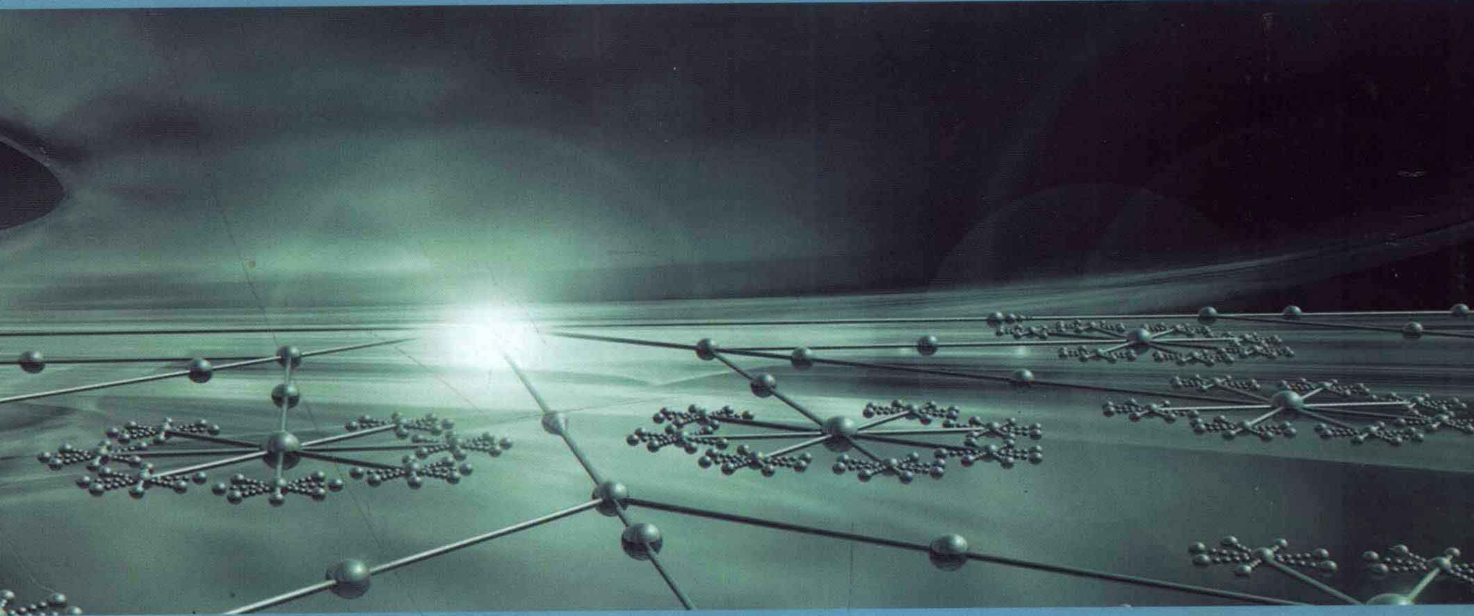
普通高等教育“十二五”规划教材



Modern Control Theory

现代控制理论

（双语教学版）

滕青芳　董海鹰　费克玲　编

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内 容 提 要

本书为普通高等教育“十二五”规划教材。

本书以状态空间法为核心，阐述了现代控制理论的基本原理及其分析和综合方法。全书共分七章，主要内容包含线性系统的状态空间描述、线性系统的运动分析、线性系统的能控性和能观性、李雅普诺夫稳定性分析、线性系统的状态综合以及二次型最优控制。本书是在作者根据我国现代控制理论课程教学要求、大量参考国际优秀原版教材，并总结近年来该课程双语教学实践经验的基础上编写而成的。

本书可作为普通高等院校自动化、电气工程及其自动化等控制类专业的现代控制理论双语教学教材，也供相关工程技术人员学习参考。

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前　言

现代控制理论是控制类专业学生必须掌握的重要基础理论，本课程无论对于控制理论研究，还是控制工程应用都有十分重要和深远的影响。该理论已广泛应用于航天航空、人造卫星、工业机器人、化工行业以及交通运输等诸多领域。

编者根据我国课程教学大纲要求，通过多年双语教学实践积累总结，在参考大量国外优秀原版专业教材和国内双语教材的基础上，编写了适合于我国本科生学习的"现代控制理论"课程英文教材。

