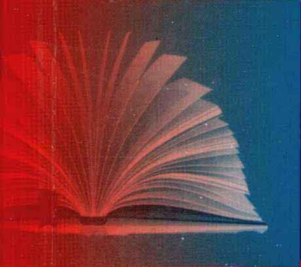

理工科核心课程双语规划教材

电动力学讲义

Lectures on Electrodynamics

钟学富　编著

中国科学技术大学出版社

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内 容 简 介

本书精选电动力学的基本内容，以讲义的形式按课堂教学顺序组织成46讲。内容包括矢量分析、静电场、静磁场、麦克斯韦方程、物质中的电磁场、电磁波的传播、辐射理论和相对论力学等。各讲内容均衡、简练，公式推导详细，附思考问题，突出重点，减轻阅读困难。

本书重点解决课堂教学的"程序化"（将科学体系变为讲授的时序）问题，可直接用作教师教案，组织课堂讲授；在适当增加内容之后，本书可作为普通大学本科或师范院校物理系电动力学课程的教材；本书还可作为参考阅读资料，帮助提高科技英语水平。

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钟学富，1939年生，四川新都人，毕业于四川大学物理系理论物理专门化。曾任中国科学院半导体研究所副研究员，硕士生导师。1987年赴美为访问学者，在密苏里大学堪萨斯城分校物理系从事固体理论研究。现居美国。

单独与合作发表中英文物理论文约30篇。主要成果包括确立半导体中一类光转化杂质模型，经实验证实并获中国科学院科技成果二等奖；首次提出在晶体场计算中考虑传导电子贡献，此概念被用于修改穆斯堡尔效应中的电场梯度公式。另外在《中国社会科学》、《哲学研究》、《光明日报》、《自然辩证法研究》等刊物发表涉及信息论和物理学的哲学问题的论文约10篇。近年来陆续出版《物理社会学》、《社会系统》、《休闲哲学》等专著，尝试将自组织及相关理论应用于社会研究，发展社会科学的演绎理论。

Preface 序 言

《经典力学讲义》和《电动力学讲义》两本教材源于1992—1998年间本人在密苏里大学堪萨斯城分校(UMKC)物理系任教时的讲稿，程度与国内物理本科的四大力学相当，但我同时还接收邻近专业如化学系的硕士和博士生。由于美国教材大体属于“包罗万象”式，教科书都是厚厚的一本，作为参考书很有查询价值，但绝对不可能在有限的学时内逐节讲授，教授不得不另写教案，以组织课堂教学，不过这却带来一系列问题。首先是讲授(听讲)和教科书之间脱节，教授“跳着讲”，学生课后“跳着读”，影响思维的连贯性。尤其是，为了和教授的讲解衔接，学生必须认真记笔记，但记笔记其实有碍专心听讲：前一个要点还没记下来，后一个又来了。为了解决这些矛盾，本人便将原本仅供自己使用、字迹潦草的教案整理出来，作为讲义发给学生。学生不必笔记，顶多在上面加点批注，省下力气专心听讲，并随时提问。讲义的内容大体上是自相包容的，假如要求不高，听讲之后，只消把不多几页真正看明白，就算达到课程的基本要求，所以学生阅读的负担不重。尤其物理系的课程，几乎处处都要“推公式”，一般教科书反而非常简略。本人斟酌情况多写两行中间步骤，便能省去学生不少时间，这大受数学底子相对薄弱的美国学生欢迎。这些体恤学生的做法，源于自己当年做学生的经验，我的老师就给我们发讲义！本人的做法其实是传承中国早年的教学思想，并将其推广到美国的课堂。

这样的讲义对中国的物理教学有什么用处？首先对教师来说，现在的教科书大多是按章节编写的，这没错，科学是严谨的思想逻辑体系。然而别忘了，教学却是要将这个逻辑结构变为“时序结构”：课，是一堂一堂上的，先讲什么后讲什么，和排程序一模一样。所以我的讲义没有章节，只有第一、第二、……，直到第 N 讲。这个编排不一定高明，甚至未必合于各校的要求，但我希望教师不要忽略这种叙述方式，而且都要能拿出自己编排的高招，包括问题的切入方式。其次是学生，假如只是为了学英文，那么，每讲内容可能正是一餐的饭量，语法、词汇都是最普通和常见的。而物理系的学生，假如还能从内容叙述和相关公式的推导中得点启发，那就算是意外的收获了。

