

◆ You-Quan Li

Lectures on Quantum Mechanics

量子力学简明教程

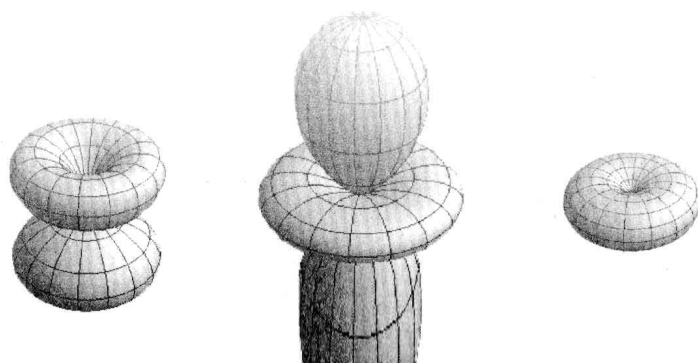


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量子力学简明教程

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Preface

Quantum theory is, perhaps, the most precisely tested and most successful theory in the history of science, which brought in great impact on our world. Quantum mechanics is one of the fundamental building-blocks of physics. It not only affects the way we think about the universe profoundly but also constitutes a basis for much of condensed-matter, nuclear and statistical physics effectively. It also has a strong influence on modern technological developments, for instance in optical and electronic devices. It is also expected to help the development of spintronics or to put a quantum computer into practise.

This book is designed to serve as a textbook rather than as a monograph. All the materials in this book is based on lecture courses given by the author to third-year undergraduates in physics department, Zhejiang University since 2002. The course is provided for undergraduate students who have the knowledge of atomic physics and are available at mathematical methods such as calculus, linear algebra, partial differential equations as well as special functions. In order to help students keep the pace with the key clue without being overwhelmed by heavy mathematical formulation, several mathematical contents that are frequently referred in the course are given as appendices. Unless they are inevitable, new postulates and concepts are attempted to be introduced as slow as possible during the progress of the course. This is based on the author's following two principles: (i) if a textbook were so perfectly arranged that

leaves nothing to be desired, it would make the readers feel that the scientific discovery is so mystical that they lose passion of creativity; (ii) if a course were so purely presented that hinges on a perfect axiomatics without mention of incomplete doctrine in history of science, it would lose the chance of training students' power of creativity. College students should cultivate their ability of capturing knowledge and attain the power of employing knowledge in addition to the simple task of absorbing knowledge. So the author tried to adopt second-person pronoun in presenting this course. According to the author's teaching experience, some mediate steps of mathematical formulation that beginners are not able to figure out frequently, are also given. In order to avoid too much content in class, some important examples are arranged as problems with solutions.

There is a saying that "physics can not be learnt by merely listening and reading, but by writing, thinking and worrying". Niels Bohr tipped off that anyone who were not shocked by quantum theory had not understood a single word. Willis Lamb addressed that anyone wanting to discuss a quantum mechanical problem had better understand and learn to apply quantum mechanics to that problem. Thus, the students are highly suggested to rethink the newly introduced concepts, to follow the necessary formulations, and to carry out the selected exercises outside class, which will benefit students most. Besides the selected problems for each chapter, several additional problems that require students' aggregative knowledge of the whole course are given after the last chapter.

Due to the limitation of course hours the author sorts some interesting and important contents in a follow-up course which includes: Dirac equation and electron spin, density matrix, quantum entanglement and quantum teleportation, the role of phases in quantum mechanics, particles in periodic potential and band structure, and approximation methods in quantum mechanics.

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The author would like to acknowledge Ling-Yun Deng for plotting the figures in the textbook. Careful reading of the manuscript by Ning-Ju Hui, Pei-Qing Jin, Li-Hua Lu, Ye-Hua Liu, Xiao-Qiang Xu, and Shan Zhu is also very much appreciated.

The present form of my lecture notes is largely benefited from the course given by Professor B.C. Qian during my undergraduate study and various quantum mechanics textbooks. The presentation in various chapters is affected by various places in the books of E. Merzbacker, L.I. Schiff, R.L. Liboff, L.D. Landau & E.M. Lifshitz, D. Atkinson & M.N. Hounkonou, as well as Chinese textbooks of B.C. Qian, R.K. Su, J.Y. Zeng and S.X. Zhou. Some of the schematic figures are plotted by referring to S.M. McMurry's book. The selection and arrangement of the contents are made on the basis of my research experience.