本书以状态空间法为核心，阐述了线性系统的基本理论及其分析和综合方法，注重基本原理和基础知识。本书内容力求做到深入浅出，在概念论述上力求清晰，在理论分析上力求严谨，在体系结构上力求完整并条理化。本书给出了使用 MATLAB 分析和解决控制问题的方法，以使读者能够掌握利用 MATLAB 进行控制系统仿真的技能，培养读者控制系统分析和设计的能力。本书对提高读者专业英语水平具有一定的作用。

本书由滕青芳教授统稿。第二、三章由滕青芳执笔，第一章由董海鹰教授执笔，第四～七章由费克玲老师执笔。全书由韩力群教授审定。本书在编写过程中参考了国际同行的相关著作，引用了他们的论述，在此特别感谢他们。同时感谢编者的研究生严伟、姚利哲、李淑芳和任苗苗，他们编辑了书中的公式、绘制了书中的图表。特别感谢韩力群教授所提出的宝贵意见和建议。感谢中国电力出版社给予的支持。

由于编者水平有限，书中疏漏和不当之处在所难免，欢迎读者批评、指正。编者衷心感谢每一位提出意见和批评的读者。

编　者

2011.6

Preface

Modern control theory is the fundamental theory for students specializing in control fields. This course is very significant and important not only for pursuing theoretical research but also for endeavoring applications in control engineering. At present, this theory has been successfully applied in many fields such as astronautics aviation, artifical satellite, industrial robots, chemical industry, as well as transportation.

According to the requirements of curriculum teaching program in our country, based on the bilingual teaching practice and experience of the authors in recent years as well as on referencing many excellent international teaching materials and domestic bilingual textbooks, this book is written to be served as a textbook for undergraduate to study curriculum Modern Control Theory.

Based on state space method, this book presents a comprehensive treatment of the analysis and synthesis of linear systems. The philosophy of the book is to emphasize basic principles and fundamental knowledge. Authors strive to explain the profound in simple terms & in clear languages. At the same time, Authors do their best to organize the book's contents in a rigorous style & in a unified and systematic way. Moreover, throughout the book MATLAB is extensively used for solving computational problems so that the readers can learn to simulate control system by means of MATLAB approach. As a result, their ability to analyze and design for control system will be cultivated and improved. Furthermore, this book is hoped to be helpful for improving professional ability as well as English proficiency of the readers.

This book is edited by professor Qingfang Teng. The chapter two and three are compiled by Qingfang Teng. The chapter one is compiled by professor Haiying Dong, and the chapter four, five, six and seven are compiled by Keling Fei. The book is reviewed by professor Liqun Han. We reference many excellent international teaching materials and domestic bilingual textbooks, and cite their statements during composing. We gratefully acknowledge the authors of those references. The book is composed and typed by many people, including the authors. We would especially like to thank authors' graduate candidates Wei Yan, Lizhe Yao, Shufang Li and Miaomiao Ren for drawing figures and editing equations shown in the text. Also the authors wish particularly to acknowledge suggestions from professor Liqun Han. The authors finally wish to thank the staff of China Electric Power Press for their advice and support during this project.

Some mistakes are inevitable in this book, and any advice of feedbacks will be appreciated. If you have any suggestions for improvements, the authors would appreciate your comments very much.

Contents

Chapter 1 Introduction to Control Systems

1.1 Historical Review

Automatic control has played a vital role in the advance of engineering and science. In addition to its extreme importance in space-vehicle systems, missile-guidance systems, robotic systems, and the like, automatic control has become an important and integral part of modern manufacturing and industrial processes. For example, automatic control is essential in the numerical control of machine tools in the manufacturing industries, in the design of autopilot systems in the aerospace industries, and in the design of cars and trucks in the automobile industries. It is also essential in such industrial operations as controlling pressure, temperature, humidity, viscosity, and flow in the process industries.

Since advances in the theory and practice of automatic control provide the means for attaining optimal performance of dynamic systems, improving productivity, relieving the drudgery of many routine repetitive manual operations, and more, most engineers and scientists must now have a good understanding of this field.

