普通高等教育“十一五”国家级规划教材

# FOUNDATION OF MODERN CONTROL THEORY

# （现代控制理论基础）

许维胜　朱劲　王中杰　主编



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# Preface

Principle of Automatic Control is a foundation module for students learning Automation and Electrical Engineering. In Chinese universities, the model is often taught in two semesters, with the transfer function based traditional control in semester one and the state-space model based modern control in semester two. The digital control and the nonlinear control are separately taught in two independent modules. Tongji University has used Regelungstechnik, written by Prof. H. Ünbehaun, as the text book for its undergraduates since 1986 and its content has been well received by the students. The book is engineering-oriented and logically structured to engage engineer and student interest. The book is divided into three volumes. The first two volumes are for undergraduates, with volume 1 covering the traditional control and volume 2 covering state-space control, digital control, and nonlinear control. The book was in German and translated into Chinese by Prof. Wu Qidi and Prof. Huang Shengle. In order to reform engineering education and foster students to excel as engineers, the experiences of German universities in engineering and technical education can provide us a valuable reference。

In recent years, Chinese universities promote teaching of engineering orientation courses in English for improving student communication skills in a globalize environment. Several popular English textbooks about the traditional control theory have been adopted by Chinese universities. However, none of them targets undergraduates for teaching the modern control theory. This motivates us to write this book about the modern control theory, following the structure of Ünbehaun's book. It covers state-space method, digital control and nonlinear control. The book was based on the authors' lecture notes and has been taught to students for three years. For publishing the notes to a book format, significant revision has been made with modification and inclusion of other additional topics, in response to the problems during the teaching. Some exercises and questions in the references were used in this book. The authors are grateful to the authors.

The book has 10 chapters to cover three main topics. The first topic is about state-space analysis and synthesis for linear systems, from Chapter 1 to 4. Chapter 1 introduces the description of linear dynamic systems and Chapter 2 covers solution of linear dynamics equations. Controllability, Observability and synthesis based on state feedback are arranged in Chapter 3 and Chapter 4, respectively.

The second topic is about linear digital control systems, from Chapter 5 to Chapter 8. Chapter 5 introduces discrete sampling systems, digital systems and computer control systems. Chapter 6 covers mathematic foundation for analysis of discrete systems, including sampling theorem, $z$ transformation and control system pulse transfer function. Chapter 7 presents discrete system analysis, including system stability and steady state response. Chapter 8 discusses linear discrete system design, including digital controller design and discretization of a continuous controller.

The third topic is about nonlinear system analysis and synthesis, from Chapter 9 to 12. Chapter 9 discusses characteristics, classification, and typical components of nonlinear systems. Describing function method, phase plan method and Lyapunov method are discussed in Chapter 10, 11 and 12, respectively. The describing function method can be used for stability analysis by linearization of a nonlinear system, which is only applicable to a nonlinear system with small perturbation. The phase plan method can be used for stability analysis of a second order system. The Lyapunov method is a universal method, widely studied for both linear and nonlinear systems.

The book is structured to engage student learning interest. It is intended to clearly present and emphasize basic concepts and physical meanings but take rigorous mathematic proofs as extended materials. This motivates the learning of general students and gives first-class students a space to study in-depth theoretical background. From our teaching experience, it was appreciated by most students and helped top student excel in the area.

The first part, linear state-space analysis and synthesis, was written by Prof. Xu Weisheng; the second part, linear discrete control systems, was written by Prof. Zhu Jin; the third part, nonlinear systems, was written by Prof. Wang Zhongjie. The book could not have been published without many people's help. Prof. Lin Feng at Wayne State University gave some valuable comments and suggestions. Prof. Jiang Ping, Prof Yao Jin, and Mr. Yi Zonggen were kind enough to proofread the book. The authors are grateful to those who supported the writing.

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**Xu Weisheng, Zhu Jin, Wang Zhongjie**
**Tongji University**
**Oct 10, 2010**

# 前 言

自动控制原理是自动化、电气工程及其自动化专业的专业基础课程。目前国内院校比较普遍的做法是将所有内容分为两个学期讲授，第一学期讲授经典部分，即以传递函数模型为基础的经典控制理论，第二学期讲授现代部分，即以状态空间模型为基础的现代控制理论。数字控制系统和非线性控制系统则另设课程讲授。同济大学自1986年以来，采用了德国H. Ünbehaun教授编写的自动控制工程作为教材，取得了较好的效果。该教材以工程性强、逻辑严密、适合工程师和大学生采用为特色。全书分三册，前一、二册主要适用于大学本科阶段。第一册主要讲授经典自动控制理论，第二册主要包括状态空间法、离散控制系统和非线性控制系统。原书用德语写成，后经吴启迪和黄圣乐两位教授翻译成中文由同济大学出版社出版。在目前工程教育改革、卓越工程师培养的背景下，德国高校的工程教学特色对我们仍然具有极为重要的参考价值。

近年来，为了更好地与国际接轨，国内高校大力推广自动控制原理的双语教学。国内出版社也引进了几部有影响的有关经典控制理论教学内容的教材。但其中关于现代控制理论的英文参考书，适用于本科教学的，目前几乎没有。基于此，我们参照H. Ünbehaun教授所主编教材第二册的结构，编写了这本现代控制理论英文教材。主要内容包括状态空间法、数字控制系统和非线性控制系统三部分内容。该教材最初是以讲义的形式印刷，并进行了三年的试用。本次出版，作者又进行了修订，根据教学中发现的问题对内容进行了调整和增删。书中有些例题和习题采纳了参考文献中的部分内容，未及一一标出，在此对原书作者表示衷心的感谢。

