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String Theory and M-Theory

A Modern Introduction

弦论和M理论导论

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Preface

String theory is one of the most exciting and challenging areas of modern theoretical physics. It was developed in the late 1960s for the purpose of describing the strong nuclear force. Problems were encountered that prevented this program from attaining complete success. In particular, it was realized that the spectrum of a fundamental string contains an undesired massless spin-two particle. Quantum chromodynamics eventually proved to be the correct theory for describing the strong force and the properties of hadrons. New doors opened for string theory when in 1974 it was proposed to identify the massless spin-two particle in the string's spectrum with the graviton, the quantum of gravitation. String theory became then the most promising candidate for a quantum theory of gravity unified with the other forces and has developed into one of the most fascinating theories of high-energy physics.

The understanding of string theory has evolved enormously over the years thanks to the efforts of many very clever people. In some periods progress was much more rapid than in others. In particular, the theory has experienced two major revolutions. The one in the mid-1980s led to the subject achieving widespread acceptance. In the mid-1990s a second superstring revolution took place that featured the discovery of nonperturbative dualities that provided convincing evidence of the uniqueness of the underlying theory. It also led to the recognition of an eleven-dimensional manifestation, called M-theory. Subsequent developments have made the connection between string theory, particle physics phenomenology, cosmology, and pure mathematics closer than ever before. As a result, string theory is becoming a mainstream research field at many universities in the US and elsewhere.

Due to the mathematically challenging nature of the subject and the above-mentioned rapid development of the field, it is often difficult for someone new to the subject to cope with the large amount of material that needs to be learned before doing actual string-theory research. One could spend several years studying the requisite background mathematics and physics, but by the end of that time, much more would have already been developed, and one still wouldn't be up to date. An alternative approach is to shorten the learning process so that the student can jump into research more quickly. In this spirit, the aim of this book is to guide the student through the fascinating subject of string theory in one academic year. This book starts with the basics of string theory in the first few chapters and then introduces the reader to some of the main topics of modern research. Since the subject is enormous, it is only possible to introduce selected topics. Nevertheless, we hope that it will provide a stimulating introduction to this beautiful subject and that the dedicated student will want to explore further.

The reader is assumed to have some familiarity with quantum field theory and general relativity. It is also very useful to have a broad mathematical background. Group theory is essential, and some knowledge of differential geometry and basics concepts of topology is very desirable. Some topics in geometry and topology that are required in the later chapters are summarized in an appendix.

The three main string-theory textbooks that precede this one are by Green, Schwarz and Witten (1987), by Polchinski (1998) and by Zwiebach (2004). Each of these was also published by Cambridge University Press. This book is somewhat shorter and more up-to-date than the first two, and it is more advanced than the third one. By the same token, those books contain much material that is not repeated here, so the serious student will want to refer to them, as well. Another distinguishing feature of this book is that it contains many exercises with worked out solutions. These are intended to be helpful to students who want problems that can be used to practice and assimilate the material.

This book would not have been possible without the assistance of many people. We have received many valuable suggestions and comments about the entire manuscript from Rob Myers, and we have greatly benefited from the assistance of Yu-Chieh Chung and Guangyu Guo, who have worked diligently on many of the exercises and homework problems and have carefully read the whole manuscript. Moreover, we have received extremely useful feedback from many colleagues including Keshav Dasgupta, Andrew Frey, Davide Gaiotto, Sergei Gukov, Michael Haack, Axel Krause, Hong Lu, Juan Maldacena, Lubos Motl, Hirosi Ooguri, Patricia Schwarz, Eric Sharpe, James Sparks, Andy Strominger, Ian Swanson, Xi Yin and especially Cumrun Vafa. We have further received great comments and suggestions from many graduate students at Caltech and Harvard University. We thank Ram Sriharsha for his assistance with some of the homework problems and Ketan Vyas for writing up solutions to the homework problems, which will be made available to instructors. We thank Sharlene Cartier and Carol Silberstein of Caltech for their help in preparing parts of the manuscript, Simon Capelin of Cambridge U. Press, whose help in coordinating the different aspects of the publishing process has been indispensable, Elisabeth Krause for help preparing some of the figures and Kovid Goyal for his assistance with computer-related issues. We thank Steven Owen for translating from Chinese the poem that precedes the preface.