An interesting history of early work on feedback control has been written by O. Mayr (1970), who traced the control of mechanisms to antiquity. Two of the earliest examples are the control of flow rate to regulate a water clock and the control of liquid level in a wine vessel, which is thereby kept full regardless of how many cups are dipped from it. The control of fluid-flow rate is reduced to the control of fluid level, since a small orifice will produce constant flow if the pressure is constant, which is the case if the level of the liquid above the orifice is constant. The mechanism of the liquid-level control invented in antiquity and still used today (for example, in the water tank of the ordinary flush toilet) is the float valve. As the liquid level falls, so does the float, allowing the flow into the tank to increase; as the level rises, the flow is reduced and, if necessary, cut off. Figure 1.1 shows how a float valve operates. Notice here that sensor and actuator are not separate devices but are, instead, contained in the carefully shaped float-and-supply-tube combination.

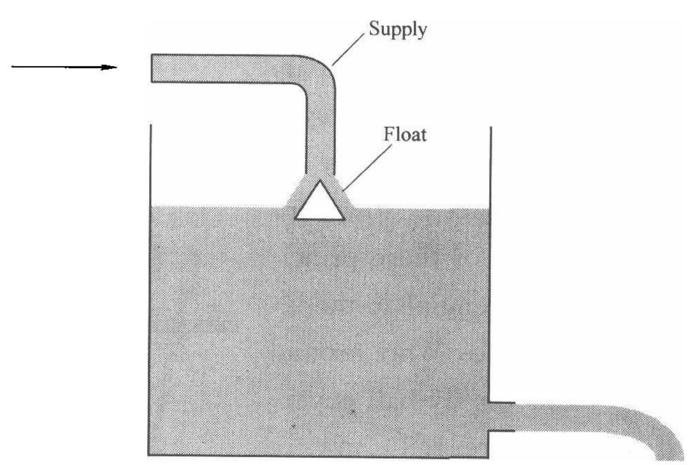

Figure 1.1 Early historical control of liquid level and flow

A more recent invention described by Mayr (1970) is a system, designed by Cornelis Drebbel in about 1620, to control the temperature of a furnace used to heat all incubator (Figure 1. 2). The furnace consists of a box to contain the fire, with a flue at the top fitted with a damper. Inside the fire box is the double walled incubator box, the hollow walls of which are filled with water to transfer the heat evenly to the incubator. The temperature sensor is a glass vessel filled with alcohol and mercury and placed in the water jacket around the incubator box. As the fire heats the box and water, the alcohol expands and the riser floats up, lowering the damper on the flue. If the box is too cold, the alcohol contracts, the damper is opened, and the fire burns hotter. The desired temperature is set by the length of the riser, which sets the opening of the damper for a given expansion of the alcohol.

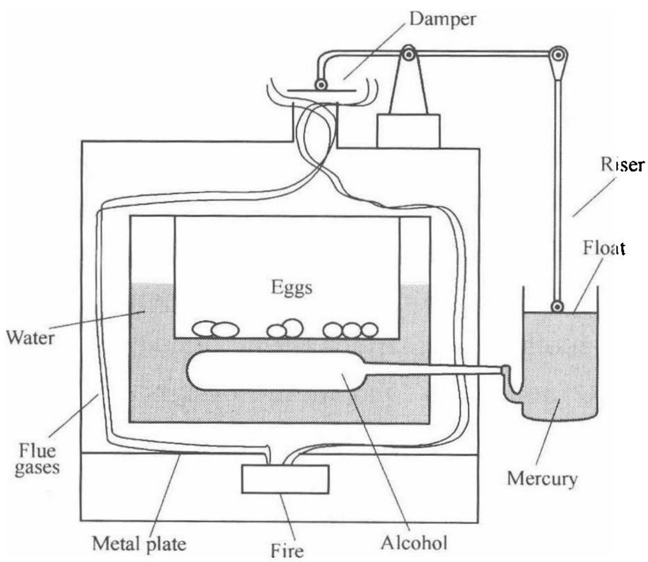

Figure 1. 2 Drebbel's incubator for hatching chicken eggs (Adapted from Mayr, 1970)