You-Quan Li

Hangzhou 2010

<http://zimp.zju.edu.cn/~tcmp>

CONTENTS

I Basic Concepts and Main Applications

1	The Discovery of Quantum Theory	3
1.1	Blackbody Radiation and Planck's Hypothesis of Energy Quanta	
	4
1.2	Photoelectric Effect and Einstein's Hypothesis of Light Quanta	
	5
1.3	The Atomic Spectra and Bohr's Model of Atoms	8
2	Wavefunction and Schrödinger Equation	13
2.1	The de Broglie Hypothesis and Davisson-Germer Experiment	
	13
2.2	Schrödinger Equation	15
2.3	Stationary Solutions	21
2.4	General Properties of Motion in One Dimension	22
2.5	Bound States in Potential Well	25

2.5.1	Square Well of Infinite Depth	26
2.5.2	Square Well of Finite Depth	28
2.6	The Harmonic Oscillator	32
2.7	Tunnelling Effect	40
3	Operators and Heisenberg Uncertainty Relation	55
3.1	Observables and Operators	55
3.2	Hermitian Operators and Their Properties	57
3.3	Some Example Operators	60
3.3.1	Momentum Operators and Their Eigenfunctions	60
3.3.2	Angular Momentum Operators and Their Eigenfunctions	62
3.3.3	Coordinate Representation and Momentum Representation	64
3.4	Evaluation of the Expectation Values	66
3.5	The Heisenberg Uncertainty Relation	68
3.5.1	Commutation Relations and Their Implications	68
3.5.2	Uncertainty Relation and the Minimum Uncertainty State	69
3.6	The Time Evolution of Expectation Values	73
4	Motion in a Centrally Symmetric Field	79
4.1	Three Dimensional Harmonic Oscillators	79
4.2	The Feature of Motion in a Centrally Symmetric Field	81
4.3	Spherical Waves and Plane Waves	83
4.4	Motion in a Coulomb Field	84

4.5	Hydrogen-like Energy Levels	88
4.6	Hydrogen Atom and Hydrogen-like Ions	91
5	States and Heisenberg Equation	97
5.1	Matrix Representation of Operators	97
5.2	States and Their Representations	100
5.3	The Heisenberg Equation	104
5.4	Algebraic Approach for Harmonic Oscillator	105
5.5	Algebraic Approach for Angular Momentum	110

II Developing Skills

6	Bound-State Perturbation and Corrections to Energy Levels	121
6.1	Nondegenerate Perturbation Theory	122
6.2	Degenerate Perturbation Theory	127
6.3	Stark Effect	132
7	Time-dependent Perturbation and Quantum Transitions	137
7.1	Perturbation Depending on Time	137
7.2	Transition Probability	138
7.2.1	Periodic Perturbation of Single Frequency	139
7.2.2	Transition to Continuous Spectrum	141
7.3	Induced Absorption and Emission	142

7.4	Einstein's Semi-phenomenological Theory for Spontaneous Emission	145
8	Scattering Theory for Elastic Collisions	151
8.1	Scattering Amplitude and Cross Section	151
8.2	Born Approximation	154
8.2.1	The Simplest Approach	154
8.2.2	Lippmann-Schwinger Equation	155
8.2.3	Scattering by Screened Coulomb Potential	157
8.3	Partial-Wave Approach	159
8.3.1	Phase Shifts and Scattering Amplitudes for Centrally Symmetric Potentials	159
8.3.2	Neutron-Proton Scattering	161
8.3.3	The Center-of-Mass Frame and the Lab Frame	163
9	Motion in a Magnetic Field	167
9.1	Schrödinger Equation for a Charged Particle in Electromagnetic Fields	167
9.2	Electrons in a Uniform Magnetic Field	168
9.3	Atoms in Magnetic Field and Zeeman Effect	171
9.4	Electron Spin	173
9.5	Spin-Orbit Coupling and the Fine Structure of Atomic Spectra	177
9.5.1	On Spin-Orbit Coupling	177
9.5.2	On the Fine Structure of Atomic Spectra	181

10	Identical Particles and Pauli Exclusive Principle	185
10.1	Permutation Symmetry and the Indistinguishability of Identical Particles	186
10.2	Noninteracting Systems, Pauli Exclusive Principle	187
10.2.1	Two-particle Systems	188
10.2.2	N-particle Systems	189
10.3	Interacting Systems	189
10.3.1	Helium Atom, Hund's Rule	190
A	Matrix and Vector Space	199
B	δ-function	203
C	Confluent Hypergeometric Function	207
D	Orbitals for d-Electrons	211
E	Lab Frame and Center-of-mass Frame	215
F	On an Integral	219
G	Time Reversal and Kramers Degeneracy	221

Part I

Basic Concepts and Main Applications

Chapter 1

The Discovery of Quantum Theory

Although quantum mechanics has brought so many benefits to our daily lives¹, it is far removed from our experience in daily lives because it was created to describe an abstract atomic world. The discovery of quantum theory forced physicists to reshape the ideas of reality, to rethink the nature of things at a much deeper level, and to revise the concepts of position and speed, as well as their notions of cause and effect. All those subtle aspects (foundations and its interpretation) not only deeply disturbed its founders in the past, but still remains dissatisfactory to some of luminaries of science even nowadays.”

Let us start from some of the crucial events in the timelines of modern physics. For more extensive contents see, for example, “the American Physical Society – A Century of Physics” at Website: <http://timeline.aps.org/APS/>.