钟学富

2011年2月18日于美国堪萨斯城

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Lecture 1 Introduction to Electrodynamics, Vector Analysis (I): Gradient

1.1 Introduction to electricity and magnetism

Electric and magnetic phenomena are widely observed in nature. There are four fundamental forces which build up our world, i.e., gravitational, electromagnetic, weak and strong interactions. Electromagnetic force is not only one of them, actually it is the most important force for us. Atoms are constructed by the Columbic attraction between nuclei and electrons. Molecules, solids and all other condensed matter are also formed on the basis of electromagnetic interaction. In fact, the electromagnetic force builds up the most of the macroscopic and microscopic world, except the nuclear structure and the motion of celestial objects.

The study on electric and magnetic phenomena is based on the concept of fields. This is an significant development in classic physics, which eliminates the mistake of action at distance in Newton's mechanics and shakes the classic concepts of space and time in this mechanics. This is actually the prerequisite of new physics, i.e., the Einstein's theory of relativity. It will be seen that the Maxwell's theory of electricity and magnetism is automatically relativistic.

In classic physics, fields are understood as continuous media. However, modern physics considers particles and fields as two aspects of matter. For example, electrons are considered as particles in classic physics, but we also have electronic field, which is described by the wave function of electrons. Electric and magnetic fields are "field" in classic physics, they are also "particles" in quantum theory, i.e., photon. This will be studied in the quantum theory of fields, which is beyond the scope of the course.

Electric and magnetic fields are considered as different fields. In fact, we have only a united electromagnetic field. Electric field and magnetic field are just two components of this field. In most cases, both fields coexist, they cannot be separated. Only in special cases, we have either sole electric field or sole magnetic field, which is the electrostatic field or the magnetic field of steady current. Electrodynamics actually can be considered as classic theory of fields.

The mathematical model of a field is obviously the function defined in a certain domain of space, therefore in electricity and magnetism we need a mathematical tool known as vector analysis, which is essentially the calculus on these functions, either scalar or vector. We shall start the study on electric and magnetic fields by introducing the mathematical tool.

1.2 Scalar and vector fields

Mathematically, any function, scalar or vector, defined in a certain domain of space can be called a field. Thus we have a scalar field and a vector field written as

$$V = V(x,y,z); \quad \boldsymbol{E} = \boldsymbol{E}(x,y,z) \tag{1.1}$$

respectively, where x, y, z are the Cartesian coordinates of the point. Obviously, a scalar field means that the property of a field is completely defined by its magnitude, while a vector field needs both its magnitude and direction. In real three-dimensional space, the direction of a vector is denoted by its directional cosines: $\cos\alpha, \cos\beta, \cos\gamma$. However, these cosines can be easily calculated from the projects of the vector on three coordinate axes. Thus, we usually write a vector in its component form and each component represents a scalar, i.e.,

$$E_x = E_x(x,y,z); \quad E_y = E_y(x,y,z); \quad E_z = E_z(x,y,z) \tag{1.2}$$

In the study of vector field, we assume that the vector algebra is already known. This includes the addition, subtraction, scalar and vector products between two vectors. Especially, two formulas of mixed product with three vectors involved are often used in our discussion below:

$$\boldsymbol{A}\cdot(\boldsymbol{B}\times\boldsymbol{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = -\boldsymbol{B}\cdot(\boldsymbol{A}\times\boldsymbol{C}) = \boldsymbol{B}\cdot(\boldsymbol{C}\times\boldsymbol{A}) \tag{1.3}$$

$$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B}) \tag{1.4}$$

The formula (1.3) indicates that three vectors in the mixed product are cycling, because the product actually represents the volume of parallelepiped spanned by three vectors.