A famous problem in the chronicles of control systems was the search for a means to control the rotation speed of a shaft. Much early work (Fuller, 1976) seems to have been motivated by the desire to automatically control the speed of the grinding stone in a wind-driven flour mill. Of various methods attempted, the one with the most promise used a conical pendulum, or fly-ball governor, to measure the speed of the mill. The sails of the driving windmill were rolled up or let out with ropes and pulleys, much like a window shade, to maintain fixed speed. However, it was adaptation of these principles to the steam engine in the laboratories of James Watt around 1788 that made the fly-ball governor famous. Figure 1. 3 shows a close-up of a fly-ball governor

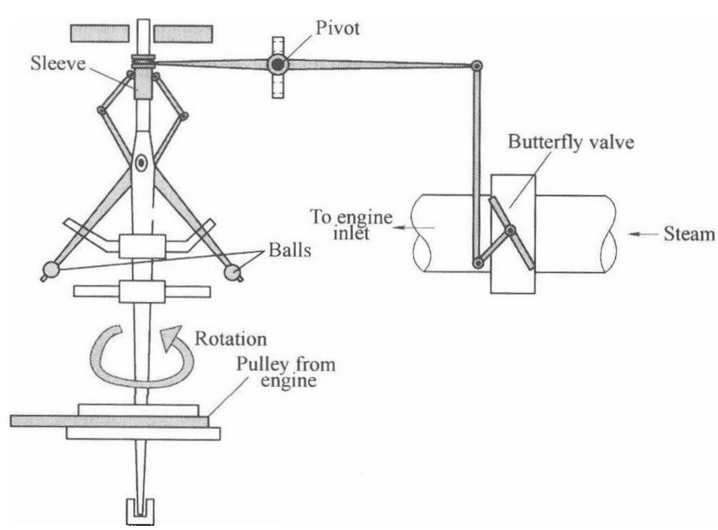

Figure 1. 3 A close-up of a fly-ball governor and a sketch of its components

and a sketch of its components.

The action of the fly-ball governor (also called a centrifugal governor) is simple to describe. Suppose the engine is operating in equilibrium. Two weighted balls spinning around a central shaft can be seen to describe a cone of a given angle with the shaft. When a load is suddenly applied to the engine, its speed will slow, and the balls of the governor will drop to a smaller cone. Thus, the ball angle is used to sense the output speed. This action, through the levers, will open the main valve to the steam chest (which is the actuator) and admit more steam to the engine, restoring most of the lost speed. To hold the steam valve at a new position, it is necessary for the fly balls to rotate at a different angle, implying that the speed under load is not exactly the same as before. We saw this effect earlier with cruise control, where feedback control gave a very small error. To recover the exact same speed in the system, it would require resetting the desired speed setting by changing the length of the rod from the lever to the valve. Subsequent inventors introduced mechanisms that integrated the speed error to provide automatic reset.

Because Watt was a practical man, like the Millwrights before him, he did not engage in theoretical analysis of the governor. Fuller (1976) has traced the early development of control theory to a period of studies from Christian Huygens in 1673 to James Clerk Maxwell in 1868. Fuller gave particular credit to the contributions of G. B. Airy, professor of mathematics and astronomy at Cambridge University from 1826 to 1835 and Astronomer Roval at Greenwich Observatory from 1835 to 1881. Airy was concerned with speed control; if his telescopes could be rotated counter to the rotation of the earth, a fixed star could be observed for extended periods. Using the centrifugal-pendulum governor, he discovered that it was capable of unstable motion- "and the machine (if I may so express myself) became perfectly wild" (Airy, 1840; quoted in Fuller, 1976). According to Fuller, Airy was the first worker to discuss instability in a feedback control system and the first to analyze such a system using differential equations. These attributes signal the beginnings of the study of feedback control dynamics.

The first systematic study of the stability of feedback control was apparently given in the paper "On Governors" by J. C. Maxwell (1868). In this paper, Maxwell developed the differential equations of the governor, linearized them about equilibrium, and stated that stability depends on the roots of a certain (characteristic) equation having negative real parts. Maxwell attempted to derive conditions on the coefficients of a polynomial that would hold if all the roots had negative real parts. He was successful only for second-and third-order cases. Determining criteria for stability was the problem for the Adams Prize of 1877, which was won by E. J. Routh. His criterion, developed in his essay, remains of sufficient interest that control engineers are still learning how to apply his simple technique. Analysis of the characteristic equation remained the foundation of control theory until the invention of the electronic feedback amplifier by H. S. Black in 1927 at Bell Telephone Laboratories.