¹also “The spectacular advances in chemistry, biology, and medicine – and in essentially every other science – could not have occurred without the tools that quantum mechanics made possible. Without quantum mechanics there would be no global economy to speak of, because the electronics revolution that brought us the computer age is a child of quantum mechanics. So is the photonics revolution that brought us the Information Age. The creation of quantum physics has transformed our world, bringing with it all the benefits – and the risks – of a scientific revolution” cited from Kleppner and Jachiw, *Science* 286 (5481) 893

1.1 Blackbody Radiation and Planck's Hypothesis of Energy Quanta

It is known that any body at any temperature above absolute zero will radiate energy at some sense. "Blackbody radiation" refers to a system which absorbs all radiation incident upon it and re-radiates energy with the characteristic of the radiating system only, independent of the type of radiation incident upon it. A century years ago, there were several theoretical approaches to expose the nature of blackbody radiation. The successive expressions for the spectral distribution of radiation are listed hereby:

- *Wien's radiation formula:*

$$\rho_T(\nu) = \alpha \nu^3 e^{-\frac{\beta}{T}\nu} \quad \text{or} \quad \rho_T(\lambda) = \alpha \frac{c^4}{\lambda^5} e^{-\frac{\beta c}{\lambda T}} \quad (1.1)$$

which was obtained by Wien in 1896 with the help of Maxwell-Boltzmann distribution under a hypothesis that the radiation frequency of molecular gases depends merely on individual velocities of the molecules. In modern notations, we write $\alpha = 8\pi h/c^3$ and $\beta = h/k_B$ with Boltzmann's constant k_B and Planck's constant h .

- *Rayleigh-Jeans law:*

$$\rho_T(\nu) = \frac{8\pi}{c^3} \nu^2 k_B T \quad \text{or} \quad \rho_T(\lambda) = \frac{8\pi}{\lambda^4} k_B T \quad (1.2)$$

which was derived from the energy equipartition law of classical kinetic theory, respectively, by Lord Rayleigh in 1900 and Jeans in 1905.

- *Planck radiation formula:*

$$\rho_T(\nu) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1} \quad (1.3)$$

or

$$\rho_T(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

which was a breakthrough work proposed by Planck in 1910.

Planck discovered that his expression for the energy density (1.3) could be derived on the basis of two postulates:

- (1) The energy in an electromagnetic mode of frequency ν is quantized, *i.e.*, it can only take one of a set of discrete values

$$\varepsilon_n = n h \nu \quad \text{with} \quad n = 0, 1, 2, \dots$$

- (2) The probability of each of these discrete values is governed by the Boltzmann distribution

$$p(\varepsilon_n) \propto e^{-\frac{\varepsilon_n}{k_B T}}$$

This is because that the mean energy of an electromagnetic mode of frequency ν is given by

$$\langle \varepsilon(\nu) \rangle = \frac{\sum_n \varepsilon_n p(\varepsilon_n)}{\sum_n p(\varepsilon_n)} = \frac{h \nu}{e^{\frac{h \nu}{k_B T}} - 1} \quad (1.4)$$

as long as the aforementioned two postulates are assumed.

Clearly, both Wien's formula and Rayleigh-Jeans' law can be deduced from the Planck formula, respectively, as limit cases $h \nu \gg k_B T$ and $h \nu \ll k_B T$. Any way, *Planck's hypothesis of energy quanta is considered as the birth of quantum theory.*

1.2 Photoelectric Effect and Einstein's Hypothesis of Light Quanta

The experimental results of photoelectric emission whose setup is sketched in Fig. 1.1 are illustrated in Fig. 1.2. From the photoelectrical I - V curves at the left panel, you can see that the photoelectric current increases with the increase of positive bias-voltage and reaches a saturation value then; it decreases with more negative bias-voltage and finally diminishes at a certain value V_0 . This implies the emergence of (a) saturation current and (b) stop voltage V_0 . From the V_0 - ν curve in the right panel, you

can notice that the stop voltage V_0 vanishes if the incident frequency ν is smaller than certain ν_0 , which means (c) the existence of red edge ν_0 . Additionally, (d) the photoelectrical current are always simultaneously observed once the cathode is illuminated by light.

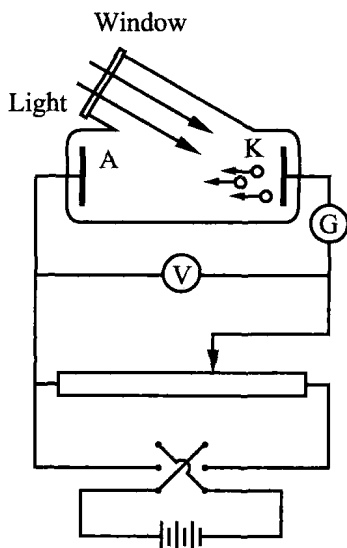


Fig. 1.1. Sketch map of the experimental setup that exhibits the photoelectric effect. The metal plate in cathode tube is illuminated with light of a given frequency and the photoelectric current is measured through the amperometer for various bias-voltages

Those experimental results manifest four features for the production of cathode rays:

- (1) For a given metal and frequency of incident radiation, the rate at which photoelectrons are ejected is directly proportional to the intensity of the incident light.
- (2) For a given metal, there exists a definite minimum frequency of incident radiation below which no photoelectrons can be emitted. This frequency is called the threshold frequency.