全书分为三部分共十章。第一部分为线性系统的状态空间分析和综合，包括第一章至第四章。第一章为动态系统的状态空间描述。第二章为线性系统状态方程的解。第三章为能控性和能观测性。第四章为状态反馈控制系统的设计。

第二部分为线性离散控制系统，包括第五章至第八章。第五章主要讲述离散采样系统、数字系统及计算机系统的概念。第六章着重阐述离散系统研究的基本数学方法，包括采样定理、$z$变换和控制系统的脉冲传递函数。第七章讲述线性离散控制系统的分析，包括系统的稳定性、稳态响应特性。第八章讨论了线性离散系统的设计，包括数字控制器直接设计方法、连续时间控制器的离散实现。

第三部分为非线性系统的分析和综合，包括第九章至第十二章。第九章首先

讨论了非线性系统的特点、分类、常见环节和表现。第十章至第十二章分别详细阐述了三种用于分析系统稳定性的方法，即描述函数法、相平面分析法和李亚普诺夫方法。描述函数法通过线性化的方法研究系统的稳定性，适用于系统非线性很小的情况。相平面方法仅适用于二阶系统的稳定性分析。李亚普诺夫方法既适用于线性系统，又适用于非线性系统，是一种比较普遍的研究方法。

在内容的编排上，本书考虑到学生的学习习惯，注意讲清重要的概念及其物理意义，而将严格的数学证明放在扩展内容中。这样既便于引导优秀学生进行较深入的理论推演，也便于大多数同学的课堂教学。实践证明，这样的安排是受同学欢迎的，也取得了较好的教学效果。

本书第一部分，即线性系统的状态空间分析和综合，由许维胜教授负责编写。第二部分，即线性离散控制系统，由朱劲副教授编写。第三部分，即非线性系统，由王中杰教授编写。美国韦恩州立大学的林峰教授为本书提出了诸多宝贵的修改意见。同时，同济大学蒋平、姚静老师、易总根研究生等，也为本书的校对和修改做了大量的工作，在此对他们的贡献表示衷心的感谢。

本书在出版的过程中得到了“十一五”教材出版基金的资助，在此一并致谢。

由于编者水平所限，书中可能存在不妥之处，敬请读者不吝赐教并指正。

许维胜　朱　劲　王中杰
同济大学
2010 年 10 月

# Contents

## Section I State Space Analysis and Synthesis of Linear Systems

## Section II Linear Discrete-time Systems

## Section III Nonlinear Systems

# Section I

## State Space Analysis and Synthesis of Linear Systems

# 1 State Space Description of Dynamic Systems

## 1.1 Introduction

Generally speaking, control theory can be divided into classical and modern control theory. Transfer function and frequency domain techniques are predominant in the classical control theory which deals primarily with linear, constant coefficient systems by use of the approaches of root locus, Nyquist analysis Bode diagram, etc. Such methods are useful for investigating single-input single-output (SISO) systems. More complex systems may have multiple inputs and/or multiple outputs (MIMO), which make the situation more intricate. The problems of design, stabilization and compensation of such systems are obviously more complicated than the ones of SISO systems, since the number of inputs and outputs increases. Clearly, analyzing such systems using classical methods would be very tedious and error-prone.

Starting in the late 1950s and early 1960s, a time domain approach using state variable descriptions, which is usually referred to as modern control theory, came into prominence and provided an alternative approach for handling MIMO systems. Its principle advantages are as follows:

- It can easily manipulate MIMO systems.
- It provides additional insights into system behaviors that transfer function analysis does not.
- It handles linear time-varying systems, even nonlinear systems.
- It is easily implemented on a digital computer.

When we are confronted with the task of analyzing and/or synthesizing MIMO systems, the time domain approach using state variables (often called state-space methods) is preferred. Once the method is chosen, developing the state-state description, which is the corresponding mathematical models of physical systems, is inevitable. In modern control theory based on state-space descriptions, the basis is the concepts of state and of state variables.

## 1.2 State and state space

**Definition of state:**

The state of a dynamic system is a mathematical structure containing a set of $n$ variables $x_1(t)$, $x_2(t)$, $\cdots$, $x_n(t)$, called state variables, such that the initial values $x_i(t_0)$, $i = 1, 2, \cdots, n$, of this set at time $t_0$ and the system inputs $u_j(t)$, $j = 1, 2, \cdots, p$, $t \geqslant t_0$, are sufficient to determine uniquely the system's future behavior of $t \geqslant t_0$. A minimum set of state variables is required to represent the system completely. The $p$ inputs, $u_1(t)$, $\cdots$, $u_2(t)$, $\cdots$, $u_i(t)$, $\cdots$, $u_p(t)$ are deterministic, *i.e.* they have specific values for all $t \geqslant t_0$.

Generally the initial time $t_0$ is taken to zero. The state variables need not be physically observable and measurable quantities; they may be purely mathematical quantities. Furthermore, we can have different choices of state variables for the same system.

**Definition of state vector:**

The state vector of a dynamic system is defined as the column vector $\boldsymbol{x}(t)$ which is composed of all the state variables $x_1(t)$, $x_2(t)$, $\cdots x_i(t)$, $\cdots$, $x_n(t)$; that is,

$$\boldsymbol{x}(t) \equiv \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \equiv \boldsymbol{x} \qquad (1\text{-}1)$$

The system state is known when all of the $n$ state variables are known, *i.e.* when $\boldsymbol{x}(t)$ is known.

For a particular system, the number of state variables, *i.e.* $n$, is fixed and is equal to the system order. It can be determined as the number of initial conditions needed to solve the system differential equations or the number of first-order differential equations needed to define the system. But as mentioned above, the choice of state variables is not unique. The only restriction on the