Shortly after publication of Routh's work, the Russian mathematician A. M. Lyapunov (1892) began studying the question of stability of motion. His studies were based on the nonlinear differential equations of motion and also included results for linear equations that are equivalent to Routh's criterion. His work was fundamental to what is now called the state-variable approach to control theory, but it was not introduced into the control literature until about 1958.

The development of the feedback amplifier is briefly described in an interesting article based on a talk by H. W. Bode (1960) reproduced in Bellman and Kalaba (1964). With the introduction of electronic amplifiers, long-distance telephoning became possible in the decades following World War I. However, as distances increased, so did the loss of electrical energy; in spite of using larger-diameter wire, increasing numbers of amplifiers were needed to replace the lost energy. Unfortunately, large numbers of amplifiers resulted in much distortion since the small nonlinearity of the vacuum tubes then used in electronic amplifiers was multiplied many times. To solve the problem of reducing distortion, Black proposed the feedback amplifier. As mentioned earlier in connection with the automobile cruise control, the more we wish to reduce errors (or distortion), the more feedback we need to apply. The loop gain from actuator to plant to sensor to actuator must be made very large. With high gain the feedback loop begins to squeal and is unstable. Here was Maxwell's and Routh's stability problem again except that in this technology the dynamics were so complex (with differential equations of order 50 being common) that Routh's criterion was not very helpful. So the communications engineers at Bell Telephone Laboratories, familiar with the concept of frequency response and the mathematics of complex variables, turned to complex analysis. In 1932 H. Nyquist published a paper describing how to determine stability from a graphical plot of the loop-frequency response. From this theory there developed an extensive methodology of feedback amplifier design described by Bode (1945) and extensively used still in the design of feedback controls.

Simultaneous with the development of the feedback amplifier, feedback control of industrial processes was becoming standard. This field, characterized by processes that are not only highly complex but also nonlinear and subject to relatively long time delays between actuator and sensor, developed proportional integral-derivative (PID) control. The PID controller was first described by Callender et al (1936). This technology was based on extensive experimental work and simple linearized approximations to the system dynamics. It led to standard experiments suitable to application in the field and eventually to satisfactory "tuning" of the coefficients of the PID controller. Also under development at this time were devices for guiding and controlling aircraft; especially important was the development of sensors for measuring aircraft altitude and speed. An interesting account of this branch of control theory is given in McRuer (1973).

An enormous impulse was given to the field of feedback control during World War Ⅱ. In the United States engineers and mathematicians at the MIT Radiation Laboratory combined their knowledge to bring together not only Bode's feedback amplifier theory and the PID control of processes but also the theory of stochastic processes developed by N. Wiener (1930). The result was the development of a comprehensive set of techniques for the design of servomechanisms, as control mechanisms came to be called. Much of this work was collected and published in the records of the Radiation Laboratory by James et al (1947).

Another approach to control systems design was introduced in 1948 by W. R. Evans, who was working in the field of guidance and control of aircraft. Many of his problems involved unstable or neutrally stable dynamics, which made the frequency methods difficult, so he suggested returning to the study of the characteristic equation that had been the basis of the work of Maxwell and Routh nearly 70 years earlier. However, Evans developed techniques and rules allowing one to follow graphically the paths of the roots of the characteristic equation as a parameter was changed. His method, the root locus, is suitable for design as well as for stability analysis and remains an important technique today.