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NOTATION AND CONVENTIONS

A	area of event horizon
AdS_D	D-dimensional anti-de Sitter space-time
A_3	three-form potential of $D = 11$ supergravity
b, c	fermionic world-sheet ghosts
b_n	Betti numbers
b_r^μ, $r \in \mathbb{Z} + 1/2$	fermionic oscillator modes in NS sector
B_2 or B	NS–NS two-form potential
c	central charge of CFT
$c_1 = [\mathcal{R}/2\pi]$	first Chern class
C_n	R–R n-form potential
d_m^μ, $m \in \mathbb{Z}$	fermionic oscillator modes in R sector
D	number of space-time dimensions
$F = dA + A \wedge A$	Yang–Mills curvature two-form (antihermitian)
$F = dA + iA \wedge A$	Yang–Mills curvature two-form (hermitian)
$F_4 = dA_3$	four-form field strength of $D = 11$ supergravity
F_m, $m \in \mathbb{Z}$	odd super-Virasoro generators in R sector
$F_{n+1} = dC_n$	$(n + 1)$-form R–R field strength
$g_s = \langle \exp \Phi \rangle$	closed-string coupling constant
G_r, $r \in \mathbb{Z} + 1/2$	odd super-Virasoro generators in NS sector
G_D	Newton's constant in D dimensions
$H_3 = dB_2$	NS–NS three-form field strength
$h^{p,q}$	Hodge numbers
$j(\tau)$	elliptic modular function
$J = ig_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$	Kähler form
$\mathcal{J} = J + iB$	complexified Kähler form
k	level of Kac–Moody algebra
K	Kaluza–Klein excitation number
$\mathcal{K}$	Kähler potential
$l_p = 1.6 \times 10^{-33}\,\text{cm}$	Planck length for $D = 4$
ℓ_p	Planck length for $D = 11$
$l_s = \sqrt{2\alpha'}$, $\ell_s = \sqrt{\alpha'}$	string length scale
L_n, $n \in \mathbb{Z}$	generators of Virasoro algebra
$m_p = 1.2 \times 10^{19} \text{GeV}/c^2$	Planck mass for $D = 4$
$M_p = 2.4 \times 10^{18} \text{GeV}/c^2$	reduced Planck mass $m_p/\sqrt{8\pi}$
$M, N, \ldots$	space-time indices for $D = 11$
$\mathcal{M}$	moduli space

N_L, N_R	left- and right-moving excitation numbers
$Q_{\rm B}$	BRST charge
$R = d\omega + \omega \wedge \omega$	Riemann curvature two-form
$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$	Ricci tensor
$\mathcal{R} = R_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$	Ricci form
S	entropy
S^a	world-sheet fermions in light-cone gauge GS formalism
$T_{\alpha\beta}$	world-sheet energy–momentum tensor
T_p	tension of p-brane
W	winding number
x^μ, $\mu = 0, 1, \ldots D-1$	space-time coordinates
X^μ, $\mu = 0, 1, \ldots D-1$	space-time embedding functions of a string
$x^\pm = (x^0 \pm x^{D-1})/\sqrt{2}$	light-cone coordinates in space-time
x^I, $I = 1, 2, \ldots, D-2$	transverse coordinates in space-time
Z	central charge
α^μ_m, $m \in \mathbb{Z}$	bosonic oscillator modes
α'	Regge-slope parameter
β, γ	bosonic world-sheet ghosts
γ_μ	Dirac matrices in four dimensions
Γ_M	Dirac matrices in 11 dimensions
$\Gamma_{\mu\nu}{}^{\rho}$	affine connection
$\eta(\tau)$	Dedekind eta function
Θ^{Aa}	world-volume fermions in covariant GS formalism
λ^A	left-moving world-sheet fermions of heterotic string
$\Lambda \sim 10^{-120} M_{\rm p}^4$	observed vacuum energy density
σ^α, $\alpha = 0, 1, \ldots, p$	world-volume coordinates of a p-brane
$\sigma^0 = \tau$, $\sigma^1 = \sigma$	world-sheet coordinates of a string
$\sigma^\pm = \tau \pm \sigma$	light-cone coordinates on the world sheet
$\sigma^\mu_{\alpha\dot{\beta}}$	Dirac matrices in two-component spinor notation
Φ	dilaton field
$\chi(M)$	Euler characteristic of M
ψ^μ	world-sheet fermion in RNS formalism
Ψ_M	gravitino field of $D = 11$ supergravity
$\omega_\mu{}^\alpha{}_\beta$	spin connection
Ω	world-sheet parity transformation
Ω_n	holomorphic n-form