1.3 Gradient of scalar field

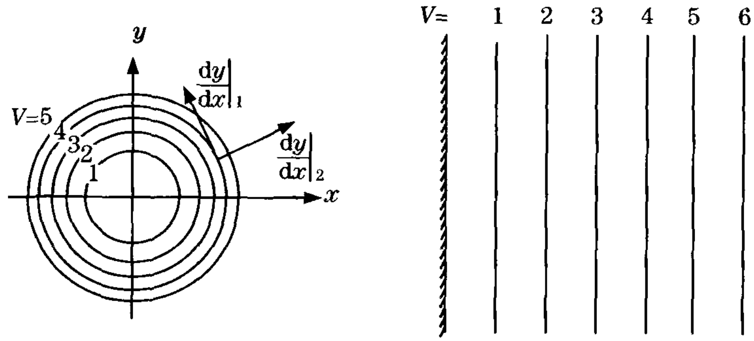
There are several quantities, known as gradient, divergence and curl, which describe the important properties of scalar and vector fields, respectively.

Gradient is the characteristic of a scalar field. The concept can be introduced as follows. We first define the equipotential or level surface of the field, which is given by

$$V(x, y, z) = C \tag{1.5}$$

where C is a constant. Note that the equation (1.5) is actually a condition or constraint imposed on the coordinates of points in the three dimensional manifold where the field exists, therefore the degree of freedom of the manifold is reduced by one. Thus equation (1.5) leads to a two dimensional manifold, i.e., a surface. For different constant C, the surface is in general different; also, each point in the field must belong to a certain equipotential surface. The great advantage of equipotential surface is that it provides an intuitive description of the field, i.e., how the property of the field (denoted by the value of the function V at each points) varies in this domain. Some important information about the field, say its symmetry, can be immediately obtained from these surfaces. This is shown in Fig. 1.1.

Although the field remains constant on its equipotential surfaces, it definitely varies from one surface to another and in general the way of variation is different for different directions. However, there are infinite directions at each point. We then need to specify a particular direction, from which the way that the field changes its value can be described. This leads to the concept of gradient. The gradient of a field $V(x, y, z)$, denoted by **grad**V or ∇V, is defined as the rate of variation of the function along the normal of the equipotential surface at the point. Obviously, gradient must be a vector, which is along the normal at the point under consideration and its magnitude indicates how rapid the field changes its value.



(a) point charge (b) uniform field

Fig. 1.1 Equipotential surfaces

We now put the definition into a mathematical form. In Cartesian coordinate system, we may define a directional derivative along any direction ds as:

$$\frac{\mathrm{d}V}{\mathrm{d}s} = \lim_{\Delta s \to 0} \frac{V(x + \Delta x, y + \Delta y, z + \Delta z) - V(x, y, z)}{\Delta s}$$
$$= \frac{\partial V}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}s} + \frac{\partial V}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}s} + \frac{\partial V}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}s} \tag{1.6}$$

where s denotes the magnitude of the corresponding vector. Note that (1.6) is obtained by expanding the numerator in Taylor series for x, y, z directions respectively and keeping only the first terms. If ds is chosen along the normal, then $\frac{\mathrm{d}x}{\mathrm{d}s}, \frac{\mathrm{d}y}{\mathrm{d}s}, \frac{\mathrm{d}z}{\mathrm{d}s}$ are the directional cosines of the normal. Thus the three partial derivatives of $V(x, y, z)$ in (1.6) can be considered as the three components of a vector:

$$\frac{\partial V(x,y,z)}{\partial x}\boldsymbol{e}_x + \frac{\partial V(x,y,z)}{\partial y}\boldsymbol{e}_y + \frac{\partial V(x,y,z)}{\partial z}\boldsymbol{e}_z = \nabla V(x,y,z) \quad (1.7)$$

This is the gradient for the scalar field $V(x, y, z)$. The formula (1.7) also provides the method how to calculate the gradient if the field V is given in a Cartesian coordinate system, i.e., one needs only to calculate the three partial derivatives as the corresponding components for the gradient. Once the gradient is obtained, an arbitrary directional derivative of the field can be calculated by the formula from (1.6):

$$\frac{\mathrm{d}V}{\mathrm{d}s} = \frac{\mathrm{d}V}{\mathrm{d}n}\frac{\mathrm{d}n}{\mathrm{d}s} = \frac{\mathrm{d}V}{\mathrm{d}n}\frac{\mathrm{d}(s\cos\theta)}{\mathrm{d}s} = \nabla V \cdot \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}s} = |\nabla V|\cos\theta \quad (1.8)$$

where θ is the angle between the direction and the normal at the point, i.e., the directional derivative is actually the projective of gradient in the given direction.