During the 1950s several authors, including R. Bellman and R. E. Kalman in the United States and L. S. Pontryagin in the USSR., began again to consider the ordinary differential equation (ODE) as a model for control systems. Much of this work was stimulated by the new field of control of artificial earth satellites, in which the ODE is a natural form for writing the model. Supporting this endeavor were digital computers, which could be used to carry out calculations unthinkable 10 years before. (Now, of course, these calculations can be done by any engineering student with a desktop computer.) The work of Lyapunov was translated into the language of control at about this time, and the study of optimal controls, begun by Wiener and Phillips during World War Ⅱ, was extended to optimizing trajectories of nonlinear systems based on the calculus of variations. Much of this work was presented at the first conference of the newly formed International Federation of Automatic Control held in Moscow in 1960. This work did not use the frequency response or the characteristic equation but worked directly with ODE in "normal" or "state" form and typical called for extensive use of computers. Even though the foundations of the study of ODEs were laid in the late 19^{th} century, this approach is now often called modern control to distinguish it from conventional control theory (or classical control theory).

Besides above mentioned optimal control of both deterministic and stochastic systems, during the year from 1960 to 1980, adaptive and learning control of complex systems were fully investigated. From 1980 to the present, developments in modern control theory centered around robust control, H_∞ control, and associated topics.

Now that digital computers have become cheaper and more compact, they are used as integral parts of control systems. Recent applications of modern control theory include such nonengineering systems as biological, biomedical, economic, and socioeconomic systems.

1.2 Modern Control Theory versus Conventional Control Theory

1.2.1 Modern Control Theory

The modern trend in engineering system is toward greater complexity, due mainly to the requirements of complex tasks and good accuracy. Complex systems may have multiple inputs and multiple outputs and may be time-varying. Because of the necessity of meeting increasingly stringent requirements on the performance of control systems, the increase in system complexity, and easy access to large scale computers, modern control theory, which is a new approach to the analysis and design of complex control systems, has been developed since around 1960, as mentioned above. This new approach is based on the concept of state. The concept of state by itself is not new since it has been in existence for a long time in the field of classical dynamic and other fields.

1.2.2 Modern Control Theory versus Conventional Control Theory

Modern control theory is contrasted with conventional control theory (or classical control theory) in that the former is applicable to multiple-input-multiple-output systems, which may be linear or nonlinear, time invariant or time-varying, while the latter is applicable only to linear time-invariant single-input-single-output systems. Also, modern control theory is essentially a time-domain approach, while conventional control theory is a complex frequency-domain approach.

1.2.3 Definitions

Before we can discuss control systems, some basic terminologies must be defined.

1. Controlled Variable and Manipulated Variable

The controlled variable is the quantity or condition that is measured and controlled. The manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally, the controlled variable is the output of the system. Control means measuring the value of the controlled variable of the system and applying the manipulated variable to the system to correct or limit deviation of the measured value from a desired value.

In studying control engineering, we need to define additional terms that are necessary to describe control systems.

2. Plants

A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation. In this book, we shall call any physical object to be controlled (such as a mechanical device, a heating furnace, a chemical reactor, or a spacecraft) a plant.

3. Processes

The Merriam-Webster Dictionary defines a process to be a natural, progressively continuing operation or development marked by a series of gradual changes that succeed one another in a relatively fixed way and lead toward a particular result or end; or an artifical or voluntary, progressively continuing operation that consists of a series of controlled actions or movements systematically directed toward a particular result or end. In this book we shall call any operation to be controlled a process. Examples are chemical, economic, and biological processes.

4. Systems

A system is a combination of components that act together and perform a certain objective. A system is not limited to physical ones. The concept of the system can be applied to abstract, dynamic phenomena such as those encountered in economics. The word system should, therefore, be interpreted to imply physical, biological, economic, and the like, systems.

5. Disturbances

A disturbance is a signal that tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called internal, while an external disturbance is generated outside the system and is an input.

6. Feedback Control

Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and that does so on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system.

1.3 Design of Control Systems

Actual control systems are generally nonlinear. However, if they can be approximated by linear mathematical models, we may use one or more of the well-developed design methods. In a practical sense, the performance specifications given to the particular system suggest which method to use. If the performance specifications are given in terms of transient-response characteristics and/or frequency-domain performance measures, then we have no choice but to use a conventional approach based on the root-locus and/or frequency-response methods (These methods have been presented in classical control theory). If the performance specifications are given as performance indexes in terms of state variables, then modern control approaches should be used (These approaches are presented in this book).