- $\hbar = c = 1$.
- The signature of any metric is 'mostly +', that is, $(-,+,\ldots,+)$.
- The space-time metric is $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$.
- In Minkowski space-time $g_{\mu\nu} = \eta_{\mu\nu}$.
- The world-sheet metric tensor is $h_{\alpha\beta}$.
- A hermitian metric has the form $ds^2 = 2g_{a\bar{b}}dz^a d\bar{z}^{\bar{b}}$.
- The space-time Dirac algebra in $D = d+1$ dimensions is $\{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu}$.
- $\Gamma^{\mu_1\mu_2\cdots\mu_n} = \Gamma^{[\mu_1}\Gamma^{\mu_2}\ldots\Gamma^{\mu_n]}$.
- The world-sheet Dirac algebra is $\{\rho_\alpha, \rho_\beta\} = 2h_{\alpha\beta}$.
- $|F_n|^2 = \frac{1}{n!}g^{\mu_1\nu_1}\cdots g^{\mu_n\nu_n}F_{\mu_1\ldots\mu_n}F_{\nu_1\ldots\nu_n}$.
- The Levi–Civita tensor $\varepsilon^{\mu_1\cdots\mu_D}$ is totally antisymmetric with $\varepsilon^{01\cdots d} = 1$.

Contents

1
Introduction

There were two major breakthroughs that revolutionized theoretical physics in the twentieth century: general relativity and quantum mechanics. General relativity is central to our current understanding of the large-scale expansion of the Universe. It gives small corrections to the predictions of Newtonian gravity for the motion of planets and the deflection of light rays, and it predicts the existence of gravitational radiation and black holes. Its description of the gravitational force in terms of the curvature of space-time has fundamentally changed our view of space and time: they are now viewed as dynamical. Quantum mechanics, on the other hand, is the essential tool for understanding microscopic physics. The evidence continues to build that it is an exact property of Nature. Certainly, its exact validity is a basic assumption in all string theory research.

The understanding of the fundamental laws of Nature is surely incomplete until general relativity and quantum mechanics are successfully reconciled and unified. That this is very challenging can be seen from many different viewpoints. The concepts, observables and types of calculations that characterize the two subjects are strikingly different. Moreover, until about 1980 the two fields developed almost independently of one another. Very few physicists were experts in both. With the goal of unifying both subjects, string theory has dramatically altered the sociology as well as the science.

In relativistic quantum mechanics, called quantum field theory, one requires that two fields that are defined at space-time points with a space-like separation should commute (or anticommute if they are fermionic). In the gravitational context one doesn't know whether or not two space-time points have a space-like separation until the metric has been computed, which is part of the dynamical problem. Worse yet, the metric is subject to quantum fluctuations just like other quantum fields. Clearly, these are rather challenging issues. Another set of challenges is associated with the quantum description of black holes and the description of the Universe in the very early stages of its history.

