVERSITY PRESS
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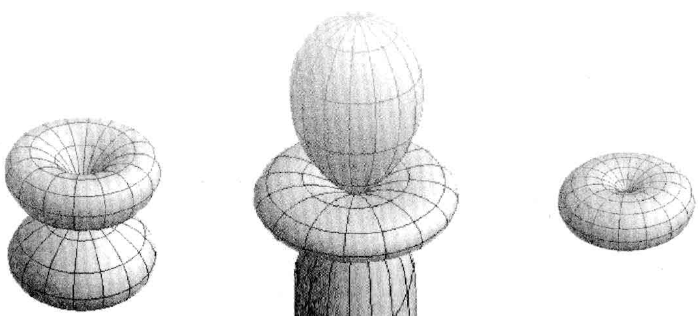

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**Lectures on Quantum Mechanics**
**量子力学简明教程**

李有泉 编著

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# Preface

Quantum theory is, perhaps, the most precisely tested and most successful theory in the history of science, which brought in great impact on our world. Quantum mechanics is one of the fundamental building-blocks of physics. It not only affects the way we think about the universe profoundly but also constitutes a basis for much of condensed-matter, nuclear and statistical physics effectively. It also has a strong influence on modern technological developments, for instance in optical and electronic devices. It is also expected to help the development of spintronics or to put a quantum computer into practise.

This book is designed to serve as a textbook rather than as a monograph. All the materials in this book is based on lecture courses given by the author to third-year undergraduates in physics department, Zhejiang University since 2002. The course is provided for undergraduate students who have the knowledge of atomic physics and are available at mathematical methods such as calculus, linear algebra, partial differential equations as well as special functions. In order to help students keep the pace with the key clue without being overwhelmed by heavy mathematical formulation, several mathematical contents that are frequently referred in the course are given as appendices. Unless they are inevitable, new postulates and concepts are attempted to be introduced as slow as possible during the progress of the course. This is based on the author's following two principles: (i) if a textbook were so perfectly arranged that leaves nothing to be desired, it would make the readers feel that the scientific discovery is so mystical that they lose passion of creativity; (ii) if a course were so purely presented that highes on a perfect axiomatics without mention of incomplete doctrine in history of science, it would lose the chance of training students' power of creativity. College students should cultivate their ability of capturing knowledge and attain the power of employing knowledge in addition to the simple task of absorbing knowledge. So the author tried to adopt second-person pronoun in presenting this course. According to the author's teaching experience, some mediate steps of mathematical formulation that beginners are not able to figure out frequently, are also given. In order to avoid too much content in class, some important examples are arranged as problems with solutions.

There is a saying that "physics can not be learnt by merely listening and reading, but by writing, thinking and worrying". Niels Bohr tipped off that anyone who were not shocked by quantum theory had not understood a single word. Willis Lamb addressed that anyone wanting to discuss a quantum mechanical problem had better understand and learn to apply quantum mechanics to that problem. Thus, the students are highly suggested to rethink the newly introduced concepts, to follow the necessary formulations, and to carry out the selected exercises outside class, which will benefit students most. Besides the selected problems for each chapter, several additional problems that require students' aggregative knowledge of the whole course are given after the last chapter.

Due to the limitation of course hours the author sorts some interesting and important contents in a follow-up course which includes: Dirac equation and electron spin, density matrix, quantum entanglement and quantum teleportation, the role of phases in quantum mechanics, particles in periodic potential and band structure, and approximation methods in quantum mechanics.

## Acknowledgements

The author would like to acknowledge Ling-Yun Deng for plotting the figures in the textbook. Careful reading of the manuscript by Ning-Ju Hui, Pei-Qing Jin, Li-Hua Lu, Ye-Hua Liu, Xiao-Qiang Xu, and Shan Zhu is also very much appreciated.

The present form of my lecture notes is largely benefited from the course given by Professor B.C. Qian during my undergraduate study and various quantum mechanics textbooks. The presentation in various chapters is affected by various places in the books of E. Merzbacker, L.I. Schiff, R.L. Liboff, L.D. Landau & E.M. Lifshitz, D. Atkinson & M.N. Hounkonnou, as well as Chinese textbooks of B.C. Qian, R.K. Su, J.Y. Zeng and S.X. Zhou. Some of the schematic figures are plotted by referring to S.M. McMurry's book. The selection and arrangement of the contents are made on the basis of my research experience.

You-Quan Li
Hangzhou 2010
http://zimp.zju.edu.cn/~tcmp

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## I Basic Concepts and Main Applications

# II Developing Skills

# Part I

# Basic Concepts and Main Applications

# Chapter 1

# The Discovery of Quantum Theory

Although quantum mechanics has brought so many benefits to our daily lives[1], it is far removed from our experience in daily lives because it was created to describe an abstract atomic world. The discovery of quantum theory forced physicists to reshape the ideas of reality, to rethink the nature of things at a much deeper level, and to revise the concepts of position and speed, as well as their notions of cause and effect. All those subtle aspects (foundations and its interpretation) not only deeply disturbed its founders in the past, but still remains dissatisfactory to some of luminaries of science even nowadays."

Let us start from some of the crucial events in the timelines of modern physics. For more extensive contents see, for example, "the American Physical Society – A Century of Physics" at Website: http://timeline.aps.org/APS/.