It is noticed that scalar field V is an arbitrary function defined in a certain region, thus the symbol ∇ in (1.7) can be considered as a vectorial differentiation operator, whose component form in Cartesian coordinates are:

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z \quad (1.9)$$

The operator ∇ (sometimes called Hamiltonian operator) will be often used throughout the discussions below.

1.4 Gradient in cylindrical and spherical coordinate systems

If a potential has cylindrical or spherical symmetry, we then use the cylindrical or spherical coordinate system. In these cases, the gradient of the field is written as

Cylindrical:

$$\nabla V(\rho,\phi,z) = \frac{\partial V}{\partial \rho}\boldsymbol{e}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\boldsymbol{e}_\phi + \frac{\partial V}{\partial z}\boldsymbol{e}_z \quad (1.10)$$

Spherical:

$$\nabla V(r,\theta,\phi) = \frac{\partial V}{\partial r}\boldsymbol{e}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\boldsymbol{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\boldsymbol{e}_\phi \quad (1.11)$$

Note that both cylindrical and spherical coordinate systems are orthogonal (i.e., the basic vectors $\boldsymbol{e}_\rho$, $\boldsymbol{e}_\phi$, $\boldsymbol{e}_z$ and $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, $\boldsymbol{e}_\phi$ in each system are perpendicular to each other. In fact, all basic vectors are defined in the same way that they point to the direction of increasing the corresponding coordinates). (1.10) and (1.11) indicate that the operator ∇ in cylindrical and spherical coordinates are written as

$$\nabla = \frac{\partial}{\partial\rho}\boldsymbol{e}_\rho + \frac{1}{\rho}\frac{\partial}{\partial\phi}\boldsymbol{e}_\phi + \frac{\partial}{\partial z}\boldsymbol{e}_z \tag{1.12}$$

$$\nabla = \frac{\partial}{\partial r}\boldsymbol{e}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\boldsymbol{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\boldsymbol{e}_\phi \tag{1.13}$$

(1.10) and (1.11) are obtained by performing the coordinate transformations from Cartesian to cylindrical and spherical systems, respectively. A general formula for this transformation is very helpful. Suppose a vector is written in two coordinate systems as

$$\boldsymbol{V} = V_x\boldsymbol{e}_x + V_y\boldsymbol{e}_y + V_z\boldsymbol{e}_z = V_\xi\boldsymbol{e}_\xi + V_\eta\boldsymbol{e}_\eta + V_\zeta\boldsymbol{e}_\zeta \tag{1.14}$$

Then its components in two coordinate systems are related by the following formulas:

$$\begin{aligned} V_x &= \frac{\frac{\partial x}{\partial\xi}V_\xi}{\Delta_\xi} + \frac{\frac{\partial x}{\partial\eta}V_\eta}{\Delta_\eta} + \frac{\frac{\partial x}{\partial\zeta}V_\zeta}{\Delta_\zeta} \\ V_y &= \frac{\frac{\partial y}{\partial\xi}V_\xi}{\Delta_\xi} + \frac{\frac{\partial y}{\partial\eta}V_\eta}{\Delta_\eta} + \frac{\frac{\partial y}{\partial\zeta}V_\zeta}{\Delta_\zeta} \\ V_z &= \frac{\frac{\partial z}{\partial\xi}V_\xi}{\Delta_\xi} + \frac{\frac{\partial z}{\partial\eta}V_\eta}{\Delta_\eta} + \frac{\frac{\partial z}{\partial\zeta}V_\zeta}{\Delta_\zeta} \end{aligned} \tag{1.15}$$

where

$$\Delta_\xi = \sqrt{\left(\frac{\partial x}{\partial\xi}\right)^2 + \left(\frac{\partial y}{\partial\xi}\right)^2 + \left(\frac{\partial z}{\partial\xi}\right)^2}$$

$$\Delta_\eta = \sqrt{\left(\frac{\partial x}{\partial\eta}\right)^2 + \left(\frac{\partial y}{\partial\eta}\right)^2 + \left(\frac{\partial z}{\partial\eta}\right)^2}$$

$$\Delta_\zeta = \sqrt{\left(\frac{\partial x}{\partial\zeta}\right)^2 + \left(\frac{\partial y}{\partial\zeta}\right)^2 + \left(\frac{\partial z}{\partial\zeta}\right)^2}$$