The most straightforward attempts to combine quantum mechanics and general relativity, in the framework of perturbative quantum field theory, run into problems due to uncontrollable infinities. Ultraviolet divergences are a characteristic feature of radiative corrections to gravitational processes, and they become worse at each order in perturbation theory. Because Newton's constant is proportional to $(\text{length})^2$ in four dimensions, simple power-counting arguments show that it is not possible to remove these infinities by the conventional renormalization methods of quantum field theory. Detailed calculations demonstrate that there is no miracle that invalidates this simple dimensional analysis.[1]

String theory purports to overcome these difficulties and to provide a consistent quantum theory of gravity. How the theory does this is not yet understood in full detail. As we have learned time and time again, string theory contains many deep truths that are there to be discovered. Gradually a consistent picture is emerging of how this remarkable and fascinating theory deals with the many challenges that need to be addressed for a successful unification of quantum mechanics and general relativity.

1.1 Historical origins

String theory arose in the late 1960s in an attempt to understand the strong nuclear force. This is the force that is responsible for holding protons and neutrons together inside the nucleus of an atom as well as quarks together inside the protons and neutrons. A theory based on fundamental one-dimensional extended objects, called strings, rather than point-like particles, can account qualitatively for various features of the strong nuclear force and the strongly interacting particles (or hadrons).

The basic idea in the string description of the strong interactions is that specific particles correspond to specific oscillation modes (or quantum states) of the string. This proposal gives a very satisfying unified picture in that it postulates a single fundamental object (namely, the string) to explain the myriad of different observed hadrons, as indicated in Fig. 1.1.

In the early 1970s another theory of the strong nuclear force – called quantum chromodynamics (or QCD) – was developed. As a result of this, as well as various technical problems in the string theory approach, string

1 Some physicists believe that perturbative renormalizability is not a fundamental requirement and try to "quantize" pure general relativity despite its nonrenormalizability. Loop quantum gravity is an example of this approach. Whatever one thinks of the logic, it is fair to say that despite a considerable amount of effort such attempts have not yet been very fruitful.

theory fell out of favor. The current viewpoint is that this program made good sense, and so it has again become an active area of research. The concrete string theory that describes the strong interaction is still not known, though one now has a much better understanding of how to approach the problem.

String theory turned out to be well suited for an even more ambitious purpose: the construction of a quantum theory that unifies the description of gravity and the other fundamental forces of nature. In principle, it has the potential to provide a complete understanding of particle physics and of cosmology. Even though this is still a distant dream, it is clear that in this fascinating theory surprises arise over and over.

1.2 General features

Even though string theory is not yet fully formulated, and we cannot yet give a detailed description of how the standard model of elementary particles should emerge at low energies, or how the Universe originated, there are some general features of the theory that have been well understood. These are features that seem to be quite generic irrespective of what the final formulation of string theory might be.

Gravity

The first general feature of string theory, and perhaps the most important, is that general relativity is naturally incorporated in the theory. The theory gets modified at very short distances/high energies but at ordinary distances and energies it is present in exactly the form as proposed by Einstein. This is significant, because general relativity is arising within the framework of a

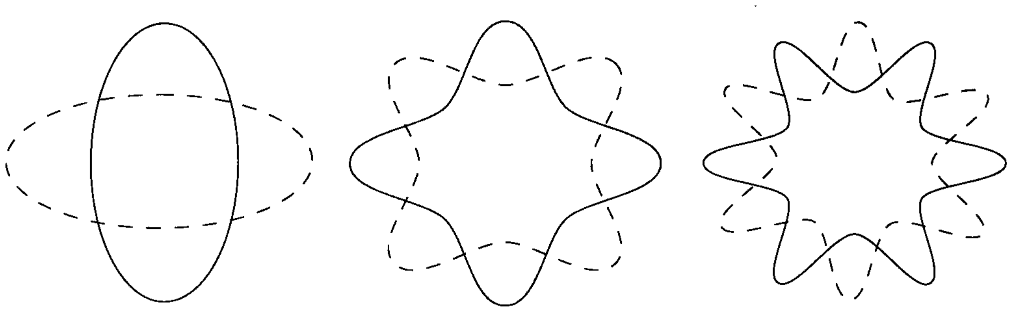

Fig. 1.1. Different particles are different vibrational modes of a string.

consistent quantum theory. Ordinary quantum field theory does not allow gravity to exist; string theory requires it.