While control system design via the root-locus and frequency-response approaches is an engineering endeavor, system design in the context of modern control theory (state-space methods) employs mathematical formulations of the problem and applies mathematical theory to design problems in which the system can have multiple inputs and multiple outputs and can be time-varying. By applying modern control theory, the designer is able to start from a performance index, together with constraints imposed on the system, and to proceed to design a stable system by a completely analytical procedure. The advantage of design based on such modern control theory is that it enables the designer to produce a control system that is optimal with respect to the performance index considered.

The systems that may be designed by a conventional approach are usually limited to single-input-single-output, linear time-invariant systems. The designer seeks to satisfy all performance specifications by means of educated trial-and-error repetition. After a system is designed, the designer checks to see if the designed system satisfies all the performance specifications. If it does not, then he repeats the design process by adjusting parameter settings or by changing the system configuration until the given specifications are met. Although the design is based on a trial-and-error procedure, the ingenuity and know-how of the designer will play an important role in a successful design. An experienced designer may be able to design an acceptable system without using many trials.

It is generally desirable that the designed system should exhibit as small errors as possible in responding to the input signal. In this regard, the damping of the system should be reasonable. The system dynamics should be relatively insensitive to small changes in system parameters. The undesirable disturbances should be well attenuated. [In general, the high-frequency portion should attenuate fast so that high-frequency noises (such as sensor noises) can be attenuated. If the noise or disturbance frequencies are known, notch filters may be used to attenuate these specific frequencies.] If the design of the system is boiled down to a few candidates, an optimal choice among them may be made from such considerations as projected overall performance, cost, space, and weight.

1.4 Outline of This Book

In what follows we shall briefly present the arrangements and contents of the book.

Chapter 1 gives introductory materials on control systems. Chapter 2 deals with mathematical modeling of dynamic systems in terms of state-space equations. Chapter 3 presents mainly basic materials for the state-space dynamic analysis of control systems. The solution of the time-invariant state equation is derived. This chapter also gives details of transient response analysis with MATLAB. Chapter 4 discusses concepts of controllability and observability. Chapter 5 presents the Lyapunov stability analysis. Chapter 6 treats the design of control systems in state space. This chapter begins with the pole-placement problems, followed by the design of state observers, and discusses with the design of observed-state feedback control system. MATLAB is utilized in solving pole-placement problems, and design of state observers. Chapter 7, the final chapter, quadratic optimal control problems are discussed in detail. Here the Lyapunov stability approach is utilized to derive the Riccati equation for quadratic optimal control. MATLAB solutions to quadratic optimal control problems are included.

Chapter 2 Modeling in State Space

In the conventional control theory we used the Laplace transform to obtain transfer function models representing linear, time-invariant physical systems described by ordinary differential equations. This method is attractive because it provides a practical approach to design and analysis and allows us to utilize block diagrams to interconnect subsystems. In this chapter we turn to an alternative method of system modeling using time-domain methods. As before, we will consider physical systems described by an nth-order ordinary differential equations. Utilizing a set of variables, known as state variables, we can obtain a set of first-order differential equations. We group these first-order differential equations using a compact matrix notation in a model known as the state variable model. The time-domain state variable model lends itself readily to computer solution and analysis.

Outline of the Chapter. Section 2. 1 introduces state variable and state space expression. Section 2. 2 gives state space representation of linear dynamic system. Section 2. 3 provides state space representation for block diagram. Section 2. 4 describes linear transformation of state space. Section 2. 5 discusses the state space representation of discrete systems. Finally, transformation of system models with MATLAB is given in section 2. 6.

2. 1 State Variable and State Space Expression

2. 1. 1 Some Basic Concept and Definitions

Before we proceed further, we must define state, state variables, state vector, and state space.

State. The state of a dynamic system is the smallest set of variables (called state variables) such that the knowledge of these variables at $t = t_0$, together with the knowledge of the input for $t \geqslant t_0$, completely determines the behaviour of the system for any time $t \geqslant t_0$.

Note that the concept of state is by no means limited to physical systems. It is applicable to biological systems, economic systems, social systems, and others.

State Variables. The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the system. If at least n variables x_1, x_2, $\cdots$, x_n, are needed to completely describe the behaviour of a dynamic system (so that once the input is given for $t \geqslant t_0$ and the initial state at $t=t_0$ is specified, the fu-