The Cartesian coordinates and cylindrical and spherical coordinates are related by:

Cylindrical:

$$x = \rho\cos\phi; \quad y = \rho\sin\phi; \quad z = z \tag{1.16}$$

Spherical:

$$x = r\sin\theta\cos\phi; \quad y = r\sin\theta\sin\phi; \quad z = r\cos\theta \tag{1.17}$$

Applying (1.16) and (1.17) to the general formulas (1.15) and noting that the components of gradient in Cartesian coordinates are given by (1.7), we get the gradient formulas in cylindrical and spherical coordinates (1.10) and (1.11).

Questions

(1) How to understand the concept of field in classic physics?

(2) What is the significance of electromagnetic interaction in the structure of matter?

(3) Scalar and vector fields are the mathematical model of a physical field, true or false?

(4) How to derive the formula of gradient in Cartesian coordinates?

(5) How to understand the vector differentiation operator ∇ ?

(6) How to transform the gradient formula to cylindrical or spherical coordinate system?

(7) What is the relationship between the gradient and a directional derivative of the field at each point?

Lecture 2 Vector Analysis (Ⅱ): Divergence

2.1 Vector integration

For a vector field $\boldsymbol{E}(x, y, z)$, there are three integrations that can be performed on it.

(1) Line integration: The integral is defined as

$$\int_{a(C)}^{b} \boldsymbol{E}(x, y, z) \cdot \mathrm{d}\boldsymbol{l} = \int_{a(C)}^{b} \boldsymbol{E}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{l} \tag{2.1}$$

where C is the curve defined in the domain where the field exists, a and b being the starting and ending points of it (Fig. 2.1),

$$\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z \tag{2.2}$$

is the position vector, which can also be used to denote the coordinates of a point. The integration (2.1) is performed along the curve C. Its element is composed by the scalar product of the field $\boldsymbol{E}$ and the line element $\mathrm{d}\boldsymbol{l}$, thus the integral should be a scalar as a whole. The physical reality corresponding to this integral can be the work done by a force, if the vector field represents a force.

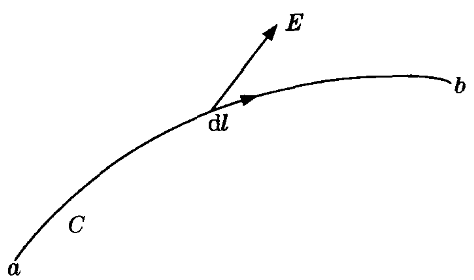


Fig. 2.1 Line integration

(2) Surface integration: The integral is defined as

$$\int_{S} \boldsymbol{E}(x, y, x) \cdot \mathrm{d}\boldsymbol{S} = \int_{S} \boldsymbol{E}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{S} \tag{2.3}$$

where S is a surface in the domain where the field exists. The integral element is the scalar product between the field $\boldsymbol{E}$ and the vectorial area element $\mathrm{d}\boldsymbol{S}$. The latter is defined as a vector in the normal direction of the area and its magnitude is given by the area itself, or $\mathrm{d}\boldsymbol{S} = \boldsymbol{n}\mathrm{d}S$, where $\boldsymbol{n}$ is a vector in the normal direction with unit length. Obviously, the integral as a whole is also a scalar as each element is. The physical reality of (2.2) can be found as the flux for a field, which is defined as the number of the force line through the area (Fig. 2.2). The density of the force line, by definition, is proportional to the strength of the field, or the magnitude of $\boldsymbol{E}$. However, the angle between the force line and the vectorial area element also matters because when they are parallel to each other, there will be no any force line passing through the area element. The effective area for the force line to pass through is determined by the project of the area element on the plane which is perpendicular to the force line. Thus the flux is calculated by the scalar product.