Yang–Mills gauge theory

In order to fulfill the goal of describing all of elementary particle physics, the presence of a graviton in the string spectrum is not enough. One also needs to account for the standard model, which is a Yang–Mills theory based on the gauge group $SU(3)\times SU(2)\times U(1)$. The appearance of Yang–Mills gauge theories of the sort that comprise the standard model is a general feature of string theory. Moreover, matter can appear in complex chiral representations, which is an essential feature of the standard model. However, it is not yet understood why the specific $SU(3) \times SU(2) \times U(1)$ gauge theory with three generations of quarks and leptons is singled out in nature.

Supersymmetry

The third general feature of string theory is that its consistency requires supersymmetry, which is a symmetry that relates bosons to fermions is required. There exist nonsupersymmetric bosonic string theories (discussed in Chapters 2 and 3), but lacking fermions, they are completely unrealistic. The mathematical consistency of string theories with fermions depends crucially on local supersymmetry. Supersymmetry is a generic feature of all potentially realistic string theories. The fact that this symmetry has not yet been discovered is an indication that the characteristic energy scale of supersymmetry breaking and the masses of supersymmetry partners of known particles are above experimentally determined lower bounds.

Space-time supersymmetry is one of the major predictions of superstring theory that could be confirmed experimentally at accessible energies. A variety of arguments, not specific to string theory, suggest that the characteristic energy scale associated with supersymmetry breaking should be related to the electroweak scale, in other words in the range 100 GeV to a few TeV. If this is correct, superpartners should be observable at the CERN Large Hadron Collider (LHC), which is scheduled to begin operating in 2007.

Extra dimensions of space

In contrast to many theories in physics, superstring theories are able to predict the dimension of the space-time in which they live. The theory is only consistent in a ten-dimensional space-time and in some cases an eleventh dimension is also possible.

To make contact between string theory and the four-dimensional world of everyday experience, the most straightforward possibility is that six or seven of the dimensions are compactified on an internal manifold, whose size is sufficiently small to have escaped detection. For purposes of particle physics, the other four dimensions should give our four-dimensional space-time. Of course, for purposes of cosmology, other (time-dependent) geometries may also arise.

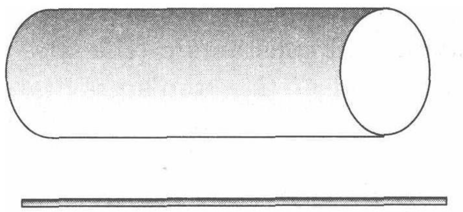

Fig. 1.2. From far away a two-dimensional cylinder looks one-dimensional.

The idea of an extra compact dimension was first discussed by Kaluza and Klein in the 1920s. Their goal was to construct a unified description of electromagnetism and gravity in four dimensions by compactifying five-dimensional general relativity on a circle. Even though we now know that this is not how electromagnetism arises, the essence of this beautiful approach reappears in string theory. The Kaluza–Klein idea, nowadays referred to as *compactification*, can be illustrated in terms of the two cylinders of Fig. 1.2. The surface of the first cylinder is two-dimensional. However, if the radius of the circle becomes extremely small, or equivalently if the cylinder is viewed from a large distance, the cylinder looks effectively one-dimensional. One now imagines that the long dimension of the cylinder is replaced by our four-dimensional space-time and the short dimension by an appropriate six, or seven-dimensional compact manifold. At large distances or low energies the compact internal space cannot be seen and the world looks effectively four-dimensional. As discussed in Chapters 9 and 10, even if the internal manifolds are invisible, their topological properties determine the particle content and structure of the four-dimensional theory. In the mid-1980s Calabi–Yau manifolds were first considered for compactifying six extra dimensions, and they were shown to be phenomenologically rather promising, even though some serious drawbacks (such as the moduli space problem discussed in Chapter 10) posed a problem for the predictive power