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[1]also "The spectacular advances in chemistry, biology, and medicine – and in essentially every other science – could not have occurred without the tools that quantum mechanics made possible. Without quantum mechanics there would be no global economy to speak of, because the electronics revolution that brought us the computer age is a child of quantum mechanics. So is the photonics revolution that brought us the Information Age. The creation of quantum physics has transformed our world, bringing with it all the benefits – and the risks – of a scientific revolution" cited from Kleppner and Jachiw, Science 286 (5481) 893

## 1.1 Blackbody Radiation and Planck's Hypothesis of Energy Quanta

It is known that any body at any temperature above absolute zero will radiate energy at some sense. "Blackbody radiation" refers to a system which absorbs all radiation incident upon it and re-radiates energy with the characteristic of the radiating system only, independent of the type of radiation incident upon it. A century years ago, there were several theoretical approaches to expose the nature of blackbody radiation. The successive expressions for the spectral distribution of radiation are listed hereby:

- *Wien's radiation formula:*

$$\rho_T(\nu) = \alpha\nu^3 \mathrm{e}^{-\frac{\beta}{T}\nu} \quad \text{or} \quad \rho_T(\lambda) = \alpha\frac{c^4}{\lambda^5}\,\mathrm{e}^{-\frac{\beta c}{\lambda T}} \tag{1.1}$$

  which was obtained by Wien in 1896 with the help of Maxwell-Boltzmann distribution under a hypothesis that the radiation frequency of molecular gases depends merely on individual velocities of the molecules. In modern notations, we write $\alpha = 8\pi h/c^3$ and $\beta = h/k_\mathrm{B}$ with Boltzmann's constant $k_\mathrm{B}$ and Planck's constant $h$.

- *Rayleigh-Jeans law:*

$$\rho_T(\nu) = \frac{8\pi}{c^3}\nu^2 k_\mathrm{B}T \quad \text{or} \quad \rho_T(\lambda) = \frac{8\pi}{\lambda^4}k_\mathrm{B}T \tag{1.2}$$

  which was derived from the energy equipartition law of classical kinetic theory, respectively, by Lord Rayleigh in 1900 and Jeans in 1905.

- *Planck radiation formula:*

$$\rho_T(\nu) = \frac{8\pi h}{c^3}\,\frac{\nu^3}{\mathrm{e}^{\frac{h\nu}{k_\mathrm{B}T}} - 1} \tag{1.3}$$

  or

$$\rho_T(\lambda) = \frac{8\pi hc}{\lambda^5}\,\frac{1}{\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}} - 1}$$

  which was a breakthrough work proposed by Planck in 1910.

Planck discovered that his expression for the energy density (1.3) could be derived on the basis of two postulates:

(1) The energy in an electromagnetic mode of frequency $\nu$ is quantized, *i.e.*, it can only take one of a set of discrete values

$$\varepsilon_n = nh\nu \quad \text{with} \quad n = 0, 1, 2, \cdots$$

(2) The probability of each of these discrete values is governed by the Boltzmann distribution

$$p(\varepsilon_n) \propto \mathrm{e}^{-\frac{\varepsilon_n}{k_\mathrm{B}T}}$$

This is because that the mean energy of an electromagnetic mode of frequency $\nu$ is given by

$$\langle \varepsilon(\nu) \rangle = \frac{\sum_n \varepsilon_n p(\varepsilon_n)}{\sum_n p(\varepsilon_n)} = \frac{h\nu}{\mathrm{e}^{\frac{h\nu}{k_\mathrm{B}T}} - 1} \tag{1.4}$$

as long as the aforementioned two postulates are assumed.

Clearly, both Wien's formula and Rayleigh-Jeans' law can be deduced from the Planck formula, respectively, as limit cases $h\nu \gg k_\mathrm{B}T$ and $h\nu \ll k_\mathrm{B}T$. Any way, *Planck's hypothesis of energy quanta is considered as the birth of quantum theory.*

## 1.2 Photoelectric Effect and Einstein's Hypothesis of Light Quanta

The experimental results of photoelectric emission whose setup is sketched in Fig. 1.1 are illustrated in Fig. 1.2. From the photoelectrical $I$-$V$ curves at the left panel, you can see that the photoelectric current increases with the increase of positive bia-voltage and reaches a saturation value then; it decreases with more negative bia-voltage and finally diminishes at a certain value $V_0$. This implies the emergence of (a) saturation current and (b) stop voltage $V_0$. From the $V_0$-$\nu$ curve in the right panel, you can notice that the stop voltage $V_0$ vanishes if the incident frequency $\nu$ is smaller than certain $\nu_0$, which means (c) the existence of red edge $\nu_0$. Additionally, (d) the photoelectrical current are always simultaneously observed once the cathode is illuminated by light.

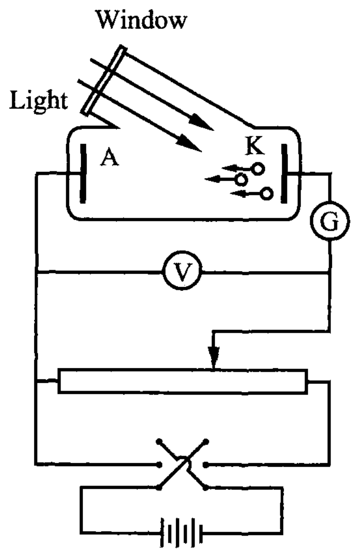


Fig. 1.1. Sketch map of the experimental setup that exhibits the photoelectric effect. The metal plate in cathode tube is illuminated with light of a given frequency and the photoelectric current is measured through the amperometer for various bia-voltages

Those experimental results manifest four features for the production of cathode rays:

(1) For a given metal and frequency of incident radiation, the rate at which photoelectrons are ejected is directly proportional to the intensity of the incident light.

(2) For a given metal, there exists a definite minimum frequency of incident radiation below which no photoelectrons can be emitted. This frequency is called the threshold frequency.