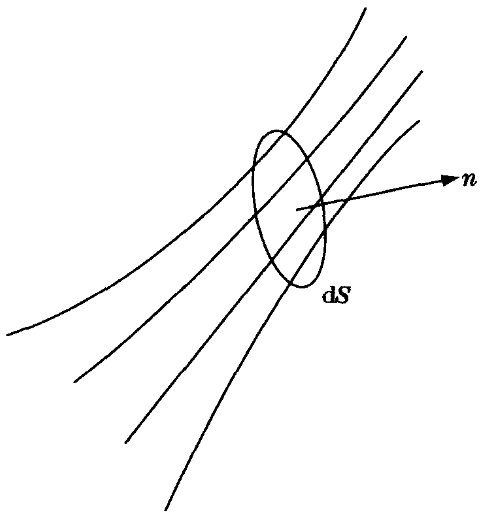


Fig. 2.2 Surface integration

(3) Volume integration: This is written as

$$\int_{\Omega} \boldsymbol{E}(x, y, x) \mathrm{d}V = \int_{\Omega} \boldsymbol{E}(\boldsymbol{r}) \mathrm{d}V \tag{2.4}$$

Note that in this case the integral element is a vector, it can be decomposed into three normal (scalar) integrals for the three components of the field, i.e.,

$$\int_{\Omega} E_x(x, y, x) \mathrm{d}V; \quad \int_{\Omega} E_y(x, y, x) \mathrm{d}V; \quad \int_{\Omega} E_z(x, y, x) \mathrm{d}V \tag{2.5}$$

Obviously, the volume integration can also be applied to a scalar field, i.e.,

$$\int_{\Omega} V(x, y, x) \mathrm{d}V \tag{2.6}$$

In all above formulas, the volume Ω is confined in the domain where the field exists.

We shall see how these integrals are used to describe the properties of a field.

2.2 Divergence

The divergence of a vector field div $\boldsymbol{E}$ is defined as

$$\nabla \cdot \boldsymbol{E} = \lim_{\Omega \to 0} \frac{1}{\Omega} \oint_S \boldsymbol{E} \cdot \boldsymbol{n} \mathrm{d}S \tag{2.7}$$

where Ω is the volume enclosed by the surface S, $\boldsymbol{n}$ is the unit vector in normal direction. For an enclosed surface, its normal is assumed to direct outward, otherwise it is said to be negative. Since the divergence is determined by the limit when the volume Ω approaches to zero, the shape of the volume should not be relevant, or Ω can be of any shape. On the other hand, as the volume approaches to zero, the divergence is actually defined at a "point" (Fig. 2.3a). We can take the divergence itself as a scalar function defined in the same domain as the field, or the divergence itself is a scalar "field" function.

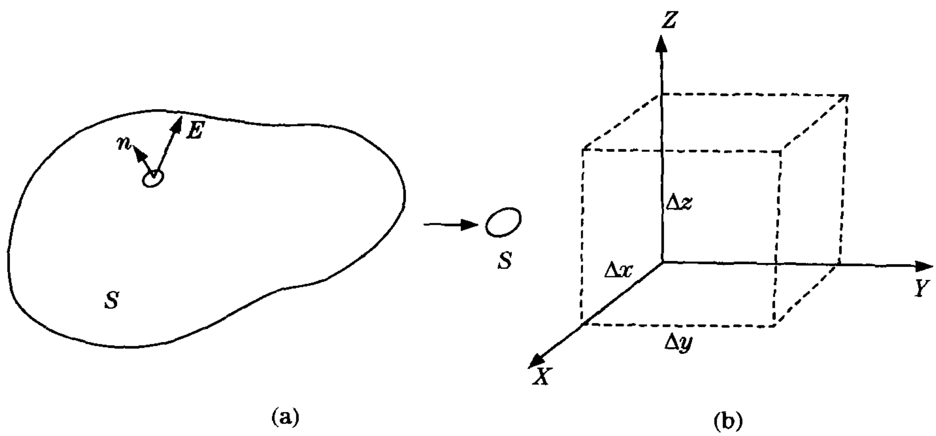


Fig. 2.3 (a) Understanding divergence; (b) Volume element

The explicit form of divergence in Cartesian coordinates can be derived as follows. Since the shape of the small volume in the definition of divergence is not relevant, we can take a rectangular parallelepiped with volume $\Delta\Omega = \Delta x \Delta y \Delta z$ and one corner at the point (x_0, y_0, z_0) for simplicity (Fig. 2.3b). In this case, the surface integration in (2.7) can be performed on the 6 sides of the volume, i.e.,